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Dynamische Systeme

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Gegenstand der Tagung waren die neuen Resultate und Entwicklungen im Gebiet der klassischen dynamischen Systeme. Themen waren unter anderem Stabilität und Instabilität von Hamilton'schen Systemen, Arnold Diffusion, Mather Theorie, Fragen der Ergodentheorie und der nicht uniformen Hyperbolizität, Komplexität von Symplektomorphismen, das Newhouse Phänomen, wie auch neue interessante Lösungen des n-Körperproblems der Himmelsmechanik. Weiter das Chord Problem in der Kontakt-Geometrie, periodische Bahnen in Magnetfeldern, und Floer Homologie.

Abstracts

The Chord problem in Contact Geometry

Casim Abbas

Let M be a three dimensional manifold with contact form λ, and let $\mathcal{L}$ be a Legendrian knot. The Chord problem is concerned with the existence of a trajectory $x(t)$ of the Reeb vector field X_λ such that $x(0), x(T) \in \mathcal{L}$ for some $T > 0$ and $x(0) \neq x(T)$. In Hamiltonian mechanics these trajectories are also known as brake–orbits which are trajectories oscillating between two states of the system with zero momentum. In the general case on a contact manifold existence could only be established for very special three manifolds like the three dimensional sphere and often only without the condition $x(0) \neq x(T)$. The subject of this talk was to introduce a new filling method by pseudoholomorphic curves developed by the author which can detect Reeb Chords on more general three dimensional contact manifolds.

Reducible and non-uniformly hyperbolic quasiperiodic Schroedinger cocycles

Artur Avila
(joint work with Raphael Krikorian)

The study of the spectral properties of the Schroedinger equation with quasiperiodic potential in dimension one is intimately related to a dynamical problem: the understanding of a certain family of cocycles (parametrized by the energy) with values in $SL(2, R)$ over a rotation of the circle. Using the wide range of techniques of the field, including complexification, renormalization, and local theory (KAM), we prove the following theorem (which can be seen as the analogue of Lyubich's "regular or stochastic" dichotomy for the quadratic family): if the potential is sufficiently smooth and if the frequency of the rotation satisfies an arithmetic condition of full measure then for almost every value of the energy the cocycle is either reducible (that is, conjugate to a constant) or non-uniformly hyperbolic. This result is connected to a rigidity statement whose proof involves getting "a priori bounds" for renormalization under convenient hypothesis. Among the spectral consequences of this result is a proof of a conjecture of Aubry and Andre on the measure of the spectrum of the Almost Mathieu operator.

Recurrence of generic diffeomorphisms

Sylvain Crovisier

We consider the generic dynamics of C^1 diffeomorphisms over a compact connected manifold M. Two important perturbation lemmas were proven in order to study this problem: Pugh's closing lemma and Hayashi's connecting lemma. We proved with C. Bonatti an other perturbation lemma, a connecting lemma for pseudo-orbits: for any generic diffeomorphism f and any two points x and y in M that may be joint by pseudo-orbits with arbitrarily small jumps, it is possible to perturb f in the C^1 topology so that x and y belong to the same orbit. This result is also true for conservative diffeomorphisms. We have then extended it with M.-C. Arnaud and C. Bonatti to symplectic diffeomorphisms. As a consequence, a generic conservative or symplectic diffeomorphism is transitive (i.e.

has a dense orbit). As an other consequence of this perturbation lemma, the chain recurrent set and the non-wandering set of a generic diffeomorphism coincide. We hope to give a generalization of Smale's spectral decomposition theorem. In this direction, we give a decomposition of the non-wandering set into pieces that are maximal weakly transitive sets and separated by a filtration.

The fixed ends problem and symmetric periodic solutions of the n-body problem

Alain Chenciner

Let $x = (\vec{r}_1, \ldots, \vec{r}_n)$ denote a configuration of n positive point masses in a Euclidean space and let $U(x) = \sum_{i<j} \frac{m_i m_j}{|\vec{r}_i - \vec{r}_j|}$ be the Newtonian potential. Using the kinetic energy metric, Newton's equations read $\ddot{x} = \nabla U(x)$ and are immediately seen to be the Euler-Lagrange equations of the Lagrangian action $\int_{t'}^{t''} (\frac{1}{2}|\dot{x}|^2 + U(x))dt$.

