Arbeitsgemeinschaft mit aktuellem Thema: MATHEMATICAL BILLIARDS Mathematisches Forschungsinstitut Oberwolfach 4.4.2010 – 10.4.2010

Organizers:

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Introduction

The billiard dynamical system describes the motion of a free particle in a domain with a perfectly reflecting boundary.

More technically, a billiard table Q is a subset of a Riemannian manifold (usually $\mathbb{R}^2$) with a piece-wise smooth boundary. We define the billiard flow as follows: the billiard ball is a point particle, it move along geodesic lines in Q with elastic collisions with ∂Q. The latter means that, at the impact point, the velocity vector of the particle is decomposed into the components, tangential and the normal to ∂Q; then the normal component instantaneously changes signs, whereas the tangential component remains the same, after which the free motion continues. In dimension two, this is the famous law of geometrical optics: the angle of incidence equals the angle of reflection.

Many mechanical systems with elastic collisions, that is, collisions preserving the total momentum and energy of the system, reduce to billiards. Perhaps the most famous example is an idealized gas made of massive elastically colliding balls. Here is an interesting lesser known example: the system of three elastically colliding point masses on a circle reduces, after fixing the center of mass, to the billiard inside an acute triangle whose angles depend on the ratios of masses. There are many physically motivated variations on billiards, such as magnetic billiards, in which free particles are subject to the action of a magnetic field.

The dynamical behavior of billiards is strongly influenced by the shape of the boundary. Billiards naturally fall into three classes: depending on whether the pieces of the boundary curve out, curve in, or are flat. In each of the cases the mathematical machinery used in the study is quite different. The presentation of talks below is organized accordingly.

The final group of talks will study outer (also known as dual) billiards, which are played outside a convex table Q in the Euclidean plane. Dual billiards are defined as follows. Fix an orientation of Q. Given a point x outside Q, draw the segment xy, with $y \in Q$, of the tangent line to Q such that its orientation agrees with that of Q. Extend this segment through y to the point $T(x)$ such that $dist(Tx, y) = dist(x, y)$. The map T of the exterior of Q to itself is the dual billiard transformation. This map is area-preserving; its definition extends to the spherical and hyperbolic geometry (in the former, outer and inner billiards are equivalent via the spherical duality). Outer billiards can be also defined in even-dimensional Euclidean spaces.

The following books are devoted to billiards: [17, 40, 64, 65].

Hyperbolic billiards

If the boundary of Q curves out, then parallel incoming orbits scatter, or disperse, producing hyperbolic behavior. A second mechanism of hyperbolicity exists: if two smooth curving in components are placed sufficiently far apart, then parallel orbits first focus, but then have time to diverge before the next collision.

The mathematical tools used to study hyperbolic billiards are the same as the ones used to study hyperbolic dynamical systems (Anosov systems, Axiom A systems, expanding maps, etc). There are serious additional difficulties, the presence of singularities (tangent orbits and orbits hitting non-smooth points of the boundary).

Elliptic billiards

The billiard in an ellipse is completely integrable: a subset of full measure in its phase space is foliated by invariant curves or caustics, corresponding to the billiard trajectories tangent to confocal ellipses and hyperbolas (note however that one leaf of this foliation is singular: this is the invariant curve consisting of the trajectories that pass through the foci of the ellipse). Similar complete integrability holds for billiards inside ellipsoids in multi-dimensional space.

It turns out that part of this structure is shared by arbitrary convex tables. Lazutkin showed that one can apply the celebrated KAM theorem to show that a set of positive measure of caustics exist for sufficiently smooth tables. Birkhoff showed that periodic orbits always exists in plane billiards with sufficiently smooth boundary with positive curvature. On the other hand, Mather proved that if the curvature vanishes at a point then the billiard possesses no caustics.

Polygonal billiards

Billiards in polygons come in two classes: rational and irrational polygons. Rational polygons are those for which the angles between sides are rational multiples of π. A rational billiard table determines a flat surface, this construction allows one to use the tools of Teichmüller theory to study rational billiards, and many deep results have been obtained this way. Most of the polygonal talks will be on rational polygons, since in the irrational case there is essentially no machinery available, other elementary geometry and computer simulation. As a result, the available results are considerably more scarce.

Dual billiards

In the first volume of the Mathematical Intelligencer, Jurgen Moser wrote an article proposing the outer billiard as a toy model to study the question if the solar system is stable or not [51]. The recent progress in the study of polygonal outer billiards will be the subject of the two talks we propose. See [19] for a survey of outer billiards.

