Arbeitsgemeinschaft Deninger–Schneider
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We will start from a simple fact, established by G.Belyi [B] around 1978: a smooth complex projective curve X can be defined over a number field (i.e. described by equations whose coefficients are algebraic numbers) if and only if it can be realized as a cover of the projective line $\mathbb{P}^1$ ramified above at most three points. In the language of compact Riemann surfaces this is equivalent to saying that there exists a meromorphic, non–constant *Belyi function* β on X, ramified at most above $0, 1, \infty$. The 'if' part is classical and goes back at least to ideas of A.Weil; it now appears as a very special consequence of the theory of the fundamental group developped by A.Grothendieck in [SGA 1]. But A.Grothendieck himself was deeply struck by the 'only if' part and its two–page long elementary proof. In fact he had somehow anticipated the result, without however neither finding a proof for it, nor even convincing himself of its validity, see [G] pp. 14-15. As he puts it (see [G] p. 15, underlined passage), the 'essential message' of this technically simple but deep result is that 'there is a profound identity between the combinatorics of finite maps on the one hand, and the geometry of algebraic curves defined over number fields on the other'. These finite maps are now called "Dessins d'enfants" in this setting, following the colourful name Grothendieck gave them and are just bipartite graphs with white vertices $\beta^{-1}(0)$, black vertices $\beta^{-1}(1)$ and edges $\beta^{-1}]0,1[$ dividing the surface into simply connected open cells. There are in fact several different descriptions of these combinatorial structures which all have their own merits; we will establish a dictionary between them. Now one can go all the way from topology to arithmetics (via analysis and algebra), as the quotation above suggests: any such bipartite graph on an orientable compact 2–manifold X defines a unique conformal structure on X and a corresponding Belyi function, hence a structure as an algebraic curve, which moreover is defined over a number field. So the dessins, which are described in a purely combinatorial way, do encode deep properties of the curves; this "code" however is quite mysterious and at the moment has been only very partly understood.

Concerning the role of the dessins for the individual curves, we will treat in the Arbeitsgemeinschaft mainly three aspects: explicit uniformization (i.e. the construction of — sometimes universal — covering groups using Belyi functions and conversely the calculation of explicit equations resulting from such covering groups), the meaning of the vertices, and the field of moduli (which is generically the minimal field of definition). The last aspect is already related to the fact that the obvious action of the absolute Galois group on the (coefficients of the defining equations of the) curves and their Belyi functions defines an action of this Galois group $G_{\mathbf{Q}} =$

$\mathrm{Gal}(\overline{\mathbf{Q}}/\mathbf{Q})$ on the set of dessins, leading — we hope — to a better understanding of the curves and $G_{\mathbf{Q}}$ as well. Belyi's original motivation was in fact the application to problems of inverse Galois theory.

What is described above is in fact but the tip of a huge iceberg which remains largely underwater, although parts are gradually becoming more visible. The 50-page long text which A.Grothendieck wrote in 1983 [G], and whose title 'Esquisse d'un programme', 'Sketch of a programme', speaks for itself, embodies in fact a fundamental and subtle renewal of his mathematical vision. In particular, roughly the first half of the text presents a program (or a dream) to study the Galois group $G_{\mathbf{Q}}$ of $\mathbf{Q}$ in a geometric, and indeed combinatorial way. This is based on the action of this group on the geometric fundamental groups of $\mathbf{Q}$–varieties via the fundamental short exact sequence established in [SGA1]. Here already one departs from the more classical Galois *linear* representations to turn to more 'nonlinear' objects. Moreover Grothendieck singles out the moduli spaces of curves as a particularly interesting collection of objects to be studied. So the ensuing 'Grothendieck–Teichmüller' theory, as it is now called, centers around two specific and classical objects, namely $G_{\mathbf{Q}}$ and the tower of all moduli spaces of (pointed, smooth or stable) curves. From this viewpoint, the projective line with three points removed appears as the first nontrivial example: namely it provides the moduli for projective lines with four marked points (using the cross-ratio). Belyi's result immediately implies that the corresponding Galois action is faithful, so that one gets an injective map from $G_{\mathbf{Q}}$ into the automorphism group of the profinite completion of F_2, the free group on two generators. This automorphism group by itself is huge, and very little is known about it. Note parenthetically the importance of completion: automorphisms of the discrete group F_2 are not so mysterious; indeed they are determined by their effect on the abelianized group, up to inner automorphism. In other words, the outer automorphism group of F_2 is just $GL_2(\mathbf{Z})$. Not so for the automorphisms of the profinite completions... Note also that "dessins" may be seen as a convenient pictorial dictionary of the profinite completion of F_2, or for that matter of the completion of $SL_2(\mathbf{Z})$, a group of which Grothendieck says it pops out magically in most important subjects of mathematics.

