Arbeitsgemeinschaft mit aktuellem Thema:

Einstein Metrics and Geometrization of 3-Manifolds

Mathematisches Forschungsinstitut Oberwolfach

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Introduction:

Thurston's Geometrization Conjecture states that any closed 3-manifold may be cut along a finite collection of essential spheres and tori into domains, each of which carries a geometric structure. A geometric structure is a complete, locally homogeneous Riemannian metric. While there are 8 possible forms of geometric structures, the three constant curvature geometries play by far the most important role; the other five are product or twisted product geometries, living on comparatively simple 3-manifolds.

The geometrization conjecture, if true, essentially solves the classification problem for 3-manifolds and gives a picture in very broad terms similar to the classification of surfaces. More precisely, it reduces the classification problem to the classification of cocompact discrete group actions on 3-dimensional space-forms; this is a much simpler issue which has been solved to a large degree, in certain respects long ago.

To give an idea of the strength of the geometrization conjecture, it implies essentially immediately the Poincaré conjecture and the spherical space-form conjecture. Since Thurston's insight that this conjecture may be true in the late 70's, it has been the central problem in the geometry and topology of 3-manifolds.

There are several currently active programs or approaches towards the proof of the geometrization conjecture.

• Thurston's original approach, based on hyperbolic geometry and the deformation theory of Kleinian groups, cf. [T2], [M], for example.

• Hamilton's approach, based on the Ricci flow. This involves the study of a non-linear parabolic flow on the space of metrics, and dates from the early 80's, cf. [H] and [CC] for a recent survey.

A third approach, developed by the first author since the early/mid 90's, is the focus of this Arbeitsgemeinschaft. The approach is to consider suitable curvature integrals on the space of metrics $\mathcal{M}$ of a 3-manifold M,

$$\mathcal{I} : \mathcal{M} \to \mathbb{R},$$

and find a "best possible" metric on M by seeking critical points or extrema of $\mathcal{I}$. The curvature integral $\mathcal{I}$ is either the Einstein-Hilbert action (total scalar curvature), or a suitable modification thereof; the critical points of $\mathcal{I}$ are constant curvature metrics, i.e. Einstein metrics in dimension 3.

Now many (or most) 3-manifolds M do not admit Einstein metrics. Essential spheres and tori in M are obstructions to the existence of such metrics, with the single exception of flat 3-manifolds. Thus, on such manifolds, $\mathcal{I}$ has no critical points on $\mathcal{M} = \mathcal{M}(M)$. It follows that sequences of metrics $g_i \in \mathcal{M}$ tending to a critical or extremal point of $\mathcal{I}$ must degenerate, i.e. $\{g_i\}$ has no convergent subsequence. The basic idea is then to relate such degenerations with the topology of the underlying 3-manifold M, in particular with essential spheres and tori in M.

The methods involved in this program are an intricate blend of:

• Riemannian geometry, in particular the tools and points of view initiated by Gromov since the early 80's, cf. [Gr2,3] and, for example, [Pe, Ch.9-11].

• Analysis on manifolds, in particular the study of 2^{nd} and 4^{th} order elliptic systems of PDE. Most prior study of elliptic PDE for metrics on manifolds has focussed on 2^{nd} order scalar equations.

• Ideas and issues in general relativity. This connection is surprising; important issues on the structure of space-times, (in 4 dimensions), for example the structure of black holes, play an important role in the program.

Thus, there are interesting relations between the equations that arise on 3-manifolds, and the vacuum Einstein equations in 4-dimensions, (via the induced equations on space-like hypersurfaces).

The lectures will discuss these methods and ideas in some detail, leading up to the current state of this approach to Thurston's conjecture. Every interested but non-expert participant should be familiar with basic background material, for instance that in [Pe, Ch.9-10], [GT, Ch.2-3 and 6-8] before the start of the meeting.

Talks:

1. **Geometrization Conjecture.**
   State and discuss the Thurston conjecture. Mention Thurston's Haken manifold theorem (the "Monster theorem"). To date, this is the only general existence theorem for Einstein metrics under topological conditions, in dimension ≥ 3.

   Then head to Conjectures I-III of [A2] - the approach to geometrization sketched above. Introduce briefly Yamabe metrics, min-max process, definition of σ-constant $\sigma(M)$; these will be discussed in more detail in Lecture 4. $\sigma(M)$ on 3-manifolds is an analogue of the Euler characteristic for surfaces. Explain the idea of geometrizing M by analysing the limiting behavior convergence/degeneration of maximizing sequences of Yamabe metrics. Introduce briefly Seifert fibered spaces, graph manifolds and their collapse along circles/tori. Mention that the geometization of graph manifolds is easy.

