

**Arbeitsgemeinschaft mit aktuellem Thema:**  
**RATIONAL HOMOTOPY IN MATHEMATICS AND**  
**PHYSICS**  
**Mathematisches Forschungsinstitut Oberwolfach**  
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**Organizers:**

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**Introduction:**

In [Sul77], Sullivan defined tools and models for rational homotopy inspired by already existing geometrical objects. Moreover, he gave an explicit dictionary between his minimal models and spaces, and this facility of transition between algebra and topology has created many new topological and geometrical theorems in the last 30 years.

When de Rham proved that  $H^*(A_{DR}(M)) \cong H^*(M; \mathbb{R})$  for the differential algebra of differential forms  $A_{DR}(M)$  on a manifold  $M$ , it immediately provided a link between the analysis on and the topology of the manifold. Sullivan suggested that even within the world of topology, there is more topological information in the de Rham algebra of  $M$  than simply the real cohomology.

In the de Rham algebra, there is information contained in two different entities: the product of forms, which tells us how two forms can be combined together to give a third one and the exterior derivative of a form. In a model, we kill the information coming from the product structure by considering free algebras  $\wedge V$  (in the commutative graded sense) where  $V$  is an  $\mathbb{R}$ -vector space. This pushes the corresponding information into the differential and into  $V$  where it is easier to detect. More precisely, we look for a cdga (for *commutative differential graded algebra*) free as a commutative

graded algebra  $(\wedge V, d)$  and a morphism  $\varphi: (\wedge V, d) \rightarrow A_{DR}(M)$  inducing an isomorphism in cohomology.

For instance, if  $G$  is a compact connected Lie group, there exists a sub-differential algebra of bi-invariant forms,  $\Omega_I(G)$ , inside the de Rham algebra  $A_{DR}(G)$ , such that the canonical inclusion  $\Omega_I(G) \hookrightarrow A_{DR}(G)$  induces an isomorphism in cohomology. This is the prototype of the process for models: namely, we look for a simplification  $\mathcal{M}_M$  of the de Rham algebra with an explicit differential morphism  $\mathcal{M}_M \rightarrow A_{DR}(M)$  inducing an isomorphism in cohomology, exactly as bi-invariant forms do in the case of a compact connected Lie group.

The first question is, can one build such a model for any manifold? The answer is yes for connected manifolds and in fact, there are many ways to do this. So, we have to define a standard way, which is called *minimal*, which means that the differentials of elements of  $V$  have no linear terms. Once we have this *minimal model* (which is unique up to isomorphism), we may ask what geometrical invariants can be detected in it. In fact, there is a functor from algebra to geometry that, together with forms, creates a dictionary between the algebraic and the geometrical worlds. *But for this we have to work over the rationals and not over the reals.* As a consequence, we have to replace the de Rham algebra by other types of forms. This new construction is very similar to the de Rham algebra and allows the extension of the usual theory from manifolds to simplicial sets (or topological spaces), which is a great advantage. Denote by  $A_{PL}(X)$  this analogue of the de Rham algebra for a simplicial set  $X$ . Since the minimal model construction also works perfectly well over  $\mathbb{Q}$ , we have the notion of a minimal model  $\mathcal{M}_X \rightarrow A_{PL}(X)$  of a path connected space  $X$ .

Conversely, from a cdga  $(A, d)$  we have a topological realization  $\langle (A, d) \rangle$  and if we apply this realization to a minimal model  $\mathcal{M}_X$  of a space  $X$  (which is nilpotent with finite Betti numbers), then we get a continuous map  $X \rightarrow \langle \mathcal{M}_X \rangle$  which induces an isomorphism in rational cohomology. The space  $\langle \mathcal{M}_X \rangle$  is what, in homotopy theory, is called a rationalization of  $X$ . What must be emphasized in this process is the ability to create topological realizations of any algebraic constructions.