Along these lines, and following the "Esquisse" again, one is naturally lead to consider the group of all automorphisms of the collection of the fundamental groups of the moduli spaces of curves, satisfying certain natural compatibility conditions of geometric origin. This essentially defines a new (not classical) object, namely the so-called Grothendieck–Teichmüller group, whose first incarnation was formally introduced by V.G.Drinfeld (in the genus 0 case). An extremely important and indeed surprising fact, predicted in the Esquisse, is that this group can be 'described' in a fairly explicit way. This is 'surprising' in particular inasmuch as automorphisms of profinite groups are for the most part terra incognita and appear indeed are rather intractable in general at present, as alluded to above for the free group on two generators. So it is a really remarkable fact that the abstractly defined Grothendieck–Teichmüller group can be described in a finite way. Now the Galois group is contained in the Grothendieck–Teichmüller group, basically because of Belyi's theorem (the faithfulness of the action mentioned above) and because arithmetic automorphisms respect $\mathbf{Q}$–structures by definition. A very much open and debated

question at present consists in comparing these two groups, which might (or not) coincide.

It may be useful to stress again some points of originality of the whole approach proposed in the "Esquisse", from a semitechnical standpoint. First again one considers (nonlinear) group actions of (profinite) arithmetic groups on (profinite) geometric fundamental groups. The fact that one operates a priori in a profinite setting is also essential; it means that one cannot a priori use the linearizing tools which are available in a pronilpotent (pro-ℓ, prounipotent) setting, such as (graded) Lie algebras and the like. Of course it is often very interesting and desirable (if only for lack of a better idea!) to go to a pronilpotent or prounipotent quotient, but one main thrust of the theory is to try and take into account the richness and mystery of the profinite completion, which for instance is necessary in order to capture the whole Galois group. And this appears as another nonlinear (non solvable) feature of the theory. Next, as mentioned above, one focuses on specific objects, and from the group theoretic viewpoint especially on the Teichmüller modular groups, namely the geometric fundamental groups of the moduli spaces of curves. These comprise the ordinary braid groups as special and relatively well–understood cases, but remain in general rather mysterious objects, especially again in the profinite setting. Here they can be contrasted in particular with the classical arithmetic groups. In particular for curves of genus g, the comparison with the symplectic groups $Sp_g(\mathbf{Z})$ is part of the classical parallel between curves and their Jacobians. Of course the Arbeitsgemeinschaft shall make these remarks more precise and detailed and this is only to hint at the fact that the path proposed by Grothendieck is distinct indeed from what he had proposed before, but also has many ramifications which connect it to more or less classical objects of study.

Exposés

1. Arithmetic Galois groups. Here, elementary facts about coverings should be explained, in particular the 'if' part of Belyi's theorem (Prop. 2.1 in [MM]), the short exact sequence for the arithmetic Galois group ([Se], (8.1)) and the action of the arithmetic Galois group on the geometric Galois group. Example $\mathbb{P}^1$ minus three points (pp. 10–13 of [MM]). More generally one can explain parts of Grothendieck's theory of the fundamental group. The fundamental short exact sequence occurs in [SGA1], chapter 9; one can also consult e.g. Grothendieck's stunning Bourbaki talk (no 182, 159), or in a softer vein the lectures given at the Luminy week on the fundamental group (December 1998; see the forthcoming book).