Theorem. *Let $x' = (\vec{r}'_1, \ldots, \vec{r}'_n)$ and $x'' = (\vec{r}''_1, \ldots, \vec{r}''_n)$ be two configurations of n positive point masses in the plane or in space, possibly with collisions. Let $x(t) = (\vec{r}_1(t), \ldots, \vec{r}_n(t))$ be a path of configurations such that $x(t') = x'$ and $x(t'') = x''$. If x is a local minimizer of the Lagrangian action among paths $y(t)$ such that $y(t') = y'$ and $y(t'') = y''$, it is collision-free on the open interval $]t', t''[$.*

This theorem is essentially due to Christian Marchal. His original proof applied only to rule out isolated collisions with a limit configuration. Using ideas of Richard Montgomery, Susanna Terracini and Andrea Venturelli which are worked out in Venturelli's thesis, I was able to complete the proof and presented it in the proceedings of the ICM 03. Marchal's idea is to show that if a minimizer x has a collision, the average of the actions of the members of a well chosen family of modified paths is lower than the original action, which is a contradiction. Through the choice of a fundamental domain, this theorem leads to non-collision assertions for periodic local minimizers of the action under symmetry constraints. Examples of application are the choreographies (Z/nZ action) and the generalized Hip-Hops ($Z/2Z$ action). Davide Ferrario and Susanna Terracini recently generalized Marchal's theorem to paths invariant under group actions which possess the so-called "rotating circle property". This increased significantly the applicability of the theorem. The theorem and its generalization were illustrated by two examples: the $Z/2Z \times Z/4Z$-symmetric Hip-Hop, discovered with Venturelli, and the D_6-symmetric Eight, discovered with Montgomery.

Interacting particles in dispersive domains

Dmitry Dolgopyat

We consider a heavy particle in a box moving under collisions with the particles of the fluid. We describe the effective dynamics of the heavy particle and present several open problems.

Mixing real analytic flows with singular spectrum

Bassam Fayad

Mixing is one of the principal characteristics of stochastic behaviour in dynamical systems. It is a spectral property and in the great majority of cases it is either a consequence of much stronger properties such as K-property or fast correlation decay which imply that the spectrum is Lebesgue. The only previously known examples where mixing was accompanied by singular spectrum were Gaussian and related systems which by their nature do not come from differentiable dynamics. In this talk we will describe geometric constructions of smooth volume preserving mixing systems where we are able to show that the maximal spectral type is singular. More specifically, we have a criterion for singular spectrum based on periodic approximations that is compatible with mixing and our examples can be obtained as analytic reparametrizations of some Liouvillean linear flows on the torus $\mathbf{T}^3$. By Host's theorem, we get by the same token examples of reparametrizations that are mixing of all orders.

Historical account on reparametrizations. In his I.C.M. address of 1954, Kolmogorov raised the problem of understanding the ergodic and spectral properties of reparametrizations of linear flows on $\mathbb{T}^n$, $n \geq 2$. Since then and starting with the work of Kolmogorov himself, this problem has been intensively studied and a surprisingly rich variety of behaviours were discovered to be possible for the reparametrized flows. We say surprisingly because at the time Kolmogorov raised the problem, some strong restrictions on the spectral type of the reparametrized flow were expected to hold, such as absence of mixed spectrum, especially in the case of real analytic reparametrizations (cf. the appendix of Fomin to the Russian version of Halmos' book on ergodic theory).

We denote by R^t_α the linear flow on the torus $\mathbb{T}^n$ given by $dx/dt = \alpha$, where $x \in \mathbb{T}^n$ and α is a vector of $\mathbf{R}^n$. Given a continuous function $\phi : \mathbb{T}^n \to \mathbf{R}^*_+$ we define the reparametrization flow $T^t_{\alpha,\phi}$ by $dx/dt = \phi(x)\alpha$. If the coordinates of α are rationally independent the linear flow R^t_α is uniquely ergodic as well as the reparametrized flow that preserves the measure with density $1/\phi$. Other properties of the linear flow may change under reparametrization. While the linear flow has discrete spectrum with the group of eigenvalues isomorphic to $\mathbf{Z}^n$, reparametrizations with continuous time change ϕ may have a wide variety of spectral properties. This follows from the theory of monotone (Kakutani) equivalence and the fact that every monotone measurable time change is cohomologous to a continuous one. However, for smooth reparametrizations the possibilities are more limited and they depend on the arithmetics of α.

