Arbeitsgemeinschaft mit aktuellem Thema:

ERGODIC THEORY AND COMBINATORIAL NUMBER THEORY

Mathematisches Forschungsinsitut Oberwolfach

October 7th - 13th, 2012

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Introduction

In 1977 Furstenberg gave an ergodic proof of the celebrated theorem of Szemerédi on arithmetic progressions, hereby starting a new field of mathematics, Ergodic Ramsey Theory. Over the years the methods of Ergodic Theory and Topological Dynamics have led to very impressive developments in the fields of arithmetic combinatorics and Ramsey theory. Besides various powerful and far reaching extensions of the Szemerédi's theorem, these methods have facilitated the recent spectacular work of Green, Tao, and Ziegler on patterns in primes. The field of Ergodic Theory has tremendously benefited as well, since the problems of combinatorial and number-theoretical nature have given a boost to the in depth study of recurrence and convergence problems. The aim of the proposed workshop is to expose wide circles of mathematicians to the beautiful results and methods of this rapidly developing area of mathematics.

Talks:

1. **Background in ergodic theory.**

   This talk should give some ergodic theory background needed for subsequent talks. Examples of measure preserving systems (rotations and homomorphisms of the torus, shift on the sequence space), ergodicity, ergodic decomposition, factors, Kronecker factor, weak mixing systems and equivalent characterizations, spectral theorem, invariant measures on compact spaces, Furstenberg correspondence principle. See [EiWa11], [Fu81a], [Walt82].

2. **Ergodic proof of (polynomial) Szemerédi's theorem I.**

   This talk should outline the ergodic proof of the theorem of Szemerédi on arithmetic progressions and its polynomial extension. Start by giving the equivalent multiple recurrence properties in ergodic theory. Sketch the proof for weak mixing and compact systems (circle rotations). Explain why isometric extensions preserve the Szemerédi property using van der Waerden's theorem. Sketch the proof for the general case. Explain the extra ingredients needed to prove the polynomial Szemerédi theorem. See [Be06b], [BL96], [EiWa11], [Fu81a], [FuKO82], [Mc99].

3. **Uniformity seminorms, related factors, and structure theorem.**

   This talk should introduce the ergodic uniformity seminorms and explain the closely related structure theorem of Host and Kra. Start by defining the uniformity seminorms and prove that the linear multiple ergodic averages are majorized by these seminorms. Sketch the proof that the seminorms define factors $\mathcal{Z}_k$ (do this for some small value of k, see Section 7 of [Kra07]). Prove that $\mathcal{Z}_1$=Kronecker using the spectral theorem. Define nilsystems and nilfactors. Give the statement (without a proof) of the theorem of Host and Kra concerning the structure of the factor $\mathcal{Z}_k$ [HK05a] (see also [Lei06] for a connection to other factors defined by Ziegler in [Zi07]) and reinterpret it as a decomposition result. Discuss briefly the relationship with the more combinatorial Gowers uniformity norms and inverse theorems [BTZ10], [TZ10].

4. **Equidistribution of polynomial sequences on nilmanifolds.**

   This talk should explain the basic qualitative and quantitative equidistribution results of polynomial sequences on nilmanifolds that will be used in subsequent talks. Start by giving the basic examples of nilsystems (rotations, Weyl, and Heisenberg). Explain the qualitative equidistribution of polynomial sequences on the circle and its extension to nilmanifolds. Give the proof in the case of linear sequences on nilmanifolds [Ta12] and explain briefly how one uses this linear result to get a polynomial extension [Lei05a]. State the quantitative equidistribution result of polynomial sequences on the circle and its extension to nilmanifolds [GT12a].

5. **PET induction and seminorm estimates.**

   This talk should explain the polynomial exhaustion technique, how it is used to prove seminorm estimates, and how such estimates help establish mean convergence results of multiple ergodic averages and in some cases identify their limit. Start by explaining the PET induction and sketch the proof of the main result from [Be87]. Mention the changes needed to get the seminorm estimates from [HK05b], [Lei05b]. Explain how one can combine such estimates with the material from lectures 3 and 4 to prove mean convergence results for multiple ergodic averages. Search in the literature to find a few cases where using this method one gets an explicit description of the limit function.

6. **Ergodic proof of the polynomial Szemerédi theorem II.**

   This talk should explain an alternate proof of the polynomial Szemerédi theorem given in [BLL08] that uses the machinery of characteristic factors and equidistribution results on nilmanifolds. Start by stating the main combinatorial result involving intersective polynomials and its ergodic counterpart. Reduce to nilsystems using results from the lectures 3 and 5. Reduce further to a (pointwise) recurrence property on nilmanifolds (Prop 2.4). State the algebraic lemma regarding intersective polynomials (Prop 3.6) without proof. Explain the proof of the key recurrence result (Prop 4.3). Mention a related open problem involving commuting transformations.