Hence, Sullivan's theory can be seen as a rational version of classical differential geometry. However, other algebraizations of rational homotopy type exist. They can take different forms and the first work in this direction historically was done by D. Quillen using differential graded Lie algebras (dgl). In [Qui69], Quillen defined a functor  $\lambda$  from the category of 2-reduced

simplicial sets to the category of dgl's. Moreover, any dgl has a realization as a simplicial set. Hence, here also any algebraic construction has a topological meaning. These two presentations are linked through the following non-commutative diagram,

$$\begin{array}{ccccc}
 & & & & \text{DGA} \\
 & & & & \uparrow u \\
 \text{CDGA} & \xleftarrow{\#} & \text{CDGC} & \xrightleftharpoons[\mathcal{C}]{\mathcal{L}} & \text{DGL} & \xrightarrow{\mathcal{A}_{\text{AH}}} & \text{DGA} \\
 & & & & \uparrow \lambda \\
 & & & & \text{Simplicial Sets} \\
 & \xleftarrow{A_{PL}} & & & & & 
 \end{array}$$

where, – “Simplicial Sets” means the category of 2-reduced simplicial sets, of finite type, – CDGA, CDGC, DGL, DGA are categories of commutative differential algebras, cocommutative differential coalgebras, differential Lie algebras or differential graded algebras, over the rationals.

The functors  $\lambda$ ,  $\mathcal{L}$ ,  $\mathcal{C}$  were introduced by Quillen in [Qui69]); the functor  $A_{PL}$  is the previous functor of piecewise linear forms of Sullivan, [Sul77]. Therefore we have two approaches to the algebraization of the rational homotopy type, one with cdga's and the other with Lie objects. In fact, as Majewski proved in [Maj00], they are equivalent: if  $X \in \text{Simplicial Sets}$ , the minimal models associated to  $A_{PL}(X)$  and  $\#(\mathcal{C}(\lambda(X)))$  are isomorphic.

The functor  $\mathcal{A}_{\text{AH}}$  is a sort of natural version of the Adams-Hilton model (introduced in [AH56] over the ring of the integers) built on the cobar-chain functor. By extending the structure with diagonal approximations, D. Anick ([Ani89]) proved that a Lie model can be built from it. Moreover this enriched model is an intermediate step in the equivalence of Majewski and the three algebraizations are equivalent.

There is also a fourth algebraic construction linked to the above, the complex of iterated integrals introduced by K. T. Chen, see [Che73]. This complex can be connected with the Hochschild complex and this will be made precise in Talk 17.

Several monographs are devoted to these theories: [BG76], [FHT01], [FOT08], [GM81], [Hai84]. [Maj00], [Tan83].

Such theories beg for applications and examples and it is possible to describe models for spheres, homogeneous spaces, biquotients, nilmanifolds, symplectic blow-ups and the free loop space. These models have geometrical applications, for instance, to complex and symplectic manifolds, the closed

geodesic problem, curvature questions, actions of tori and the Chas-Sullivan loop product. The focus of this Arbeitsgemeinschaft is the relationship between Rational Homotopy Theory and Geometry with a natural extension to Physics via string topology. As such, the Arbeitsgemeinschaft will be of interest to geometers and topologists whose interests lie in the often murky world between these two subjects.

## Talks:

There will be two survey talks given by experts in rational homotopy and in geometry. Talk 1, Fundamentals of Geometry, will be given by Wilderich Tuschmann and Talk 18, Differential Modules and Applications, will be given by Yves Félix. All other talks need volunteers.

### 1. Fundamentals of Geometry

This talk should introduce basic topics of geometry. In particular, *sectional curvature* should be defined and various results and questions concerning it should be surveyed, including the consequences of assuming nonpositive, nonnegative or positive sectional curvature. For example, results about finiteness of the number of homeomorphism or diffeomorphism types under sectional curvature (as well as volume or diameter) hypotheses should be discussed and Bott's conjecture that nonnegative sectional curvature implies ellipticity could be mentioned. References are: [Pet98, dC92, BC64, Gro03, Zil07, Fuk06, Tus02, Gro09, Wil07, PT99, Gro81, Esc82, KPT10, Wil07], [FOT08, Chapter 6].

### 2. Sullivan models

This talk presents the fundamental algebraic topological tool for studying rational homotopy type, the *Sullivan model*. The basic ideas will be illustrated by considering a toolkit of geometrical examples used in later expositions: Lie Groups, Homogeneous Spaces and Biquotients. For coherence with later talks, this talk must contain:

- Models of Lie groups, homogeneous spaces, principal  $G$ -bundles; see for instance [FOT08, Pages 71 and 83-84].
- Models of biquotients, introduced and studied in [Esc92], [Sin93],

[KZ04]. (See also [FOT08, Page 137] for the construction of their models and explicit examples.)

- Models of nilmanifolds, see [FOT08, Page 118].