If α is Diophantine and the function ϕ is C^∞ then the reparametrized flow is smoothly isomorphic to a linear flow. This was first noticed by Kolmogorov, and later Herman found sharp results of that kind for the finite regularity case. Kolmogorov also knew that for a Liouvillean vector α a smooth reparametrization could be weakly mixing, or equivalently have a continuous spectrum. Later, Shklover proved existence of analytic weakly mixing reparametrizations of some Liouvillean linear flows on $\mathbb{T}^2$; his result being optimal in that he showed that for any analytic reparametrization ϕ other than a trigonometric polynomial there is α such that $T^t_{\alpha,\phi}$ is weakly mixing. About the same time A. Katok found a general criterion for weak mixing. For a Liouvillean α the reparametrized flow often has a continuous spectrum. Specifically, B. Fayad showed that for any Liouvillean translation flow R^t_α on the torus $\mathbb{T}^n$, $n \geq 2$, the generic C^∞ reparametrization of R^t_α is weakly mixing.

Discrete and continuous spectra are not the only possibilities: B. Fayad, A. Katok and A. Windsor have proved that for every $\alpha \in \mathbf{R}^2$ with a Liouvillean slope there exists a strictly positive C^∞ function ϕ such that the flow $T^t_{\alpha,\phi}$ on $\mathbb{T}^2$ has a mixed spectrum since it has a discrete part generated by only one eigenvalue. They also construct analytic examples for a more restricted class of Liouvillean α. Recently, M. Guenais and F. Parreau achieved analytic reparametrizations of linear flows on $\mathbb{T}^2$ that have an arbitrary number of eigenvalues. They even construct an example of a reparametrization of a linear flow on $\mathbb{T}^2$ that is isomorphic to a linear flow on $\mathbb{T}^2$ with "exotic" eigenvalues, i.e. not in the span of the original eigenvalues. Nevertheless, unlike continuous and discrete spectra, it is possible to formulate some regularity conditions on the Fourier coefficients of ϕ to preclude a mixed spectrum for all α and have the following dichotomy: $T^t_{\alpha,\phi}$ either has a continuous spectrum or is L^2 isomorphic to a constant time suspension.

Reparametrizations and mixing. A. Katok showed that on the torus $\mathbb{T}^2$ a reparametrization having some finite fixed regularity of any linear flow has a simple spectrum, a singular maximal spectral type, and can not be mixing. The latter conclusion was extended by A. V. Kochergin to Lipschitz reparametrizations. The argument is based on a Denjoy–Koksma type estimates which fail in higher dimension as shown by J.-C. Yoccoz in an appendix to his thesis. B. Fayad showed that there exist analytic mixing reparametrizations of linear flows. Recently Kochergin showed that for Hölder reparametrizations of some Diophantine linear flows on $\mathbb{T}^2$ mixing is also possible (oral communication). On another hand, a restricted display of mixing features can lead to pathological examples such as topologically mixing flows with a discrete spectrum.

The mixing examples obtained by reparametrizations of linear flows belong to a variety of slowly mixing systems, also including the mixing flows on surfaces constructed by Kochergin in the seventies, for which the characteristics of the spectrum remain undetermined. Modifying the construction of mixing analytic reparametrizations on $\mathbb{T}^3$ it is possible to maintain mixing while the spectrum is forced to be singular.