   State Conjectures I-III, and indicate why they imply the geometrization conjecture. Via [GL, Thm.8.1], show that Conjectures I-II imply the geometrization of irreducible 3-manifolds with infinite π_1, and so in particular the Hyperbolization Conjecture.

   References: [A2, §4], [Be, Ch.6C], [T2], [GL], [Sch].

2. **Cheeger-Gromov Theory (L^∞ case).**
   Overall issue: convergence/collapse of sequences of metrics. Definition of convergence of metrics in $C^{k,\alpha}$ and $L^{k,p}$ topologies; convergence is always convergence modulo diffeomorphisms. Discuss the behavior of sequences of metrics with uniformly bounded curvature in L^∞, (use of

harmonic coordinates). This is done via the thick-thin decomposition of a Riemannian manifold. Convergence of sequences of metrics on the uniformly thick part, collapse on the (arbitrarily) thin part. Explain the general idea of F-structure, without getting into technicalities.

Tie this together and state the general trichotomy for behavior of sequences of metrics with uniformly bounded curvature: for a subsequence, one has either

convergence, collapse, or formation of cusps,

(as in the basic picture in Teichmuller theory on surfaces). All limits are complete; limit structure depends on choice of base point. Convergence is in the $C^{1,\alpha}$ and $L^{2,p}$ topologies.

In dimension 3: collapse occurs exactly on graph manifolds. Discuss Seifert fibered 3-manifolds, their collapse along circles, and unions of Seifert fibered spaces - graph manifolds, with toral collapse in glueing region. Mention that graph manifolds are closed under connected sums, [So], [Gr2, Appendix 2]

References: [A1, §1], [A2, §5], [A3, §2], [CG1,2], [F], [Gr2], [Pa], [Pe, Ch.10], [Ro, §3,4].

3. **Cheeger-Gromov Theory (L^2 case).**
L^∞ bounds on curvature are not natural in terms of analysis; they lead for example to bad Euler-Lagrange equations. It's more natural to consider L^2 bounds (or possibly L^p bounds).

Sobolev embedding in dimension 3, $L^{2,2} \subset L^{1,6} \subset C^\alpha$, $\alpha < \frac{1}{2}$, allows one to work with L^2 bounds on curvature, (curvature involves 2 derivatives of metric).

State the definition, and discuss the L^2 curvature radius ρ, cf. [A3, §3], although use the slightly modified definition in [A8, §2]. This is an important quantity in the later lectures, relating the degree of curvature concentration to a distance function. Also state and discuss volume radius ν. Observe that ρ is essentially preserved under coverings.

State and discuss results showing that the L^∞ Cheeger-Gromov theory extends to an L^2 theory in dimension 3, in particular within the L^2 curvature radius. Important difference: limits may not be complete, (lack of general volume comparison theorem, Gallot's example). Use of volume radius in place of injectivity radius.

When $\nu << \rho$, the collapse may be unwrapped in ρ-balls by passing to covering spaces, i.e. the F-structure is injective. This situation is special to dimension 3.

References: [A3, §3 through Thm.3.19], c.f. also [A4,Remark 5.5], [Ga], [Ro, §3,4]

4. **Curvature Integrals I: Einstein-Hilbert action.**
   This is the simplest, (most natural), curvature integral. The Euler-Lagrange equation $z = 0$ of 2^{nd} order. The foundation of gravity/general relativity.

   Discuss Yamabe metrics, the space $\mathcal{C}$ of Yamabe metrics, min-max process, definition of $\sigma(M)$. Introduce the linearization L of scalar curvature and its adjoint

$$L^{*}u = D^{2}u - \Delta u - ur.$$

   Discuss main aspects of the analysis of the scalar curvature functional on $\mathcal{C}$ in [A7, §2], cf. also [Be, Ch.4]; in particular the two splittings of z, Theorems 2.2, 2.10 and the bound (2.38) of [A7].

   Show that Yamabe metrics realizing $\sigma(M)$ are Einstein when $\sigma(M) \leq 0$, and state this remains an open problem when $\sigma(M) > 0$.

   Emphasize that the potential u is global - it cannot be determined from the local geometry. The set $\{u = 0\}$ is important, and related to the horizon - introduced in later lectures.

   References: [A7, §2], [Be, Ch.4], [Sch]

5. **Curvature Integrals II: Scalar curvature.**
   Turn to integrals involving scalar curvature s which are bounded below, so can try to seek minima. There are several reasons for preferring such integrals (see e.g. discussion in [A8, §2]), but one reason is the difficulty of understanding the global nature of the potential u in the case of the Einstein-Hilbert action, (horizon problem).