### 3. Group Actions

This talk will survey various interactions between group actions and rational homotopy theory. These interactions arise in many geometric contexts. Specific topics that will be discussed include: the equivariant minimal model for finite group actions (to be used for the discussion on  $A$ -invariant geodesics), the Borel construction and its model, the toral rank  $\text{rk}_0(X)$  and the basic homotopy Euler characteristic bound  $\text{rk}_0(X) \leq -\chi_\pi(X)$ , Halperin's toral rank conjecture and Hamiltonian actions in symplectic geometry. References are: [Tri78, Opr84, AH78, AP86, Hal85, Rau98, Hsi75, AP93, JL04, CJ97, DS88, AB84, LM03, All98, Ste08], [FOT08, Chapters 3 & 7].

### 4. Geodesics and the Free Loop Space I

This talk introduces geodesics and briefly surveys known results which lead to a connection with rational homotopy theory. For instance, topics can include the proof in 1898 by Hadamard that each nontrivial conjugacy class of  $\pi_1(M)$  contains a closed geodesic that is the shortest closed curve representing an element in the conjugacy class. In 1951, Lusternik and Fet proved that each compact Riemannian manifold contains at least one closed geodesic. Also, the description of closed geodesics as critical points of the energy functional on the analytic free loop space  $LM := M^{S^1}$  later. The link to rational homotopy then comes from the fundamental Gromoll-Meyer theorem which will be a focus of this lecture. The Theorem relates the growth of the Betti numbers of the free loop space to the The Closed Geodesic Problem: Does every compact Riemannian manifold  $M$  of dimension at least two admit infinitely many geometrically distinct geodesics? In fact, by work of Gromov, the number of distinct geodesics (for certain metrics and under a length constraint) is bounded below by Betti numbers. References are: [dC92, Pet98, BC64, Bot82, LF51, Kli78, GM69, Gro73, LS34, Gro78, BZ82, BH84, Ban93, Fra92, Hin93, GH09], [FOT08, Chapter 5].

## 5. Geodesics and the Free Loop Space II

This talk will describe the model for the free loop space and prove the Sullivan-Vigué Poirrier theorem that a compact simply connected Riemannian manifold whose rational cohomology algebra requires at least two generators has infinitely many geometrically distinct closed geodesics. This then solves the Closed Geodesic Problem for a large class of spaces. A geodesic  $\gamma(t)$  is called  $A$ -invariant for an isometry  $A$  if there exists some  $T \in \mathbb{R}$  such that  $\gamma(t + T) = A(\gamma(t))$  for all  $t \in \mathbb{R}$ . For example, on the sphere  $S^2$  with the usual metric and for the antipodal map  $A$ , the great circles are  $A$ -invariant geodesics. When  $M$  is a flat torus and  $A$  is the involution  $A(x, y) = (y, x)$  then the  $A$ -invariant geodesics are the lines in the square  $[0, 1]^2$  that are parallel to the ascending diagonal. This talk will also give analogues of the results surrounding the Closed Geodesic Problem for  $A$ -invariant geodesics. In particular, analogues of the free loop space (and its model), of the Gromoll-Meyer Theorem (Tanaka's Theorem) and of the Sullivan-Vigué Poirrier Theorem will be discussed. References are [VPS76, GH82, GH82, Gro73, Gro74, GHVP78, Tan82], [FOT08, Chapter 5].

## 6. Geodesic Flows

Let  $M^n$  be a closed smooth  $n$ -manifold. The *geodesic flow* on  $M$  is a flow  $\phi: SM \times \mathbb{R} \rightarrow SM$  on the sphere bundle  $SM$  associated to the tangent bundle  $TM$  given by  $\phi_t(x, v) = \phi(x, v, t) = (\gamma_v(t), \dot{\gamma}_v(t))$ , where  $v$  is a unit tangent vector at  $x \in M$  and  $\gamma_v$  is the unique unit speed geodesic on  $M$  starting at  $x$  in the direction  $v$ . The *topological entropy* of the flow  $\phi$ ,  $h_{\text{top}}(\phi)$ , is a quantity that measures the complexity of a flow and because the geodesic flow depends only on the Riemannian metric  $g$ , the topological entropy  $h_{\text{top}}(\phi)$  of the geodesic flow  $\phi$  associated to  $g$  is denoted by  $h_{\text{top}}(g)$ . This talk is devoted to an exposition of the fundamental result of Gabriel Paternain: If  $(M, g)$  is a simply connected closed Riemannian manifold with  $h_{\text{top}}(g) = 0$ , then  $M$  is rationally elliptic. In particular, if the geodesic flow is completely integrable as a Hamiltonian system and has periodic integrals, then  $M$  is elliptic. References are: [Pat99, Pat92, PP94], [FOT08, Chapter 5].