Talks:

Hyperbolic billiards

1. **Introduction to hyperbolic billiards.**
   In this talk the main ideas and tools of hyperbolic billiards will be introduced. Invariance of phase volume (this is the first talk), mechanisms

of hyperbolicity (scattering, focusing), positive Lyapunov exponents, construction of stable and unstable manifolds, cone fields and/or continued fractions. The main ideas of the local ergodicity theorem should be presented. The main reference is the book [17].

2. **Boltzmann-Sinai ergodic hypothesis.**
   At the end of the 19th century Boltzmann worked on the foundations of statistical mechanics and made his famous ergodic hypothesis [63]. In the 1970 Sinai proposed a concrete version of this hypothesis, N-balls move without friction on the n-dimensional torus and collide elastically; he proved ergodicity of this model for $N = n = 2$. Recently Simanyi proved ergodicity in general [59, 60, 61]. In this talk the main ideas of the proof should be presented, noting that local ergodicity should already be presented in the first hyperbolic talk.

3. **Geometric approach to semi-dispersing billiards.**
   In the gas of N hard balls in open space, is the number of collisions uniformly bounded? The solution of this long-standing problem (the answer is affirmative) was obtained about 10 years ago by D. Burago, S. Ferleger and A. Kononenko using ideas of Alexandrov geometry, see [11, 12, 13]. In this talk, the main ideas of the proof will be explained.

4. **Billiards with external force.**
   Consider the following mechanical device: an inclined board with interleaved rows of pegs. A ball dropped in moves under gravity and bounces off the pegs on its way down. Chernov and Dolgopyat have studied an idealized version of this model and have proven recurrence (which is a surprise) and given the limit laws for the rescaled velocity and position [16].

Elliptic billiards

5. **Existence and non-existence of caustics.**
   This talk will introduce ideas of KAM theory in the context of billiards. The content of the talk includes the string construction, Lazutkin's theorem on the existence of caustics for billiards with sufficiently smooth strictly convex boundary [41], Mather's theorem on non-existence of caustics for convex billiards with a flat point on the boundary [45] and its quantitative version [29]. One may also describe Berger's theorem

on multi-dimensional caustics [6]. See also [64, 65], and [18] for introduction to KAM theory.

6. **Periodic billiard orbits in a convex table with smooth boundary.**
   The first, classical, result to be described is Birkhoff's theorem that, for every period and every rotation number, there exist at least two distinct billiard trajectories. This result holds for a more general class of area preserving twist maps, see [37]. Secondly, in recent work by Farber and Tabachnikov, Birkhoff's theorem was generalized to multi-dimensional billiards. The approach is via Morse theory and involves algebraic topology of cyclic configuration spaces, see [21, 22].
   One can also mention Ivrii's conjecture that the set of periodic billiard orbits in the Euclidean plane has zero measure. This conjecture is proved only for small periods: 3 for inner, and 3 and 4 for outer, billiards. Recently, Baryshnikov and Zharnitsky applied ideas of sub-Riemannian geometry to this problem; their approach is interesting and promising: see [4, 28, 69].

7. **Birkhoff conjecture and Hopf rigidity.**
   The billiard in an ellipse is integrable: its caustics are confocal ellipses and hyperbolas. G. Birkhoff conjectured that ellipses are characterized by integrability of the billiard transformation. A similar conjecture holds for outer billiards. This question remains open, and only partial results are known. In particular Bialy has shown that if the whole phase space is foliated by caustics then the table is a disc [7, 71], see also [64, 65].

8. **Interpolating Hamiltonians and length spectrum.**
   The length spectrum of a billiard table is the set of lengths of periodic billiard trajectories. To study periodic trajectories near the boundary of the billiard table, one uses the technique of interpolating Hamiltonians [48, 52, 58]. According to this theory, up to derivatives of all orders, the billiard ball map is included into a Hamiltonian flow. This makes it possible to obtain an asymptotic expansion of the length spectrum near the boundary as a series in negative even powers of the period; the coefficients are integral geometric quantities related to the boundary curve. A similar expansion holds for outer billiards, see [64, 66].

The same techniques yield asymptotic results on approximation of convex domains by polygons, proved previously by other methods [42, 46].

Polygonal billiards

9. **Introduction to rational polygons.**
   In this talk the main tools of the trade will be introduced: the flat surface associated with a rational billiard, flat metrics, quadratic differentials, definition of the Veech group. If time permits, one could prove the minimality of the billiard flow for directions without saddle connections. The main references are [44, 62, 72].

10. **Existence of periodic orbits.**
    This talk will survey various results on the existence and density of periodic orbits in rational and irrational billiards ([9, 26, 31, 43, 44, 64]). The speaker should explain the existence of perpendicular periodic orbits in rational polygons [8, 26] and then go into the existence results in irrational triangles of Schwartz [53, 32, 54].