On the conjugacy problem for measured foliations on higher genus surfaces

Giovanni Forni

An orientable foliation on an orientable compact surface M is a foliation $\mathcal{F}(\eta) := \eta = 0$ defined by a 1-form η on M with a finite set $\Sigma(\eta)$ of saddle singularities. Two orientable foliations $\mathcal{F}(\eta)$, $\mathcal{F}(\hat\eta)$ are (smoothly) conjugate if there exist a diffeomorphism $\phi : M \to M$ and a strictly positive function $\alpha : M \to \mathbf{R}^+$ such that $\hat\eta = \alpha\, \phi^*(\eta)$. An orientable foliation $\mathcal{F}(\eta)$ is a measured foliation, i.e. it has a smooth transverse invariant measure iff the 1-form η is closed. The space of (orientable) measured foliations can be endowed with a (Lebesgue) measure class via the map into the relative cohomology $H^1(M, \Sigma(\eta); \mathbf{R})$ given by the so-called Katok's fundamental class.

In this talk we outline an approach based on renormalization and on the Nash-Moser implicit function theorem to proving the following result on the local conjugacy problem for measured foliations on surfaces of genus higher than 2:

There exist integers $r >> \gamma > 3$ such that for almost all closed 1-form η and for all $s > r$ there exists a closed local submanifold $N^s(\eta)$ in the space of all 1-forms on M such that the following holds.

(a) The closed 1-form $\eta \in N^s(\eta)$;

(b) for every 1-form $\hat\eta \in N^s(\eta)$ the foliation $\mathcal{F}(\hat\eta)$ is $W^{s-\gamma}$-conjugate to $\mathcal{F}(\eta)$, i.e. the conjugating diffeomorphism ϕ and function α belong to Sobolev spaces of order $s - \gamma$;

(c) the local manifold $N^s(\eta)$ has finite codimension (roughly proportional to s) in the space of all 1-forms on M with a finite set of saddle singularities;

(d) the tangent space $T_\eta N^s(\eta)$ of the manifold $N^s(\eta)$ coincides with the kernel of the space of all basic currents of Sobolev order up to s for the measured foliation $\mathcal{F}(\eta)$;

(e) the local manifold $N^s(\eta)$ is locally invariant under the action by pull-back of the group of diffeomorphisms and the multiplicative action of the group of strictly positive functions on M on the space of all 1-forms.

We remark that if M is the 2-dimensional torus the above result is a classical theorem by Kolmogorov (in the analytic category) and Moser (in finite differentiability). It is in fact one of the simplest applications of the KAM method to dynamical systems. A significant difference between the result for the torus and the above extension to higher genus surfaces is that in the torus case the codimension of the submanifold $N^s(\eta)$ is independent of s (it is in fact equal to 1 for all $s > r$).

On the geodesic flow on surfaces of non-positive curvature

FEDERICO RODRIGUEZ HERTZ

Let S be a surface of non-positive curvature of genus bigger than 1 (i.e. not the torus). We prove that any flat strip in the surface is in fact a flat cylinder. Moreover we prove that the number of homotopy classes of such flat cylinders is bounded. A flat strip is an open set, bounded by two geodesics, where the curvature vanishes.

The theorem is a first step in order to answer the following

Problem. *Call $K = \{x \in S \mid \mathfrak{K}(x) = 0\}$ where $\mathfrak{K}$ is the curvature of g. Let γ be a geodesic in S. Then if $\gamma \subset K$ then γ is a closed geodesic. Moreover there are only finitely many homotopy classes of such geodesics.*

In the talk we gave an outline of the proof of the theorem that uses techniques of complex analysis, and we also discussed some consequences of the theorem and the problem. A preprint can be found in arXiv:math.DS/0301010.

On the Newhouse phenomenon (of infinitely many coexisting sinks)

VADIM KALOSHIN

Consider the space of C^r diffeomorphisms (smooth invertible self-maps) of a compact surface M (e.g. S^2 or T^2) $\mathrm{Diff}^r(M)$ with $r \geq 2$. A sink of $f : M \to M$ is a periodic point $x \in M$ which attracts all points from its neighbourhood (as in your kitchen). Points attracted to x are called basin of attraction of x. In the 60-th Thom conjectured that a generic diffeomorphism cannot have infinitely many coexisting sinks. Indeed, each sink has an open basin of attraction and infinitely many of those seems too much. In the 70-th Newhouse constructed an open set of diffeomorphisms $N \subset \mathrm{Diff}^r(M)$ such that a generic diffeomorphism in N does have infinitely many coexisting sinks. It is an amazing phenomenon, called Newhouse phenomenon. It disproves Thom's conjecture and is a significant obstacle to describe ergodic properties of surface diffeomorphisms. We discuss this phenomenon and closely related results of Benedicks-Carleson, Mora-Viana, Wang-Young, Morreira-Yoccoz. The main result indicates that in some sense this phenomenon has "probability zero". This is a restricted particular case of the so-called Palis conjecture.