## 7. Formality and Kähler Manifolds

This property is one of the first important applications of Sullivan's

theory. The existence of a Kähler metric on a compact manifold  $M$  imposes strong constraints on the homotopy type of  $M$ . For instance, the even Betti numbers of  $M$  are nonzero and the odd Betti numbers of  $M$  are even (this is a consequence of the *Hard Lefschetz property*). In [DGMS75], the four authors prove that the rational homotopy type is also very special: a compact Kähler manifold is a *formal space*. (As a consequence, the only nilmanifolds that admit a Kähler metric are tori.) This talk should give the definition of formality [FOT08, Page 92] and the characterization of formal nilmanifolds, as done by Hasegawa [Has89] or [FOT08, Page 120]. The proof of formality for Kähler manifolds uses an interesting trick, called the  $\partial\bar{\partial}$ -Lemma and this should be given. An alternative proof is given in [BG88], where the authors use another feature of compact Kähler manifold, the Hard Lefschetz property. In [Bla56], Blanchard noticed that the Hard Lefschetz property implies the vanishing of the derivations of negative degree in the cohomology of  $M$  and used it to study the degeneracy of the Serre spectral sequence of fibrations whose fibre has cohomology satisfying the Hard Lefschetz property. A summary of these properties and the proofs of the main results are recalled in [FOT08, Section 4.2.3]. The talk should also contain the proof of Blanchard's result on derivations. This will be related to concrete curvature questions in the second talk on curvature.

## 8. Spectral Sequences and Models

There is a synergy between models and spectral sequences. Spectral sequences arise naturally from the structure of models, as for instance the odd spectral sequence in the study of the cohomology algebra of homogeneous spaces  $G/T$  with  $T$  a maximal torus. But also, a model built from the  $r$ -stage of a particular spectral sequence coming from a cdga  $(A, d)$  can be deformed in a model of  $(A, d)$  itself ([FOT08, Theorem 4.56], [HT90]). This deformation carries with it new information as is illustrated by the filtered model ([FOT08, Definition 4.56], [HS79]) and by the Frölicher spectral sequence and the Dolbeault model of complex manifolds ([FOT08, Sections 4.3 and 4.4], [NT78], [Tan94], [Pit89], [Pit88]). This talk will center on these examples of the interplay between spectral sequences and models.

## 9. Formality and Symplectic Manifolds I

As symplectic geometry and topology developed in the 70's and 80's, a typical approach to understanding the qualities of symplectic manifolds was to compare them to Kähler manifolds. Indeed, the first question asked was whether every compact symplectic manifold is Kähler. Thurston provided the first counterexample, a 4-dimensional nilmanifold (which, in fact, had been known to Kodaira). It proved to be much harder to see that simply-connected symplectic manifolds were not always Kähler and this relied on the fact that the cohomology of a Kähler manifold satisfies the Hard Lefschetz property. Since Kähler manifolds are formal spaces, the question arose whether this was also the case for symplectic manifolds. This question has elicited a tremendous amount of work. This talk will survey the relationships among the qualities of being symplectic, obeying the Hard Lefschetz property and being formal. Besides basic results such as the Kodaira-Thurston manifold, the formality of simply-connected symplectic homogeneous spaces and Massey products in symplectic manifolds, the work of Fernandez, Munoz and Cavalcanti on the relation of the Hard Lefschetz property to formality may be mentioned. References are: [McD84, PS09, LO94, RT00, TO97, BT00, Cav07a, CFM08, BT98, Mer98], [FOT08, Chapter 4].