7. **Structure of multiple correlation sequences and applications.**

This talk concerns a decomposition result for multiple correlation sequences and related applications to quantitative improvements of Szemerédi's theorem given in [BHK05]. Start by giving a brief sketch of the proof of the decomposition result for multiple correlation sequences (this is Theorem 1.9 in [BHK05], but see also an alternate proof given in [Lei10], and an extension to the non-ergodic case given in [Lei12]). Explain the ergodic correspondence principle (Proposition 3.1). Give a brief sketch of the proof of the positive results for lower bounds of multiple intersections with small length (Theorem 1.2) and the negative results of Theorems 1.3 and 2.1.

8. **Euclidean Ramsey theory.**

   This talk concerns applications of ergodic theory to Euclidean Ramsey theory. Start by giving a detailed sketch of the proof of Theorem B from [FuKW90] concerning triangular configurations that can be found in thickened sets of positive density in the plane. Explain why the thickening is needed for co-linear points and mention the open problem regarding non-degenerate triangles (see [Bou86], or Section 4 in [Gra94]). Mention the generalization of this result to arbitrary finite configurations [Zi06] (see also [Zi99] for a simpler case) and explain briefly its proof.

9. **Recurrence for random sequences.**

   This talk concerns random sequences of integers and related recurrence and convergence properties. Start by giving the definition of a random sequence of integers. Give the proof of the single convergence and recurrence result from [Bou88] involving random "non-lacunary" sequences (see also [Bos83]). Your proof should preferably be based on the exposition that appears in [RW95]. Explain briefly the progress made towards the corresponding multiple recurrence and convergence problems and state some related open questions [FrLW11].

10. **Recurrence for Hardy sequences.**

    This talk concerns Hardy sequences and related recurrence and convergence properties. Start by defining the class of Hardy sequences and state some key properties they satisfy. The main result to be proved is that non-polynomial Hardy sequences that do not grow very fast are good for single recurrence (the precise statement appears in the appendix of [FrW09]). Prove this by combining the spectral theorem and the equidistribution results of [Bos94] (explain the proof of this result), except for a few cases that can be treated as in the appendix of [FrW09] (you can skip this argument). Explain how tools from lectures 3, 4, and 5 can be combined to prove a generalization of this result to multiple recurrence and state some related open problems [Fr10].

11. **Recurrence for generalized polynomials.**

    This talk should include various examples of recurrence and multiple recurrence for generalized polynomials, and also explain the connection between generalized polynomials and nil-rotations. See [BHa96], [BM10], [BHM06], [BL07].

12. **Markov processes and Ramsey theory for trees.**

    This talk concerns the article [FuW03] where novel recurrence properties for Markov processes are established which in turn imply an analogue of Szemerédi's theorem for binary trees.

13. **Convergence results involving multiple averages.**

    Survey the history of convergence results of multiple averages and explain the recent proof of Walsh to convergence of nilpotent multiple averages. See [Au10], [H09], [Ta08], [Walsh12], [Zo12].

14. **Non-abelian group actions: Szemerédi and van der Waerden theorems for commuting actions of non-commutative groups.**

    This talk concerns the amenable Roth theorem and two of its applications: a van der Waerden - type theorem and a non-commutative Schur theorem. See [BHi93], [BM07], [BMZ97].

15. **Non-abelian group actions: Noncommuting actions of $\mathbb{Z}$ (beyond nilpotency).**

    This talk concerns some negative recurrence and convergence results for non-commuting actions of $\mathbb{Z}$ (failure of Roth Theorem [BL02], [BL04]) and some positive convergence results (pointwise convergence for cubic and polynomial averages [As10], [CF12]).

16. **Topological dynamics and Ramsey theory.**

    A talk on this topic could consist of dynamical formulations and sketches of proofs (or special cases) of Schur's, Hindman's, van der Waerden's theorem, the polynomial van der Waerden theorem, the nilpotent van der Waerden theorem, and the polynomial Hales-Jewett theorem. See [Be00], [BL96], [BL99], [Fu81a], [FuW98], [Lei98].

17. **Ultrafilters and coloring problems.**

    This talk concerns applications of ultrafilters to partition Ramsey theory. Start by formulating Hilbert's lemma, Schur's theorem and Hindman's theorem. Give some background on ultrafilters (definition, extending the operations from $(\mathbb{N}, +)$ and $(\mathbb{N}, \times)$, existence of idempotent ultrafilters etc). Prove Hindman's theorem with the help of idempotent ultrafilters a la Poincaré recurrence. Talk about minimal idempotents and central sets. See [Be96], [Be06a], [BHi90], [Fu81a], [HS98].

