Mathematisches Forschungsinstitut Oberwolfach

Report No. 27/2003

Mini-Workshop: Groupoids and Stacks in Physics and Geometry

June 29th – July 5th, 2003

This meeting was organized by Alberto Cattaneo (Zurich) and Ping Xu (Penn State). It focused on a variety of topics on groupoids and stacks and their applications in mathematical physics, bringing together mathematicians and mathematical physicists working on symplectic geometry, Lie groupoids and string theory.

The aim of the workshop was to create interaction between researchers from different areas and to bring young researchers to this new subject. There were many lively discussions after the talks. The talks given at the meeting covered the following topics:

(1) Poisson sigma modules and non-linear gauge theory.
(2) Groupoids and deformation quantization.
(3) Lie groupoids and differential stacks.
(4) Gerbes and quantum field theory.
(5) Twisted K-theory.

Abstracts

Integration of twisted Dirac brackets and presymplectic groupoids

Henrique Bursztyn

We extend the correspondence between Poisson manifolds and symplectic groupoids to the realm of [twisted] Dirac structures.

Coisotropic submanifolds in Poisson geometry and branes in the Poisson sigma model

Alberto S. Cattaneo

Coisotropic submanifolds play a fundamental role in symplectic geometry as they describe systems with symmetries (”first-class constraints”) and provide a method to generate new symplectic spaces (”symplectic reduction”).

Their generalizations to Poisson manifolds also carry naturally induced foliations such that the leaf spaces are again Poisson. They are the general framework to study symmetries in the Poisson world.

The Poisson sigma model is a topological string theory having a Poisson manifold as its target. Its path-integral quantization (with suitable boundary conditions) yields Kontsevich's deformation quantization of the target Poisson manifold.

Coisotropic submanifolds of the target Poisson manifold turn out to label the possible boundary conditions (”branes”) of the Poisson sigma model, the previous case corresponding to choosing the whole manifold as its coisotropic one.

The aim of this talk is to describe work in progress on the role of coisotropic submanifolds in the classical, Hamiltonian version of the Poisson sigma model and to describe how this is related to ”dual pairs” (the morphisms of a category whose objects are Poisson manifolds and that seems to be natural if one has quantization in mind). This generalizes the construction of the symplectic groupoid of a Poisson manifold (which corresponds again to choosing the coisotropic submanifold to be the whole Poisson manifold).

Some properties of the Seiberg-Witten map for a time-dependent background

Bianca Letizia Cerchiai

In this talk the Seiberg-Witten map for a time-dependent background is discussed. The algebra studied here is related through T-dualities and a twist to a null-brane orbifold and it is an example of the Lie algebra type, as the noncommutativity parameter is linear in the coordinates. The equivalence map between the Kontsevich star product for this background and the Weyl-Moyal star product for a background with constant noncommutativity parameter is also calculated. The method used to solve the Seiberg-Witten equations is cohomological. It is based on the introduction of a BRST operator and ghosts. This allows the determination of a coboundary operator and then of the corresponding homotopy operator. It would be interesting to generalize this approach to a BV formalism and study its relation to the underlying algebroid structure.

Courant Algebroids

Alexander Chervov
(joint work with Paul Bressler)

This paper is devoted to studying some properties of the Courant algebroids: we explain the so-called "conducting bundle construction" and use it to attach the Courant algebroid to Dixmier-Douady gerbe (following ideas of P. Severa). We remark that WZNW-Poisson condition of Klimcik and Strobl (math.SG/0104189) is the same as Dirac structure in some particular Courant algebroid. We propose the construction of the Lie algebroid on the loop space starting from the Lie algebroid on the manifold and conjecture that this construction applied to the Dirac structure above should give the Lie algebroid of symmetries in the WZNW-Poisson σ-model, we show that it is indeed true in the particular case of Poisson σ-model.

Relations between symplectic groupoid and deformation quantization

Benoit Dherin

Abstract: We start considering the trivial symplectic groupoid over $\mathbb{R}^d$ associated to the null Poisson structure. Its multiplicative structure can be expressed in terms of a generating function. We address the problem of deforming such a generating function in direction of a non-trivial Poisson structure so that the multiplication remains associative. We prove that such a deformation is unique under some reasonable conditions and we give the explicit formula for it. This formula turn out to be the semi-classical approximation of Kontsevich's deformation quantization formula. For the case of a linear Poisson structure, the deformed generating function reduces exactly to the BCH formula of the associated Lie algebra. The methods used to prove existence are interesting in their own right as they come from an at first sight unrelated domain of mathematics: the Runge-Kutta theory of the numeric integration of ODE's.

Why stacks maybe aren't the right objects to put strings on

Andre Henriques

I will try to compare stacks (= orbifolds) and non-commutative ringed spaces. The goal is to show that some constructions don't make sense for the first ones and do make sense for the second ones. In particular, I'll talk about crepant resolutions of surface singularities. This work was inspired and follows ideas of David Berenstein.