   Thus, turn to integrals of the form $\mathcal{S}^{2}, \mathcal{S}^{2}_{-}, \mathcal{S}^{p}$, the $L^{2}(L^{p})$ norm of s or of $s^{-} = \min(s, 0)$. Go through the proof that the infimum of such functionals equals $|\sigma(M)|$, when $\sigma(M) \leq 0$, [An8, Prop.3.1]. Restate Conjectures I-II in terms of minimizing $\mathcal{S}_{-}^{2}$. (Conjecture III must be done via the Einstein-Hilbert action).

State the Euler-Lagrange equations for $\mathcal{S}^2$ and $\mathcal{S}_{-}^{2}$ - 4th order. All critical points of $\mathcal{S}_{-}^{2}$ are constant curvature, [A8, Lem.1.3]; this is unknown for $\mathcal{S}^2$ when the scalar curvature changes sign. Relation of these equations with that for Einstein-Hilbert action. The (now local) potential $-\frac{s^-}{\sigma}$, (or $-\frac{s}{\sigma}$), in the Euler-Lagrange equation corresponds to u from Lecture 4.

Discuss: It is very difficult to minimize scalar curvature integrals - one has very little control on the behavior of a general minimizing sequence. Need to find a "controlled" minimizing sequence.

References: [A8, §1,2], [Be, Ch. 4]

6. **Curvature Integrals III: Full curvature.**
   Gromov's Minimal Volume; minimizing $|R|_{L^\infty}$ on unit volume metrics. Minimizers are complete $C^{1,\alpha}$ Riemannian manifolds, but in general might not be smoother. (No good Euler-Lagrange equation for critical points).

   Consider instead $\mathcal{R}^2$ - the L^2 norm of the Ricci (or Riemann) curvature. Existence of minima in dimension 3, geometric decomposition of M w.r.t. $\mathcal{R}^2$ (and related curvature integrals). State the Euler-Lagrange equations - an elliptic 4th order system. Structure of minimizers - C^∞ regularity, completeness of minimizers, graph manifold structure of ends.

   The geometric decomposition w.r.t. $\mathcal{R}^2$ does *not* give geometrization of M in general. In particular, this is the case on reducible 3-manifolds, $M = M_1 \# M_2$, M_i hyperbolic.

   Need to pass to scalar curvature functionals, regularized by perturbing them in the direction of the L^2 norm of curvature.

   References: [A3, §3-§5], [Gr2], [BCG], [Bes]

7. **The functional $I_{\varepsilon}{}^{-}$ and Minimizers** $(\Omega_\varepsilon, g_\varepsilon)$. Introduce and motivate the basic functional

$$I_{\varepsilon}{}^{-} = \varepsilon v^{1/3} \int |z|^2 + (v^{1/3} \int (s^-)^2)^{1/2},$$

$s^- = \min(s, 0)$, s the scalar curvature and z the trace-free Ricci curvature, $\varepsilon > 0$.

Existence of minimizers and the geometric decomposition $(\Omega_\varepsilon, g_\varepsilon)$ w.r.t. $I_\varepsilon{}^-$. Basic properties of minimizers - state the Euler-Lagrange equations, regularity, completeness. $I_\varepsilon{}^-(g_\varepsilon) \to |\sigma(M)|$ as $\varepsilon \to 0$, when $\sigma(M) \leq 0$. Behavior on graph manifolds.

The main issue now is to understand the behavior of

$$(\Omega_\varepsilon, g_\varepsilon), \quad \text{as} \quad \varepsilon \to 0.$$

This gives a family of metrics, in ε, forming a minimizing family for $\mathcal{S}^2_-$ as $\varepsilon \to 0$; an optimal such family. Although the full EL equations are complicated, emphasize the importance and form of the trace equation, an elliptic equation for the potential u. Idea is to carry out analysis on the scalar curvature, as a potential *function*.

Discuss the basic estimates on the potential u (or $\tau = -u$) in [A8, Thms.3.11,3.12], showing that $u \to const$, (or $u \to 1$), almost everywhere; discuss also [A8, Prop. 3.13].

References: [A8, §3].

8. **Proof of Conjectures I-II for Tame 3-Manifolds.**
Definition of tame 3-manifolds. M^3 tame implies that

$$\rho(x) \geq \rho_o,$$

for all $x \in (\Omega_\varepsilon, g_\varepsilon)$ as $\varepsilon \to 0$, where $\rho_o = \rho_o(M)$ depends only on M, c.f. [A9, Lem.1.5]. Sketch basic idea in proof of this; (details require work in later lectures).