11. **Coding billiards 1.**
    One codes the billiard orbit by the sequence of sides it hits. In this talk the speaker will give a survey of the known results: directional complexity for rational polygons [34, 64], sub-exponential complexity in the general case [27], and the relation between complexity and generalized diagonals [15].

12. **Coding billiards 2.**
    This talk is on Sturmian sequences and their recent generalization. Consider a regular n-gon and code as in the previous talk, except that parallel sides have the same symbol. The case of the square is classical, the arising sequences are called Sturmian sequences and have many beautiful properties [2, 64]. Recently Smillie and Ulcigrai have obtain some of the same properties for regular n-gons [57].

13. **Ergodicity of polygonal billiards.**
    In this talk one will outline the proof of the celebrated theorem of Kerckhoff, Masur, Smillie [1, 23, 38, 44]: the billiard in a rational polygon is uniquely ergodic for almost every direction.

14. **Billiards in genus 2.**
Veech polygons are rational polygons that have optimal billiard properties: each direction is either uniquely ergodic or completely periodic. For the polygons for which the associated flat surface has genus two, these billiard systems have been recently well understood: McMullen [47] and Calta [14].

15. **Square covered infinite billiards and translation surfaces.**
Several recent articles have begun the study of the geometry and dynamics of infinite square tiled billiards and infinite square tiled translation surfaces [3, 33, 35, 36]. In this talk one should present one of the following two results. 1) The paper [35] in which the "standard" staircase surface is well understood for a dynamical point of view: the dynamics is recurrent, the Radon invariant measures can be described explicitly and a Veech dichotomy is proven. These results follow from the fact that this staircase is a a $\mathbb{Z}$-cover of the torus. 2) Alternatively one could present the results of [36] on the classic Ehrenfest wind-tree model. This model is a $\mathbb{Z}^2$-extension of a genus two surface, thus the recurrence is substantially more difficult to prove and other properties seem unattainable at this time.

16. **Security, or finite blocking**.
A polygon, or more generally, a Riemannian mainfold is called secure if for any two points $x, y \in P$ there is a finite set of points $F \subset P \setminus \{x, y\}$ such that every billiard orbit from x to y passes through a point of F. The flat surfaces with the finite blocking property have been described in [24, 49, 50]. One can also mention the result that Birkhoff billiards are not secure [68].

Outer billiards

17. **Polygonal outer billiards: stable and periodic orbits.**
J. Moser asked whether the orbits of polygonal outer billiards may escape to infinity. For a class of polygons, called quasi-rational, one has the stability result: all orbits stay bounded, [25, 39, 70], see also [19]. The class of quasi-rational polygons includes lattice polygons and regular ones. In the first part of this talk, the definition of quasi-rational polygons will be given and the proof of the stability theorem will be

outlined.
The second part of the talk concerns periodic orbits in polygonal outer billiards. Whereas the existence of periodic billiard trajectories in polygons is open, a similar problem for polygonal outer billiards has a positive solution, see [67]. Also the complexity of polygonal outer billiards is polynomial, see [30, 5]; for conventional polygonal billiards this is an open problem.

18. **Solution to Moser's problem: outer billiards on kites.**
Recently, R. Schwartz solved Moser's problem: for some polygons, the orbits of outer billiard may escape to infinity. Schwartz analyzed the outer billiard dynamics for a class of quadrilaterals with axial symmetry; such quadrilaterals are called kites. This talk will describe the work of Schwartz that is now available in a book form, [55, 56].
One can also mention another example: a semi-circular outer billiard table [20]. In this system, there exist open domains which spiral away to infinity, unlike the much more complicated behavior of escaping orbits in outer billiards on kites. The same analysis applies to some other outer billiard tables and impact oscillators described in [10].

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Participation:

The idea of the Arbeitsgemeinschaft is to learn by giving one of the lectures in the program.

If you intend to participate, please send your full name and full postal address to

`tabachni@math.psu.edu` **and** `troubetz@iml.univ-mrs.fr`

by February 21, 2010.

You should also indicate which talk you are willing to give:
First choice: talk no. ...
Second choice: talk no. ...
Third choice: talk no. ...

You will be informed shortly after the deadline if your participation is possible and whether you have been chosen to give one of the lectures.

The Arbeitsgemeinschaft will take place at Mathematisches Forschungsinstitut Oberwolfach, Lorenzenhof, 77709 Oberwolfach-Walke, Germany. The institute offers accomodation free of charge to the participants. Travel expenses cannot be covered. Further information will be given to the participants after the deadline.