## 10. Formality and Symplectic Manifolds II

McDuff's example of a simply connected symplectic non-Kähler manifold was constructed via a symplectic blow-up and this type of construction has contributed greatly to our understanding of the differences between symplectic and Kähler manifolds. Babenko and Taimanov first analyzed the blow-up from the Massey product point of view and showed that non-formal symplectic manifolds existed. Lambrechts and Stanley later analyzed the rational homotopy type of the symplectic blow-up in detail and their model gives a clear picture of Massey product existence and the models that accompany this analysis form the main topic of this talk. This then will allow a model analysis of the McDuff example in terms of its non-formality. If time allows, the Fernandez, Munoz and Cavalcanti blow-up examples of compact simply connected symplectic manifolds which satisfy Hard Lefschetz, but which are non-formal may be discussed. References are:



[LS08b, LS05, Cav07b, FM08, FM05, CFM08, Mer98], [FOT08, Chapter 8; pp. 318-327 and 330-339]

## 11. Curvature I

As mentioned in the Fundamentals of Geometry talk, it is often the case that bounds on sectional curvature lead to finiteness of the number of diffeomorphism types of manifolds satisfying them. Similar questions can be asked about rational homotopy types with a view toward a better understanding of the boundary between geometry and topology. With this in mind, let  $\mathcal{M}_{\kappa \leq \text{sec} \leq \lambda}^{\leq D}(n)$  denote the class of closed  $n$ -manifolds with sectional curvature and diameter obeying  $\kappa \leq \text{sec} \leq \lambda$  and  $\text{diam} \leq D$ . Karsten Grove asked the following question: Does the sub-class of  $\mathcal{M}_{\kappa \leq \text{sec} \leq \lambda}^{\leq D}(n)$  consisting of simply connected manifolds contain finitely many rational homotopy types? This question was answered in the negative by Fang and Rong as well as Totaro. This talk will focus on the Fang-Rong approach since it uses rational homotopy theory in a paradigmatic way. Namely, Fang and Rong create the right algebra to answer the question and then realize the algebra geometrically by using minimal models and basic geometric results of Eschenburg. If time allows, the approach of Totaro may be discussed. This uses biquotients to construct manifolds with the right curvature properties and then shows that a sufficient number of these also have the right algebra to answer the question. References are: [FR01, Tot03, Esc82, Ron95, Gro81, Wil07], [FOT08, Chapter 6].

## 12. Curvature II

Consider the following question: Given a vector bundle  $\mathbb{R}^n \rightarrow E \rightarrow M$ , where  $M$  is a closed manifold with nonnegative sectional curvature, does  $E$  admit a metric with nonnegative sectional curvature? Ozäydin and Walschap answered the question negatively by giving the first examples of vector bundles over compact nonnegatively curved manifolds (in fact, oriented  $\mathbb{R}^2$ -vector bundles over  $T^2$ ) whose total spaces have no metrics of nonnegative curvature. A more general approach was initiated by Belegradek and Kapovitch using rational methods to give an obstruction called *splitting rigidity* to the existence of such a metric on the total space of a vector bundle. The main applications come when splitting rigidity is shown to follow from the vanishing of negative-

degree cohomology derivations — the famous result of Blanchard for Kähler manifolds. This talk should introduce the general problem of finding nonnegative curvature metrics on total spaces of bundles and then focus on the Belegradek-Kapovitch approach. References are: [BK01, BK03, PW06, ÖW94, CG72, Wil07], [FOT08, Chapter 6].

13. **Poincaré Duality and Models I.** (*Realization of a model whose cohomology satisfies Poincaré duality.*) Let  $X$  be a simply connected rational space. The two talks on Poincaré duality address the question of whether  $X$  contains a manifold in its rational homotopy type if its cohomology algebra  $H^*(X; \mathbb{Q})$  satisfies Poincaré duality. This first talk discusses a theorem of Barge [Bar76] and Sullivan [Sul77] giving conditions under which the existence of manifolds in the rational homotopy type of an  $n$ -dimensional Poincaré space  $X$  is assured. If  $n \neq 4k$ , the answer is always positive. If  $n = 4k$ , some conditions are required involving the signature of the quadratic form on  $H^{2k}$ . Roughly, this result can be stated as follows: conditions that are necessary for the realization of the model of  $X$  by a closed manifold are also sufficient. A recent nice account of the simply connected case and an extension to the non-simply connected case has been given in [Su09].

14. **Poincaré Duality and Models II**

*A model which satisfies Poincaré duality.* This second talk on duality addresses the situation that, in a certain sense, is the reverse of that discussed in the first talk. More precisely, if  $M$  is an  $n$ -dimensional closed simply connected manifold, does there exist a finite dimensional commutative differential graded algebra  $(A, d)$  which satisfies Poincaré duality at the cochain level and which is connected to the Sullivan model of  $M$  by a quasi-isomorphism? Such a model was conjectured some years ago by S. Halperin. A result related to this question was obtained by J. Stasheff [Sta83] which proves that the rational homotopy type of  $M$  is determined by its  $(n - 1)$ -skeleton (see also [Umb89] and [AHL]). The final positive answer was given by P. Lambrechts and D. Stanley [LS08a]. Applications to the rational homotopy type of configuration spaces of two points [LS04] can also be considered in this talk.