An example related to the Conley Index

Krystyna Kuperberg
(joint work with G. Kozlowski and K. Wójcik)

We construct a dynamical system (an $\mathbb{R}$-action) on $\mathbb{R}^3$ whose only compact invariant set A consists of a fixed point and such that no isolating block for A is a 3-cell. There are handle-body isolating blocks for A, but the smaller the isolating block, the more handles are needed.

It is known that an isolated compact invariant set A of a C^1 flow on a manifold possesses an isolating block that is a manifold with boundary. If A is a one-point set and the manifold is $\mathbb{R}^3$, does there always exist an isolating block for A that is a handle-body?

On small divisors in infinite dimensions

Thomas Kriecherbauer

We prove the existence of families of multiphase travelling wave solutions (periodic in time, quasi-periodic in space) for infinite lattices of particles with non-linear nearest-neighbour interactions. The construction is based on an approach to KAM theory in infinite dimensions which was introduced by W. Craig and C.E. Wayne and further extended by J. Bourgain. The main technical issue in this approach is to establish bounds on the inverses of parameter dependent matrices which grow at most subexponentially with the size of the matrices for non-resonant parameters and to control the size and the geometry of the set of resonant parameters. We explain the multiscale analysis used to address this issue.

An example of hyperbolic diffusion

Mark Levi

We demonstrate the existence of motions with unbounded energy for billiards whose walls move periodically with time, under the crucial assumption that the walls are concave. It turns out, furthermore, that the energy approaches infinity exponentially.

The solutions gain energy by always bouncing between the approaching walls. The existence of such solutions depends on our ability to prescribe arbitrarily the sequence of collisions; this freedom comes from the hyperbolic structure of the associated "frozen" problem and depends crucially on the assumption of concavity of the walls. The construction of such "shadowing" solutions is done by a variational approach of minimizing the action for a prescribed sequence of collisions. We construct a box (in configuration space) which is invariant under the (negative) gradient flow of the action. The uniqueness of this minimizer follows from the positive definiteness of the Hessian; this positivity, in turn, depends on the assumption of concavity.

Geometric mechanisms for instability in Hamiltonian systems

Rafael de la Llave
(joint work with A. Delshams and T. M. Seara)

We present several mechanisms for diffusion in Hamiltonian systems. They are based on identifying geometric objects and how do they fit together. The techniques used are standard in the geometric theory of Hamiltonian dynamical systems: normally hyperbolic manifolds, averaging theory, KAM theory and obstruction properties. It is to be noted that we only need assumptions about hyperbolicity of certain objects but there is no need for convexity. We first discuss a mechanism overcoming the large gap problem. The observation is that, whenever there are resonances that cause gaps, we can also find secondary tori or stable manifolds of lower dimensional tori which dovetail in the gaps. Incorporating them in the transition chains allows to obtain diffusion of order 1 for generic polynomial perturbations of penduli cross rotators, a model which indeed presents gaps. Preprints are available from www.ma.utexas.edu/mp_arc. The preprint $\#03-137$ is a short sketch of the steps and $\#03-182$ presents full details. We also present a mechanism based on the existence of normally hyperbolic laminations. When applied to the problem of geodesic flows + periodic potentials, this mechanism establishes the existence of orbits of Hausdorff dimension larger than one on which the diffusion is linear. In surfaces of negative curvature, the only potentials for which there is no diffusion are trivial.