Characteristic classes for principal G-bundles over groupoids

Camille Laurent

The notion of a principal bundle over a groupoid generalizes the notion of a principal bundle over a manifold and that of a group homomorphism. It also includes equivariant principal bundles and principal bundles over an orbifold. In this talk, we will discuss the Chern-Weil construction of characteristic classes in this general context, which unifies various constructions of Chern-Weil maps including a recent one due to Bott-Tu on Chern-Weil map of equivariant principal bundles.

What is a quantum gerbe?

Jouko Mickelsson

I reviewed how gerbes occur in quantum field theory. In particular, the case of fermions in external gauge fields and with varying boundary conditions was discussed in detail. When the gauge group is a compact Lie group G the gerbe comes from the loop group bundle over G, with total space equal to the space of smooth vector potentials on the circle, assuming that the physical space is the circle; there are corresponding constructions in higher dimensions. The Dixmier-Douady class of the gerbe can be computed using families index theo rem. In the case of a circle and $G = SU(n)$ it is simply an integer k times the basic class in $H^3(SU(n), Z)$. The integer k is related to the chiral anomaly . For chiral fermions the gauge symmetry is broken which leads to an extension of the group of gauge transformations. In the 1-dimensional case this extension is an affine Kac-Moody group and the integer k defining the DD class is equal to the level of the representation of the affine Kac-Moody group.

The case of supersymmetric Wess-Zumino-Witten model was also discussed. The quantization leads again to a loop group gerbe, which is that data defining a twist in the K-theory class given by supersymmetry generators. There should be connection to the Freed-Hopkins-Teleman theorem on equivalence of equivariant twisted K-theory on G and the Verlinde algebra.

Finally, I discussed some ideas about gerbes on quantum groups.

Bundle 2-gerbes

Danny Stevenson

There has been recent interest in constructing geometric objects, so called 'p-gerbes' or 'p-line bundles' which provide a geometric realization for classes in $H^{p+2}(M; Z)$. 2-gerbes are category theoretic objects introduced by Breen in his study of a degree 3 non-abelian sheaf cohomology. For the case p=2 Brylinski and McLaughlin studied a certain class of 2-gerbes and developed differential geometric notions such as connections and curvature. In this talk I shall introduce another class of geometric objects called 'bundle 2-gerbes' which also realize classes in $H^4(M; Z)$. I shall also describe a theory of connections and curvature for these objects and relate them to the 2-gerbes of Brylinski and McLaughlin.

Lie algebroid homomorphisms and gravity

Thomas Strobl

Given a Lie algebroid E, we consider topological theories where the set of classical solutions are or contain Lie algebroid morphisms from T Sigma to E up to homotopy. Here Sigma is some arbitrary manifold, which we want to regard as spacetime. We clarify the additional structures needed so that one can define what we call a reasonable theory of E-gravity on Sigma. It is identified with an E-Riemannian foliation (a generalization of a Riemannian foliation, which is needed if E is some standard Lie algebroid). Known 2d and 3d gravity models are particular examples of the much more general framework.

Twisted K-theory of Differentiable Stacks

Jean-Louis Tu

We develop twisted K-theory for stacks, where the twisted class is given by an S^1-gerbe over the stack. General properties, including the Mayer-Vietoris property, Bott periodicity, and the product structure $K^i_\alpha \otimes K^j_\beta \to K^{i+j}_{\alpha+\beta}$ are derived. Our approach provides a uniform framework for studying various twisted K-theories including the usual twisted K-theory of topological spaces, twisted equivariant K-theory, and the twisted K-theory of orbifolds. We also present a Fredholm picture, and discuss the conditions under which twisted K-groups can be expressed by so-called "twisted vector bundles". Our approach is to work on presentations of stacks, namely *groupoids*, and relies heavily on the machinery of K-theory (KK-theory) of C^*-algebras.

S^1-gerbes over differential stacks

Ping Xu

In this talk, I will review some basic construction of S^1-gerbes over differential stacks from the view point of groupoids. In particular, we will discuss a geometrical construction of Dixmier-Douady class in terms of pseudo-connections and pseudo-courvatures. As an example, we we present, for a compact simple Lie group G, an infinite dimensional model of S^1-gerbe over the differential stack G/G whose Dixmier-Douady class corresponds to the canonical generator of the equivariant cohomology $H^3_G(G, Z)$. Applications to mementum map theories are disscussed. In particular, this yields a pre-quantization of q-Hamiltonian spaces in the sense of Alekseev-Malkin-Meinrenken.

Twisted equivariant K-theory for simple Lie groups and Verline algebras

Bin Zhang

In this talk I will discuss the geometric construction of twisted equivariant K-theory for simple Lie groups. I will also discuss my understanding of the connection between twisted equivariant K-theory and Verlinda algebras (after Freed).

Edited by Ping Xu

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