18. **Ultrafilters and ergodic theory.**

    This talk concerns uses of ultrafilters in ergodic theory. Start by defining p-limits and comparing them with Cesaro limits. Give an enhanced form of Sárközy's theorem via p-limits. Characterize mild mixing with the help of idempotent ultrafilters. Explain briefly how this theory is applied in proving a general (non-amenable) Roth theorem. Explain what essential idempotents are and how they are used to prove Szemerédi's theorem for generalized polynomials. See [Be96], [Be06a], [BD08], [BM98], [BM07], [BMZ97].

19. **Ultrafilters, nonstandard analysis, and characteristic factors.**

    This talks concerns uses of ultrafilters and nonstandard analysis in simplifying proofs of various convergence results and inverse theorems in ergodic theory. Start by explaining the use of ultraproducts to construct nonstandard models of mathematics. Loeb measure. Nonstandard analysis proof of the ergodic theorem. Towsner's nonstandard analysis proof of convergence for multiple commuting shifts. Nonstandard equidistribution, and connections with single-scale equidistribution (as per Tao's book). More ambitiously, some discussion of how nonstandard analysis is used in the Green-Tao-Ziegler and Camarena-Szegedy approaches to the inverse conjecture for the Gowers norms. See [CS11], [KK97], [Sz12], [Ta12], [To07].

20. **Sumset phenomenon and ergodic theory.**

    This talk concerns uses of ergodic theory to study properties of sumsets of infinite sets. Start by explaining the result of Jin [J02] who showed by methods of non-standard analysis that whenever A and B are sets of integers having positive upper Banach density, the sumset $A + B$ is piecewise syndetic. In [BFW06] Bergelson, Furstenberg, and Weiss proved with the help of ergodic theory that $A + B$ must be piecewise Bohr. The general result for amenable groups was obtained in [BBF10]. An interesting new result was recently obtained by Griesmer [Gri12] who generalized the result from [BFW06] to cases where A has Banach density 0. Another interesting recent paper due to Beiglböck [Bei11] utilizes ultrafilters to recover the results obtained in [BBF10]. See also [J04].

21. **Van der Corput sets.**

    In [KM78] Kamae and Mendès France shoed that many sets of recurrence have a stronger property which they called the van der Corput property. A set D of positive integers is a van der Corput set (or a vdC set) if for any sequence $(x_n)_{n\in\mathbb{Z}}$ of real numbers, and for each $d \in D$, the sequence $(x_{n+d} - x_n)_{n\in\mathbb{Z}}$ is uniformly distributed mod 1, in which case the sequence $(x_n)_{n\in\mathbb{Z}}$ itself is also uniformly distributed mod 1. While Kamae and Mendès France showed that many familiar sets of recurrence are vdC sets, Bourgain showed in [Bou87] that not every set of recurrence is a vdC set (see also [Fo91]). Many quantitative results on vdC sets can be found in [R84]. The connections between various notions of recurrence and vdC sets in $\mathbb{Z}^d$ are explored in [BL08].

22. **Green-Tao, Tao-Ziegler theorems.**

    This talk should describe the ergodic theoretic part of the proof in [GT08] that the primes contain arbitrarily long arithmetic progressions with the analytic number theoretic part as a black box. Discuss first the transference principle, with the new proof by Gowers [Go08] and Reingold, Trevisan, Tulsiani, Vadhan [RTTV08]. Describe the new ingredients needed for the polynomial theorem in [TZ10].

23. **Möbius and dynamics.**

    This talk concerns Sarnak's conjecture for disjointness of the Möbius flow to deterministic systems and its connection to Chowla's conjecture [Sa]. State the Green-Tao result on orthogonality of the Möbius function to nilsequences [GT12b], and explain its motivation. Explain the Bourgain-Sarnak-Ziegler orthogonality criterion and give some of its applications [BSZ11], [Zi11].

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Participation:

The idea of the Arbeitsgemeinschaft is to learn by giving one of the lectures in the program.

If you intend to participate, please send your full name and full postal address to

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by **August 20th, 2012** at the latest. You should also indicate which talk you are willing to give:

First choice: talk number
Second choice: talk number
Third choice: talk number

You will be informed shortly after the deadline if your participation is possible and whether you have been chosen to give one of the lectures.

The Arbeitsgemeinschaft will take place at Mathematisches Forschungsinstitut Oberwolfach, Lorenzenhof, 77709 Oberwolfach-Walke, Germany. The institute offers board and lodging free of charge to the participants. However, travel expenses cannot be covered. Further information on the conference and on how to arrive at the Oberwolfach conference center will be given out to the participants, shortly after the deadline for applications.