A Computer-Assisted Proof of Saari's Conjecture for the Planar Three-Body Problem

Richard Moeckel

The five central configurations of the three-body problem give rise to solutions where the bodies rotate rigidly around their centre of mass. For these solutions, the moment of inertia of the bodies with respect to the centre of mass is clearly constant. Saari conjectured that these rigid motions are the only solutions with constant moment of inertia. The talk described an unusual proof for the planar problem which makes use of both symbolic algebraic computations and geometric computations involving polyhedra. First, the problem is reduced to showing that a certain system of three polynomial equations in three unknowns has a finite number of solutions. A finiteness test from the theory of sparse polynomial systems (or BKK theory) is applied. This involves constructing the Minkowski sum of the Newton polytopes of the three polynomials and then testing many reduced polynomial systems, one for each face of the Minkowski sum polyhedron.

Growth of maps, distortion in groups and symplectic geometry

Leonid Polterovich

We present applications of Floer homology to symplectic dynamics. The results include lower bounds on the growth rate of symplectic maps and obstructions to symplectic actions of discrete groups. In particular, we prove that certain lattices do not admit non-trivial actions on surfaces by area-preserving maps, and thus confirm some cases of the 1986 Zimmer conjecture. We discuss a multi-dimensional symplectic version of the Zimmer program.

Critical points for surface diffeomorphisms

Enrique Pujals
(joint work with Federico Rodriguez Hertz)

For $f \in \mathrm{Diff}^r(M^2)$ with $r \geq 2$ we study the following map G acting on the unit tangent bundle $(TM)_1$,

$$(TM)_1 \to (TM)_1, \quad G(v) = \frac{Df(v)}{|Df(v)|}.$$

It is shown that G is non-uniformly hyperbolic, and using this we introduce a notion of critical point. We then prove some dynamical consequences related to the presence or absence of such critical points.

Aubry–Mather type results for PDE's

Paul Rabinowitz
(joint work with Edward Stredulinsky)

The study of such results was initiated by J. Moser and extended by V. Bangert. We describe results motivated by the work of Bangert in the setting of

$$-\Delta u + F_u(x,u) = 0, \quad x \in \mathbb{R}^n$$

where

(F_1) $F \in C^2$ in its arguments
(F_2) F is 1-periodic in $x_1, \ldots, x_n$ and in u
(F_3) F is even in $x_1, \ldots, x_n$.

First a variational minimization argument is used to find the basic heteroclinic solutions obtained in a different fashion by Bangert. Then constrained minimization arguments are used to obtain more complex homoclinic and heteroclinic solutions. In somewhat the spirit of work on dynamical systems of Mather, "renormalized" functionals are required to keep critical values finite.

Closed orbits of a charge in a magnetic field

Felix Schlenk
(joint work with Urs Frauenfelder)

We study the motion of a unit charge on a closed Riemannian manifold subject to a magnetic field and prove that for almost every small enough number c there exists a closed contractible orbit on the energy level E_c. The proof uses methods of Hofer geometry which imply a general energy-capacity inequality.

Floer Homology and the Loop Product for the Free Loop Space

Matthias Schwarz
(joint work with Alberto Abbondandolo)

We consider Floer homology for the cotangent bundle of a given Riemannian manifold (M^n, g) and the associated geodesic flow Hamiltonian $H\colon T^*M \to \mathbb{R}$, $H(q,p) = \frac{1}{2}|p|_g^2$. The associated Floer homology $HF_*(T^*M, \omega_{Liouville}, H)$ carries additionally a ring structure, the so-called pair-of-pants ring structure $m_{pop}\colon HF_k \times HF_l \to HF_{k+l-n}$, where $n = \dim M$ and HF_* is graded by the Conley-Zehnder index.

The main result is a ring isomorphism with the homology of the free loop space ΛM of M, with the loop product structure $*_{loop}$ as defined by Chas-Sullivan.
Theorem. *There exists an explicit ring isomorphism*

$$\Phi\colon \left(HF_*(T^*M, \omega_{Liouville}, H), m_{pop}\right) \to \left(H_*^{sing}(\Lambda M), *_{loop}\right).$$

This theorem reproves the result due to Viterbo of the module-isomorphism ignoring the ring structure, and extends it as an isomorphism result for the two algebras.