Apply the L^2 Cheeger-Gromov theory. As $\varepsilon \to 0$, metrics either converge to compact limit metric on M, form cusps in M, or collapse. Smoothness of convergence to limits. All limits (compact or cusps) are of constant curvature. Discuss main ideas in proofs of Props. 2.1-2.5 of [A9], but avoid technicalities in proof of [A9, Prop.2.7].

Go through proof that tori in cusps are incompressible in M in some detail; this is the analogue of the Thurston cusp closing theorem. Discuss the geometrization of graph manifolds, via the geometry of the collapse.

References: [A9, §2], [T1,2], [Gr1].

Lecture 8 explains the occurence of tori in the decomposition of M into geometric pieces. The remaining lectures address curvature blow-up and the occurence of essential 2-spheres.

9. **Static Vacuum Einstein Equations in Relativity.**
These equations on a 3-manifold N are

$$L^*u = 0, \quad \Delta u = 0,$$

and give a vacuum solution ($Ric = 0$) to the Einstein equations on the warped product $\mathbb{R}\times N$, $d\theta^2+u^2g_M$; (the signature Riemannian/Lorentzian makes no difference here).

The main (canonical) solution is the Schwarzschild metric on $N = \mathbb{R}^+ \times S^2$. (Note the S^2!) Discuss the basic geometry of this metric. State and discuss the black hole uniqueness theorem for static vacuum solutions. Emphasize the importance of the hypotheses: smoothness up to horizon, asymptotically flat. Many examples of solutions not smooth up to horizon and/or not AF, (Weyl solutions).

State the generalized Lichnerowicz theorem [A6, Thm.0.1], for static space-times and in general for stationary space-times. Sketch proof in the special case where the potential u is bounded above or below via Yau's Liouville theorem [Y] in [A7, Thm.A.1]. State also the corresponding apriori curvature bound, [A6, Thm.0.2], [A7, Thm 3.3] and related estimates in [A8, Remark 3.15], [A10, Prop.1.1].

References: [A6], [A7], [EK], [BM].

10. **Introduction to Curvature Blow-up.**
Here the issue is to understand the behavior of sequences of metrics where the curvature becomes unbounded.

Rescalings of a metric or sequence of metrics, simple examples, role of base points and diffeomorphisms, (convergence in pointed Gromov-Hausdorff topology). Rescaling by size of curvature to make curvature bounded - so can apply Cheeger-Gromov theory to obtain limits. Observe that large rescalings model degeneration of metrics on small scales. Need for strong convergence/regularity boost to obtain anything meaningful; the condition $|R| = 1$ is not preserved in limits, (loss of regularity), without this.

Possibility of multiple scales of curvature blow-up, resulting incompleteness of limits and problems with obtaining limits when work with L^{∞}. Use instead the L^2 curvature radius ρ to obtain limits within ρ-balls, i.e rescaling of ρ to 1. Unwrapping in covering spaces in case of collapse.

Discuss the model of a sequence of metrics crushing the essential 2-sphere to a point in $M \# M$; the rescalings or blow-ups converge to the (isometrically doubled) Schwarzschild metric, cf. [A7, Ex.6.2]. Discuss also [A7, Prop.3.1] - blow up limits of Yamabe metrics are solutions of the static vacuum Einstein equations.

References: [A7, §1,3,6], [A8, §0,2,4].

11. **Structure of Blow-up Limits I.**
Behavior of Euler-Lagrange equations for $(\Omega_{\varepsilon}, g_{\varepsilon})$ under blow-ups: $\alpha = \varepsilon/\rho^2$. The three limits,

static vacuum Einstein equations, $\mathcal{Z}_c^2$ equations, $\mathcal{Z}^2$ equations,

according to behavior of α. Geometric meaning of the $\mathcal{Z}_c^2, \mathcal{Z}^2$ equations or minimizers. State regularity/smoothness of convergence to limits, (strong convergence); this requires use of cutoff s^- in $I_{\varepsilon}{}^-$ as opposed to I_{ε}. Possible incompleteness of limits unless work at maximal scale.

Discuss model example of crushing S^2 : limit is $\mathcal{Z}_c^2$ solution. Describe the Schwarzschild metric also as a $\mathcal{Z}_c^2$ solution. Multiple end examples.

References: [A8, §4, §6]

12. **Structure of Blow-up Limits II: Non-Existence Results.**

State and discuss following results, indicating basic ideas in proofs, or proofs in special cases:

• No complete static vacuum solutions, (from Lecture 9).

