Mathematisches Forschungsinstitut Oberwolfach

Report No. 28/2003

Mini-Workshop: Henri Poincaré und die Topologie

June 29th – July 5th, 2003

The main aim of this mini-workshop was to survey Poincaré's work on topology and its influence (Krömer, Robadey, Scholz, Volkert); this got an unexpected actuality by the proof of Poincaré's conjecture (by Perelman) which is broadly discussed in these days.

Related themes like algebraic geometry (Brigaglia), projective geometry (Nabonnand) and the theory of the continuum (Cantu) were also treated. Another important topic of the work ship was Poincaré's role in the (pre)-history of relativity theory (Gray, Rowe, Walter) and his contributions to theoretical physics in general (Mawhin).

Further on we discussed the state of the art of the publication of Poincaré's correspondence with mathematicians, in particular the on-line publication of his letters (Walter).

During this mini workshop it became clear that it is a fascinating task to integrate the numerous facets of Poincaré's life and work, including his role as one of the leading scientists of his time writing also for a general public, into a more complete and detailed picture.

Three evening talks on themes of general interest (Böttcher, Mawhin, Volkert) were presented to round up our program.

The organizers want to thank all participants of the mini-workshop for their engagement in talks and discussions, in particular F.Böttcher for preparing this "Tagungsbericht", the administration and the staff of the MFO for the excellent conditions of our stay there and the participants of the mini-workshop on "exotic homology spheres" for the interesting joint sessions of our two mini-workshops.

Jean Mawhin, Philippe Nabonnand, Klaus Volkert

Abstracts

From Poincaré to Lefschetz via Italian Geometers: The Impact of Algebraic Topology into Algebraic Geometry

Aldo Brigaglia

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Veronese and his Theory of Continua

Paola Cantu

Having proved that the Axiom of Archimedes was implied by the principle of continuity, Giuseppe Veronese built in 1891 a non-Archimedean geometrical continuum using synthetic means. The existence of a system of linear quantities containing infinitely small as well as infinitely great quantities was heavily criticized by Cantor, Vivanti, Schoenflies, though it was readily accepted by Stolz, Bettazzi, Hilbert. This paper aims at showing that the mathematical theory of continuity and the geometry of hyperspaces contained in Veronese's main work *Fondamenti di geometria* were strongly influenced by his "philosophical" conceptions of space and intuition. Geometry is considered as a science that is neither experimental as physics nor abstract as mathematics: its objects are partly abstracted from experience and partly produced by thought; its premises are empirical, half-empirical and abstract. A four-dimensional space and a non-Archimedean continuum can be intuited, though not empirically. They can be intuited through an abstract intuition that one develops with time, experience and geometrical practice.

A Talk based on Gray's review of Galison's New Book: *Poincaré's Map and Einstein's Clocks*

Jeremy Gray

Galison's forthcoming book will undoubtedly generate much debate. I gave a summary of its arguments, traced the various 'intersecting arcs' upon which Galison situates both Poincaré and Einstein, and discussed some of the merits and problems of the book. Among the merits are a rich description of Poincaré's work at the Bureau des Longitudes. Among the difficulties is the refusal to attach weights to the various factors supposed to be at work. Influence, significant influence, is suggested by the weight of information provided, but it is not rigorously argued for, merely offered in juxtaposition to other factors. Nor am I convinced that the operationalisation of time is the radical departure from Newtonian time that Galison suggests, and the quick treatment of relativistic time seems to me to be part of the rhetorical devices used to promote his conclusions.

The full review of the book will appear in Nature in the autumn of 2003.

The roots of category theory in algebraic topology

Ralf Krömer

It is well known that the roots of category theory lay in algebraic topology; the issue of the present contribution is to examine closer these roots. Starting up with the intention of the category concept, there is coming the idea to ones mind that category should have emerged of some study of arrows. There was indeed a strong tendency to study mappings from the early 30ies on (Lefschetz fixed point theorem, What are the homotopy classes of mappings between given spaces?). Part of the reason to introduce the concept of homology group (replacing the numerical invariants in original combinatorial topology) was that study of mappings, and indeed so because a mapping of one space into another induces a group homomorphism between the corresponding homology groups. One more historical reason for the introduction of the concept of homology group was the intent to have a homology theory for general spaces; there one had to introduce homology groups because numerical invariants didn't work, the resulting groups being not necessarily finitely generated. This said, one might conjecture (and the folklore history indeed suggests) that the homology functor was the construction which led Eilenberg-Mac Lane to introduce the functor concept in general. But this official history is not altogether correct because Eilenberg-Mac Lane actually did introduce the concept of functoriality (in the 1942 paper "Group extensions and homology") in a slightly different context: They wanted to express homology with respect to infinite cycles and an arbitrary coefficient group G by expressions involving G and cohomology with respect to finite cycles and integer coefficients. In modern terms, their result for complexes K reads

$$0 \to \mathrm{Ext}(H_{q+1}(K,\mathbb{Z}),G) \to H^q(K,G) \to \mathrm{Hom}(H_q(K,\mathbb{Z}),G) \to 0,$$