A family of spatial periodic solutions for $2N$ bodies with equal masses

Andrea Venturelli
(joint work with Susanna Terracini)

By minimizing the Lagrangian action functional, we prove the existence of a family of spatial periodic solutions for $2N$ bodies with equal masses interacting with a gravitational Newtonian force. Solutions of this kinds were discovered numerically in 1983 by Davies, Truman and Williams, and in 1999 by Hoynant. We give a rigorous analytic proof of their existence. The conditions we impose on the set of loops where we minimize the action functional are both symmetry conditions and topological conditions. It's one of the very few cases where it is possible to find new solutions by minimizing the action with topological conditions. The greatest difficulty in this variational approach is to show that minimizers are collision free. Our proof uses a method well known in nonlinear analysis called the "blow-up", that allow us to reduce to a planar Kepler problem. The simplest solutions of our family are, for the 4 body problem, the Hip-Hop solution found in a previous work with Alain Chenciner, and the first example of spatial choreography.

Symplectic intersections and Hamiltonian Dynamics

Zhihong Jeff Xia

Special submanifolds, such as Lagrangian submanifold, isotropic and coisotropic submanifolds, in a symplectic manifold have some special intersection properties; these intersection properties have interesting consequences in Hamiltonian dynamics. We first apply the intersection theory for Lagrangian submanifolds to obtain the existence and persistence of homoclinic points for hyperbolic periodic points and (normally) hyperbolic invariant tori for near integrable symplectic diffeomorphisms. We also obtain chaotic invariant sets near these homoclinic points when they have certain weak transversalities. We then prove an intersection theorem for isotropic and coisotropic submanifolds in symplectic manifolds and apply the theorem to the intersections of the strong stable manifold and the centre unstable manifold of points on a normally hyperbolic invariant manifold.

Edited by Felix Schlenk

Participants

Prof. Dr. Casim Abbas
abbas@math.msu.edu
Department of Mathematics
Michigan State University
A202 Wells Hall
East Lansing MI 48824 – USA

Dr. Artur Avila
artur@math.sunysb.edu
Mathématiques
Collège de France
(Annexe)
3, rue d'Ulm
F-75005 Paris Cedex 05

Prof. Dr. Victor Bangert
bangert@mathematik.uni-freiburg.de
Mathematisches Institut
Universität Freiburg
Eckerstr.1
D–79104 Freiburg

Prof. Dr. Alain Chenciner
chencine@imcce.fr
ASD
IMCCE
77, avenue Denfert Rochereau
F-75014 Paris

Dr. Kai Cieliebak
kai@mathematik.uni-muenchen.de
Mathematisches Institut
Universität München
Theresienstr. 39
D–80333 München

Prof. Dr. Sylvain Crovisier
Sylvain.Crovisier@u-bourgogne.fr
Laboratoire de Topologie-UMR 5584
Département de Mathématiques
Université de Bourgogne
B.P. 47870
F-21078 Dijon Cedex

Prof. Dr. Walter Craig
craig@math.mcmaster.ca
Department of Mathematics and
Statistics
Mc Master University
1280 Main Street West
Hamilton, Ont. L8S 4K1 – CANADA

Prof. Dr. Dmitry Dolgopyat
dolgop@math.psu.edu
Department of Mathematics
Pennsylvania State University
University Park, PA 16802 – USA

Dr. Dragomir Dragnev
dragnev@cims.nyu.edu
dragnev@usc.edu
Department of Mathematics
University of Southern California
DRB 1042 W 36th Place
Los Angeles, Ca 90089-1113 – USA

Dr. Bassam Fayad
fayadb@math.univ-paris13.fr
Département de Mathématiques
Institut Galilee
Université Paris XIII
99 Av. J.-B. Clément
F-93430 Villetaneuse

Prof. Dr. Giovanni Forni
gforni@math.nwu.edu
Dept. of Mathematics
Lunt Hall
Northwestern University
2033 Sheridan Road
Evanston, IL 60208-2730 – USA

Prof. Dr. John Franks
john@math.nwu.edu
Dept. of Mathematics
Lunt Hall
Northwestern University
2033 Sheridan Road
Evanston, IL 60208-2730 – USA