2. Belyi's theorem and its variants. The proof for the existence of Belyi functions β (see above) on curves defined over number fields can be found at many places [B], [MM] (Section 1.2.4), [Sn2]. "Dessins", "clean dessins", "regular" (= "Galois") and "uniform" or "semiregular" dessins (i.e. if the ramification index of β is constant in every fibre of β^{-1}) should be defined and it should be explained how they are related to "Triangulations" [SV1], "Fatgraphs" (= graphes épais [BaIt]), "Maps" and "Hypermaps" [JS1-3] and how dessins can be described in a non–graphic way by (hyper–)cartographic groups, i.e. the monodromy groups of the corresponding Belyi functions. Automorphism groups of dessins and the corresponding curves, and their relation to the cartographic groups (Prop. 1 in [BaIt]). Belyi's theorem says moreover that

curves over number fields always can be written as a quotient $\Gamma\backslash\mathcal{H}$ where $\mathcal{H}$ denotes the upper half plane and Γ a finite index subgroup of a cocompact Fuchsian triangle group Δ ([CIW], [CoWo], Cor. 3 in [JS3]) Finally, it should be explained how dessins determine a conformal structure, hence giving an equivalence between topological, analytical, algebraic and arithmetic coverings ([MV], Section 5 of [JS1]).

3. Examples. Galois actions. First examples of dessins can be taken from [Bi] and [Sn2]. It should be proved that the absolute Galois group acts faithfully on dessins in genus 1 and on trees in genus 0 [Sn2]. Nice examples for the action in genus 1 can be found in [SSy]. Example 5 of [Wo1] considers an easy non–abelian action. Ex. 3 of [JS3] treats generalized Fermat curves. Invariants for the Galois actions on dessins [JSt].

4. Moduli fields and fields of definition. We should learn what these terms mean for coverings (i.e. here for Belyi pairs consisting of a curve and a Belyi function on it) and for the curves themselves [DE], [Wo2]. Weil's cocycle condition [W] for fields of moduli being fields of definition. It should be explained that this condition is satisfied for two extreme cases: if the curve has only the trivial automorphism and if the curve has "many" automorphisms, i.e. a Belyi function giving a normal covering of $\mathbb{P}^1$ and a regular dessin ([CH]; [Sa] contains an easy new proof not based on [W]). It is not so easy to construct examples for which the moduli field is different from the minimal field of definition, see e.g. Thm. 2 and remark (c) of [Ea]. In a more cohomological and general vein, one may refer and explain part of [DD].

5. Uniformization, equations and Galois actions made explicit. Uniformization of (punctured) curves by (in general non–congruence) subgroups of the elliptic modular group [Bi], Cor. 1/2 in [JS3]. For uniform and in particular for regular dessins — canonically associated to curves with many automorphisms [Wo2] — and suitable cocompact triangle groups Δ (see talk 2.), Γ is even the universal covering group [Wo2], examples are available in Section 6 of [Wo3]. Along the lines of [St1] it should be explained how the canonical model of a curve with regular dessin can be calculated if the genus is not too high. Galois actions and fields of definition of curves with regular dessins in the example of Macbeath–Hurwitz curves [St2].

6. Vertices of dessins may have a special meaning for the curve: For regular dessins and under additional conditions, they are Weierstrass points, see in particular Thm. 6 of [SWa], and in general dessins induce nontrivial morphisms of the curve into Shimura varieties mapping vertices to CM points [CoWo].

7. Further Galois invariants were constructed recently, using objects of analytic origin attached to Riemann surfaces, like the so-called Strebel differential. They were used in particular to separate simple looking dessins (i.e. show that they are not Galois conjugate) which could not be separated previously. This lecture can be based in particular on [Z1,2], see also [StWo].

8. Rigidity and the inverse Galois problem. Based for instance on part of [MM] or [Se], this lecture should explain the basics of the rigidity method, which originated some twenty years ago from the work of several authors (G.Belyi, M.Fried, H.Matzat, J.Thompson) and enables one to obtain so-called regular realizations of many finite groups over $\mathbf{Q}$ or at least some abelian extension of it.

9. $\mathbb{P}^1$ minus three points and polylogarithms. In this last lecture on the projective line minus three points, one should give glimpses of the connection with the differential equations and iterated integrals which define polylogarithms. There is actually a strong connection between this body of work and the budding Grothendieck–Teichmüller theory. Covering the content of [Hain] will not uncover all ramifications but will open the way to further reading and discoveries.