• No complete $\mathcal{Z}^2$ solutions. Show this leads to existence of a maximal blow-up scale or degree of curvature concentration (α bounded above).

• No complete $\mathcal{Z}^2_c$ solutions with free S^1 action. Show this leads to convergence to limits, when limits are complete $\mathcal{Z}^2_c$ solutions, (no collapse).

Combine these results to state [A8, Theorem B]; blow-up limits of $(\Omega_\varepsilon, g_\varepsilon)$ at maximal scale exist and are complete $\mathcal{Z}^2_c$ solutions.

Properties of these limit blow-ups: $s \geq 0$, minimize $\mathcal{Z}^2$ among comparison metrics with $s \geq 0$ and larger volume.

References: [A4], [A8, §5,0].

13. **Asymptotically Flat Manifolds I.**

Give the precise definition of an asymptotically flat (AF) end. AF blow-up limits. The mass term m is given by $m = lim_{r\to\infty} \int_{S(r)} |du|$, where u is the potential, (when $u > 0$ on the end). State the Sphere Conjecture of [A9]: there exists a complete $\mathcal{Z}^2_c$ blow-up limit which has an AF end.

Go through the proof of the result that the Sphere Conjecture $\Rightarrow$ Conjectures I, II in some detail; this is a cut and paste or comparison argument similar in spirit to the incompressibility of tori in Lecture 8. Thus, relate curvature blow-up to existence of essential 2-spheres in M.

Conclusion: Conjectures I and II follow if can find an AF end of a complete $\mathcal{Z}^2_c$ solution, (blow-up limit).

References: [A9, §3]

14. **Asymptotically Flat Manifolds II.**

State and discuss the generalized black hole uniqueness theorem [A6, Thm.0.3]; ends of static vacuum solutions, with compact (or pseudo-compact) horizon, are either AF or parabolic (collapsing at ∞). Discuss some of the main ideas of the proof. This result and proof is background for [A8, Theorem C]. State and discuss this theorem, provide examples.

Explain difficulties in proceeding. The main issue is the compactness of the level sets of the potential u. This is hard to establish. As formal analogues of $\mathcal{Z}^2_c$ solutions, discuss examples of Weyl solutions (of the static vacuum Einstein equations), with non-compact horizon and so non-compact levels of the potential.

References: [A6], [A8]

15. **σ-Tame Manifolds.**

Definition of σ-tame. State and discuss the Main Theorem of [A10], and then describe some of the main ideas in the proof - see in particular §0, §1 and the Remarks in [A10]. Thus, one wants to find base points y_ε and scales $\lambda_\varepsilon = \rho(y_\varepsilon) \to 0$ such that the rescaled metrics $g'_\varepsilon = \rho(y_\varepsilon)^{-2} \cdot y_\varepsilon$ based at y_ε converge to a limit incomplete *flat* manifold F, with some compact "separated" boundary components. The $\mathcal{Z}^2_c$ equations are replaced by their linearizations at the flat metric. There are 2-spheres in F surrounding such separated boundary components, analogous to 2-spheres in AF ends, and one proves these are essential as in Lecture 13.

To carry this out, one needs to study in detail the value distribution theory/potential theory of u; upper and lower mass estimates in Theorem 3.5 and Corollary 5.1 of [A10].

References: [A10]

16. **Geometrization and Long-time Evolution in Relativity.**

This is another (4th) approach toward geometrization, probably the most difficult, but possibly also the most interesting. The idea is to analyse the long-time behavior of space-like (constant mean curvature) hypersurfaces M^3 in a vacuum space-time M^4. Here the evolution is by the Einstein flow, a non-linear hyperbolic system. The asymptotic behavior to the future may lead to the geometrization of M^3. Essential spheres in M may be related to black holes in the space-time.

References: [A11, §0], [FM1,2], [Re]

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Participation:

The idea of the Arbeitsgemeinschaft is to learn by giving one of the lectures in the program.

If you intend to participate, please send your full name and full postal address to *both* organizers at

`anderson@math.sunysb.edu`   `lohkamp@math.uni-augsburg.de`

by July 12, 2002, at the latest.

You should also indicate which talk you are willing to give:
First choice: talk no. ...
Second choice: talk no. ...
Third choice: talk no. ...

You will be informed shortly after the deadline if your participation is possible and whether you have been chosen to give one of the lectures.

The Arbeitsgemeinschaft will take place at Mathematisches Forschungsinstitut Oberwolfach, Lorenzenhof, 77709 Oberwolfach-Walke, Germany. The institute offers accomodation free of charge to the participants. Travel expenses cannot be covered. Further information will be given to the participants after the deadline.