## 15. String Topology I

In [CS99], Chas and Sullivan defined a loop product whose construction can be sketched as follows. Let  $M$  be a compact closed oriented  $n$ -dimensional manifold. Observe first that, if  $LM$  is the free loop space on  $M$ , composition of loops can be done on the subspace  $LM \times_M LM$  consisting of pairs of loops based at the same point; this means we have a composition map  $c_M: LM \times_M LM \rightarrow LM$ . Recall now that if  $P$  is of dimension  $q$  and  $P \hookrightarrow M$  is an embedding of closed oriented manifolds, then there is an umkehr map

$$H_q(M) \cong H^{n-k}(M) \xrightarrow{i^*} H^{n-k}(P) \cong H_{k+q-n}(P)$$

where the two isomorphisms are Poincaré duality. Chas and Sullivan observe that a tubular neighborhood of the inclusion  $\iota: LM \times_M LM \rightarrow LM \times LM$  can be obtained by pulling-back a tubular neighborhood of the diagonal  $\Delta: M \rightarrow M \times M$  and they obtain an umkehr map  $\iota_!: H_*(LM \times LM) \rightarrow H_{*-n}(LM \times_M LM)$ . By definition, the *loop product* is the composition  $(c_M) \circ \iota_!: H_*(LM \times LM) \rightarrow H_{*-n}(LM)$ . By re-grading,  $\mathbb{H}_*(LM) := H_{*+n}(LM)$ , the vector space  $\mathbb{H}_*(LM)$  acquires the structure of an associative, commutative graded algebra. Moreover, from this product, Chas and Sullivan define a bracket on the (desuspended) equivariant homology of  $LM$ ,  $\mathbb{H}_*^{S^1} := H_{*+n}(ES^1 \times_{S^1} LM)$ . These structures are invariant by a homotopy equivalence between two smooth and closed manifolds which is orientation-preserving [CKS08]. The development of properties needs a cochain representative and this is done in different forms in several papers, [CS99], [Mer04], [CJ02], [McC06], [Laz08], [Tam09].... This list is not exhaustive; a global overview is contained in the notes of lectures given by R. Cohen and A. Voronov [CV06]; see also [Sul07]. The first talk should be devoted to the five first sections of [FTVP07] whose paradigm is a dual version of the loop product and string bracket in terms of cdga's over  $\mathbb{Q}$ , with explicit examples of computation.

## 16. String Topology II

The previous products and brackets give rise to rich structures such as Gerstenhaber and Batalin-Vilkovisky algebras. This second talk is devoted to their definition and construction on chain models as is

described in [Che07], for instance. Related papers are [CS04, FMT05, Men04, CT07, FT08, Tra08, Tra02, VG95].

**17. Chen’s Iterated Integrals and higher Hochschild chain complex**

K. T. Chen’s theory of iterated integrals is a geometric way to obtain information about the rational homotopy type of manifolds and simplicial sets. In particular, this point of view is well-suited to the study of spaces of loops, see [Che73] or [Mer04]. This talk is concerned with a generalization to the higher Hochschild chain complex (introduced in [Pir00]) as it appears in Section 2 of [GTZ10].

**18. Differential Modules and Applications**

Many results of rational homotopy are using the notion of differential modules over a cdga. This talk will present basic definitions and properties of these modules and give illustrations of their use in Geometry and Topology.

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## Participation:

The idea of the Arbeitsgemeinschaft is to learn by giving one of the lectures in the program.

If you intend to participate, please send your full name and full postal address to both John Oprea and Daniel Tanré at

`j.oprea@csuohio.edu` and `Daniel.Tanre@univ-lille1.fr`

by February 13, 2011 at the latest.

You should also indicate which talk you are willing to give:

First choice: talk no. ...

Second choice: talk no. ...

Third choice: talk no. ...

You will be informed shortly after the deadline if your participation is possible and whether you have been chosen to give one of the lectures.

The Arbeitsgemeinschaft will take place at Mathematisches Forschungsinstitut Oberwolfach, Lorenzenhof, 77709 Oberwolfach-Walke, Germany. The institute offers accommodation free of charge to the participants. Travel expenses cannot be covered. Further information will be given to the participants after the deadline.