10. Braids. The projective line minus three points classifies the set of projective lines minus *four* (distinct) points: to four points one associates their cross-ratio, which is distinct from 0, 1 and ∞ because the points are assumed to be distinct. In this lecture, one moves to higher dimension, keeping the genus equal to 0, i.e. one explores the basics of the configurations of points on the sphere (projective lines). One should learn about the presentation and first properties of Artin braid groups, sphere braid groups, mapping class groups (i.e. Teichmüller modular groups) in genus 0, together with a detailed review of the low dimensional (dim. 1 and 2) cases. In particular one should give a detailed description of the (two dimensional) moduli space of spheres with 5 distinct (ordered or unordered) points. Possible references include [Bir], [K], [D], [PS], [LoSn3].

11. Braids and more inverse Galois theory. In this second lecture on inverse Galois theory, one should see the role of the action of the braid groups (probably with just a few strands) in a nonrigid situation, connecting with the basics of Hurwitz spaces. This could be based again on [MM] and papers by Fried-Völklein referenced there. Perhaps the best approach is to actually treat just one nontrivial example in details, with -say- covers of the projective line with four ramification points.

12. Braided categories and Grothendieck–Teichmüller group. Here one should first explain McLane's coherence conditions ([D], [K] , [PS]) and then move to the original definition of the Grothendieck-Teichmüller group GT by Drinfeld [D], perhaps following also [I1,2], [Sn]. One should explain the connection between the first two relations in GT, the hexagonal relations and the the automorphisms of the projective line minus three points and the fact that they are satisfied for elements of the Galois groups, see the appendix to [I2]. One should give the formulas for the action of GT on braid groups, and why it carries over to genus 0 Teichmüller modular groups.

13. The Grothendieck–Teichmüller group again. In this second introductory lecture on GT, one should discuss in more details the pentagonal relation, which 'lives at the second level', in the sense that it comes from a *two* dimensional moduli space, namely that of sphere with five marked points. Here again one can consult [I1,2], sketching the proof that the absolute Galois group of $\mathbf{Q}$ is indeed included in GT. One should explain the action of GT on why McLane's coherence relations embody in the genus 0 setting the 'two level principle' put forward in the Esquisse: namely the action on all the braid groups (or genus 0 Teichmüller modular groups) is governed by the first two levels (spheres with 4 and 5 marked points); see also [LoSn3], [Sn3].

14. The moduli space of curves. Here we 'add genus', moving to the general moduli spaces of curves. This lecture is devoted to the definitions and basic geometric properties of these spaces, from a complex analytic and hyperbolic viewpoints. One can follow part of the first

chapter of [PS] and draw from [FLP], the beginning of [H] and especially [IT]. One goal would be to explain what are the Fenchel-Nielsen coordinates. It would be good to say something of the algebro-geometric viewpoint, using e.g. [Bai], [DM], or one of the many gentle expositions of the latter paper.

15. The Teichmüller modular groups and geometry at infinity. Here one should first redefine the Teichmüller modular groups as fundamental groups of the moduli spaces of curves and explain why they coincide with the topologically defined 'mapping class groups', see e.g. [PS]. They are generated by a finite number of Dehn twists; see [Hu] for the purely topological exposition. Dehn twists correspond to 'loops at infinity' on moduli spaces. One should state some basic properties of the compactifications of the moduli spaces, in terms of Riemann surfaces with nodes ([IT], [PS]) or/and stable curves ([DM], [Des]).

16. Complexes of curves and GT. The combinatorial description of the geometry at infinity by means of complexes of curves; see [H] and also [FLP] and [Lo]. The simple connectivity of Hatcher's complex of curves and the relations in the Grothendieck–Teichmüller group for moduli spaces of curves of all genera; locality properties of the Grothendieck–Teichmüller and Galois actions [HLS]. Looking back at the genus 0 case; braided categories as a special case [PS].

17. Cell decomposition of the moduli spaces. A little more hyperbolic and conformal geometry for the moduli spaces of curves. Cellulation of the moduli spaces of curves, following [Pe] and [BE], see also [PS], Chapter 1. Back to the projective line with three points removed: curves with many automorphisms and dessins as centers of the cells.

18. Loci of curves with automorphisms. Description of the locus of curves with given automorphism groups. Singularities and orbifolds. Elements of finite order in the Teichmüller modular groups; see [GH] and earlier references by the second author of that paper,see also [Lo]. See also [P1], [P2] for a more algebro-geometric viewpoint.

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