(the sequence being exact), so the group on the right is the quotient of the homology group in question and the group on the left (called the group of group extensions of G by H_{q+1}). It was in the tentative to transfer this result from complexes to spaces that they became interested in functoriality of Hom and Ext (because the passage to the Cech-type homology and cohomology theories of the space involves refinements of coverings and transformations of the corresponding "nerve" complexes). At no place in this first paper is there mention of functoriality of (Co-)Homology (i.e. the induced mappings between homology groups of different spaces are not even mentioned); the only thing that interests are functors from groups to groups (the constructions Hom and Ext and their group-theoretical relation to free groups), the results being applicable to homology because of the fact that homology groups are free. It is for this reason that in the subsequent paper (which was the announcement of a longer paper, the 1945 paper on category theory) the functor concept is only defined from Groups to Groups, not covering Homology.

The Foundation of Mechanics before and after Poincaré: Belgian Reactions to the Foucault's Pendulum Experiment

Jean Mawhin

In 1851, Foucault made his famous experiment of the pendulum to "demonstrate" the rotation of the Earth. Because of the relative character of motion, the interpretation of this experiment was somewhat delicate, as observed already by Mach and Duhamel. This lead to a first controversy between the Belgian mathematicians Joseph-Marie De Tilly and Paul Mansion.

Around the beginning of the twentieth century, Poincar discussed the principles of mechanics, following and developing the arguments of Mach and Duhamel. This viewpoint was strongly defended in Belgium by Ernest Pasquier.

New demonstrations in Paris and in Brussels, to celebrate the fiftieth anniversary of Foucault's experiment, were the occasion of new discussions and polemics, in particular between Lucien Anspach and Ernest Pasquier. Surprisingly, the arguments of Anspach, from the Universit Libre de Bruxelles, were definitely less rational than those of Pasquier, from the Universit Catholique de Louvain.

Le théorème fondamental de la géométrie projective et la continuité
Phillippe Nabonnand

v. Staudt was the first to state the fundamental theorem of the projective geometry. Unfortunately, his proof was incomplete as Klein and others pointed out. Klein's aim was to found non-Euclidean and Euclidean geometry on the projective geometry. So, he needed that projective geometry was itself founded independently of Euclidean geometry and rigorously. Particularly, Klein tried in 1873 to correct this "Lücke" in v. Staudt's proof in introducing a new axiom of continuity that requests the continuity of the projective domains and the continuity of the sequence of the harmonic element. This tentative was the beginning of a stream of works about the fundamental theorem of the projective and the axioms of continuity in projective geometry. For example, Lüroth and Zeuthen proposed a proof that only needs the continuity of projective domains. In 1880, Darboux proved that projective transformations of the real projection geometry (that preserve harmonic groups) are necessarily continuous (Darboux'proof was also not complete and Klein corrected it). The main goal of this talk is to show that it is false to show that Klein said many mistakes concerning this question, as many historians of mathematics say, and that, on the contrary, Klein's contributions were relevant and fruitful.

The Link between Analysis Situs and Geometrisation in Poincaré's Writing
Anne Robadey

In his thesis on Poincaré's memoir "Sur les courbes définies par une équation différentielle" (1881), C. Gilain suggests that the geometrisation of the problem by Poincaré is not a mere geometric illustration of analytic properties. Rather, the geometric representation plays a theoretical role which issues in the explicit introduction of Analysis situs in the 3rd and 4th parts of the memoir.

This analysis seems to me to be relevant in the study of an other paper by Poincaré, and I have presented in my talk some new evidence of the importance of the geometrisation in Poincaré's use of Analysis situs. My study concentrated on a few pages of Poincaré's "sur les lignes géodésiques des surfaces convexes" (1905), and I showed how, there too, the geometrisation is deeply associated with the use of a result of Analysis situs. This link is manifested in the very form of Poincaré's writing, and its significance lies in the fact that this occurs in a proof which is mainly an analytical proof, where the topological result is only quoted at the end to complete the argumentation, and could have been used without the accompanying geometrisation of the problem.

So topology really appears in these texts as a branch of Geometry, and the use of topological arguments seems to be subordinated, in Poincaré's practice here, to a preliminary geometrisation of the problem.

