

Workshop on Definability and Decidability Problems in Number Theory.

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Abstract. Hilbert's tenth problem (H10 for short) asked for an algorithm to decide solvability of Diophantine equations in the integers. Building on work of M. Davis, H. Putnam and J. Robinson, Matiyasevich showed in 1970 that every recursively enumerable set can be realized as a Diophantine set, and therefore H10 has a negative answer. This result did not resolve the analogous decidability question for $\mathbb{Q}$, other number fields, or their rings of integers. In 1949, using the first-order undecidability of $\mathbb{Z}$ (already known by then), Julia Robinson succeeded in showing that the full theory of $\mathbb{Q}$ was undecidable by constructing a first-order definition of $\mathbb{Z}$ in $\mathbb{Q}$. Similarly, an existential definition of $\mathbb{Z}$ in $\mathbb{Q}$ would provide a negative solution to the analogue of H10 over $\mathbb{Q}$. The search for such a definition or a proof that it does not exist is a major theme within the subject. In the fifties, Julia Robinson initiated the study of definability and decidability over number fields and infinite algebraic extensions of $\mathbb{Q}$, while Raphael Robinson took the first steps towards the understanding of the first-order theory over function fields and analytic structures.

In the last 20 years or so, considerable progress has been achieved on these problems thanks to the discovery of striking connections with other fields of mathematics, such as algebraic geometry, anabelian geometry, Nevanlinna theory and recursion theory. However, many fundamental questions remain wide open due to our lack of understanding of Diophantine sets. This workshop will serve as an opportunity for researchers in related fields to bring together their ideas and expertise in order to make progress on these matters, and to make more widely known the techniques and recent developments in the area.