Poincaré's Views on the Principle of Relativity and "Poincaré's Silence"
David Rowe

In connection with the principle of relativity, Poincaré never mentioned Einstein's name. Nor did Einstein refer to any of Poincaré's works even though he was familiar with his discussion of the principle of relativity in Science and Hypothesis (1902), a work that was discussed in the "Olympic Academy" in Bern before Einstein published his famous paper of 1905. Afterward mathematicians like Minkowski, Hilbert, and Weyl took a leading role in promoting relativity theory before it caught fire in November 1919. Poincaré, who independently took significant steps in the direction of Einstein's and Minkowski's ideas, remained curiously silent about their work even after others like Max Planck were proclaiming its revolutionary significance. Shortly after Minkowski's death in January 1909, Poincaré delivered the first series of Wolfskehl lectures in Göttingen. His closing lecture on the new mechanics dealt with the possibility that mass increases with velocity thereby making it impossible to accelerate bodies beyond the speed of light. Nevertheless, citing Laplace's argument that gravitation must necessarily propagate at a far greater velocity than c, he concluded that it would be premature to declare that classical physics was dead. Young Hermann Weyl was unimpressed by Poincaré's performance and looked forward to the visit of H. A. Lorentz, who was scheduled to be the second Wolfskehl lecturer. Some years after Poincaré's death in 1912, Felix Klein remembered Poincaré's silence and speculated that he might have merely been returning the favour, since Minkowski had ignored Poincaré's contributions in his famous Cologne lecture "Raum und Zeit." After 1919 relativity became almost exclusively associated with Einstein's name, which had a polarizing effect on many Germans. The representatives of "Deutsche Physik" clamoured for action when Hitler seized power and with only mild protests (only Max von Laue resisted forcefully) the theory of relativity and its author were quickly condemned as anti-German. Wilhelm Lenz, a student of Sommerfeld, tried to rehabilitate relativity by calling attention to Poincaré's priority regarding several of its key concepts. This effort to "Aryanize" the theory failed, of course.

The Concept of Manifold in and around the Work of Poincaré

Erhard Scholz

Poincaré introduced manifolds in a collection of methods; he gave two definitiions complementing each other (regular zero-sets, local parametrization with "analytical" continuation) and used several methods of construction, the most important of which used(locally regular) cell complexes. It was discussed how Poincaré worked with this methods going slightly beyond the formal rules stated explicitly.

In the second part of the talk the rise of three different lines in the development of the manifold concepts in the early 20th were presented:

– the beginnings of the axiomatic appraoch to the manifold concept (Hilbert 1902, Hausdorff 1912/1914, Weyl 1913, H.Kneser 1926)

— the combinatorial approach from (Dehn/Heegard 1907, Tietze 1908, Steinitz 1908, Weyl 1923ff.) to the "h-manifolds" of van Kampen, Vietoris e.a. at the end of the 1920s,

— Brouwer's simplicial (later: PL-) approach of 1911ff.

The debate in the late 1920s on the different approaches to understand manifolds was related to the broader foundational issue of "continuum", in particular van der Waerdens review of the situation at the DMV meeting of 1929. The short story of the first phase of the elaboration of the concept of manifold ended with O. Vebeln and J.H.C Whitehead's axioms of manifolds of different structures, characterized by a groupoid of regular coordinate transformations.

Poincaré's Way to his Homology Spere

Klaus Volkert

In my talk I discussed Poincaré's work on the classification of closed (mostly orientable) three-manifolds beginning with an announcement of his great paper (of 1895) on "Analysis situs" in 1892. The first stage in this developpment was the introduction of the fundamental group in 1892/1895 and the construction of a series of examples with same Betti-numbers but non-isomorphic fundamental groups (the so-called sixth example in the paper of 1895 - the result is announced in 1892 with a false Betti-number in dimension 2). The way in which this series is constructed (as homogeneous spaces belonging to a sub-group of affine mappings of ordinary three-space) was a technique known to Poincaré from his work on automorphic functions in the beginnings of the 1880's. In establishing the fact that the fundamental groups are not isomorphic Poincaré had to do a considerable work in combinatorial group as well. There raised a question: Is the fundamental-group strong enough to determine a closed 3-mainfold from the point of view of analysis situs?

The first and the second complement of the Analysis-situs-series (1899/1900) are devoted to the development of the technique of incidence-matrizes. Here Poincaré paid attention to the torsion elements (by the way he proved the fundamental theorem on finitely generated Abelian groups here) and he raised the question whether or not the Betti-numbers plus the torsion-coefficients were sufficent to determine a closed manifold. A closer analysis of the examples he had constructed in 1895 would have convinced Poincaré that the answer is definitely no to this question.

But we had to wait until the fifth complement (1904) in which Poincaré came back to his question. Here he constructed his now so-called homology sphere that is a closed 3-manifold with vanishing homology but with a finite non-trivial fundamental group. The technique by which this example is constructed is that of Heegard's diagrams: Poincaré was the first to use it in a sophisticated way. The fifth complement contains many interesting ideas; for example we find there elements of the later so-called Morse-theory and a detailed study of systems of closed curves on a surface of genus 2. But one question remained open: Is it possible that a closed 3-manifold has a trivial fundamental-group without being a thee-sphere? Questions like that can often be found in Poincaré's papers which were written in the style of an inner dialogue. But this question was particular intriguing: It became known as Poincaré's conjecture being one of the biggest problems in modern mathematics.

More informations can be found in the author's book "Das Homöomorphieproblem insbesondere der 3-Mannigfaltigkeiten in der Topologie 1892 - 1935" (Paris: Kimé, 2002) = Cahier spécial 4 of the journal "Philosophia scientiae".

National and Professional Traditions in the History of Relativity: The Case of Minkowskian Relativity

Scott Walter

Contemporary understanding of the theory of special relativity as a certain constraint on the form of natural law may be traced to Hermann Minkowski's 1908 lecture in Cologne, "Raum und Zeit". Minkowski's view met immediately with hostility and mistrust from physicists, followed by neglect, such that at the time of his death in January, 1909, its success was far from assured. The eventual triumph of Minkowski's view is linked to three factors: (1) the announcement by A. H. Bucherer of measurements confirming the relativistic prediction of the deflection of cathode rays, (2) a broad diffusion of Minkowski's lecture encouraging mathematicians to take up study of relativity theory, and (3) Arnold

Sommerfeld's publication, in 1910, of a 4-dimensional vector formalism. My bibliometric study shows that in the period from 1909 to 1916, research papers using a 4-dimensional approach were primarily the work of theoretical physicists and mathematicians, in almost equal numbers. It is suggested here that this engagement by mathematicians with the theory of special relativity led to the adoption of Minkowski's view of this theory by Einstein and other leading theorists.

Evening Talks

Émilie Du Châtelet (1706 – 1749) - Passion for the Sciences

Frauke Böttcher

In the history of mathematics and sciences female scientists as Laura Bassi, Maria Gaetana Agnesi and Émilie Du Châtelet are considered as the great exceptions. Often their scientific seriousness is questioned. This concerns their personal attitude towards the sciences they were doing and even their scientific works.

For that reason the aim of the talk is to show that Émilie Du Châtelet was a serious learner and scientist of mathematics and natural sciences. She worked in these knowledge fields until her premature death in 1749. Only some days before she died she finished her frensh translation of the *Principiae* of Isaac Newton.

In the talk her attitude towards scientific and mathematical subjects will be illustrated by her biography. Quotations from her correspondance with her scientific teachers Maupertuis and Clairault will present a women eager to learn with great scientific interest. This learning ends up in her book "Institutions des physiques" (1740). In this book she worked out the philosophical position of Leibniz and Wolff as one of the first persons in France.

The importance of the sciences for her life conception and her conception of happyness is transmitted in her texts "Discours sur le bonheur" (1746/47) and the preface of the "Institutions". They will be presented along the following leitmotif: "l'amour de l'étude est de toutes les passions celle qui contribue le plus à notre bonheur." (Du Châtelet "Discours sur le bonheur" 1997 (new edition), S. 52)

Henri Poincaré: a biographical sketch

Jean Mawhin

Born in Nancy on April 29 1854, Henri Poincar died unexpectedly in Paris on July 17 1912. Considered as the first mathematician of his time, his contributions to astronomy, theoretical physics and philosophy of science are of first importance. He is without discussions considered as the father of fuchsian functions, of the theory of chaos and of algebraic topology.

He has discovered independently of Einstein the mathematics of special relativity, and historians of science still debate passionately to know if he has to beconsidered as a co-founder of this theory. no substantial biography of Poincar. The aim of this talk is to give some insight on the personality, the carreer and the main achievements of this exceptional scientist.

Seifert and Threlfall: living in Oberwolfach 1945

Klaus Volkert

In this talk some informations about the lives and the work of Herbert Seifert (1907- 1996) and William Threlfall (1889 - 1949), two well known topologists (cf. their joint textbook "Lehrbuch der Topologie" (1934)), were given - partly based on non-published material out of their "Nachlass ". Further on we studied passages out of Threlfall's diary concerning their stay in Lorenzenhof from september 1944 to summer 1945, in particular concerning the last days befor the end of the war in the Black forest (april 1945). Hopefully more of the material will be published soon (by the Academie of Heidelberg).

Edited by Frauke Böttcher

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