Prof. Dr. Victor Ginzburg
ginzburg@math.uchicago.edu
ginzburg@math.ucsc.edu
Department of Mathematics
University of California Santa Cruz
Santa Cruz, CA 95064 – USA

Prof. Dr. Boris Hasselblatt
BHasselb@tufts.edu
Dept. of Mathematics
Tufts University
Medford, MA 02155 – USA

Prof. Dr. Helmut W. Hofer
hofer@cims.nyu.edu
Courant Institute of
Mathematical Sciences
New York University
251, Mercer Street
New York, NY 10012-1110 – USA

Prof. Dr. Vadim Y. Kaloshin
kaloshin@caltech.edu
Mathematics 253 - 37
California Institute of Technology
Pasadena, CA 91125 – USA

Prof. Dr. Anatole B. Katok
katok_a@math.psu.edu
Department of Mathematics
Pennsylvania State University
303 McAllister Building
University Park, PA 16802 – USA

Prof. Dr. Svetlana Katok
katok_s@math.psu.edu
Department of Mathematics
Pennsylvania State University
University Park, PA 16802 – USA

Prof. Dr. Andreas Knauf
knauf@mi.uni-erlangen.de
Mathematisches Institut
Universität Erlangen
Bismarckstr. 1 1/2
D–91054 Erlangen

Dr. Thomas Kriecherbauer
Thomas.Kriecherbauer@ruhr-uni-bochum.de
Mathematisches Institut
Ruhr Universität Bochum
Universitätsstraße 150
D–44780 Bochum

Prof. Dr. Krystyna Kuperberg
kuperkm@mail.auburn.edu
Dept. of Mathematics
Auburn University
Parker Hall
Auburn, AL 36849-5310 – USA

Dr. Janko Latschev
janko@math.unizh.ch
Institut für Mathematik
Universität Zürich
Winterthurerstr. 190
CH-8057 Zürich

Prof. Dr. Patrice Le Calvez
lecalves@math.univ-paris13.fr
Departement de Mathematiques
Universite Paris 13
Inst. Galilee
Avenue Jean-Baptiste Clément
F-93430 Villetaneuse

Prof. Dr. Mark Levi
levi@math.psu.edu
Department of Mathematics
Pennsylvania State University
University Park, PA 16802 – USA

Prof. Dr. Rafael de la Llave
llave@math.utexas.edu
Dept. of Mathematics
University of Texas at Austin
RLM 8.100
Austin, TX 78712-1082 – USA

Prof. Dr. John N. Mather
jnm@math.princeton.edu
Department of Mathematics
Princeton University
Fine Hall
Washington Road
Princeton, NJ 08544-1000 – USA

Prof. Dr. Richard B. Moeckel
rick@math.umn.edu
School of Mathematics
University of Minnesota
127 Vincent Hall
206 Church Street S. E.
Minneapolis, MN 55455 – USA

Dr. Eugene Neduv
zhenia@math.columbia.edu
neduv@CIMS.nyu.edu
Department of Mathematics
Columbia University
Code 4406
2990 Broadway
New York NY 10027 – USA

Yaron Ostrover
yaronost@post.tau.ac.il
School of Mathematical Sciences
Tel Aviv University
Ramat Aviv
Tel Aviv 69978 – ISRAEL

Prof. Dr. Ricardo Perez-Marco
ricardo@math.ucla.edu
Department of Mathematics
UCLA
405, Hilgard Ave.
Los Angeles, CA 90024 – USA

Prof. Dr. Yakov B. Pesin
pesin@math.psu.edu
Department of Mathematics
Pennsylvania State University
University Park, PA 16802 – USA

Prof. Dr. Leonid V. Polterovich
polterov@post.tau.ac.il
Tel Aviv University, Dept. of Maths
Raymond and Beverly Sackler
Faculty of Exact Sciences
Ramat-Aviv
Tel Aviv 69978 – ISRAEL

Prof. Dr. Jürgen Pöschel
poschel@mathematik.uni-stuttgart.de
Institut für Analysis, Dynamik
und Modellierung
Universität Stuttgart
Pfaffenwaldring 57
D–70569 Stuttgart

Prof. Dr. Enrique R. Pujals
enrique@impa.br
Instituto de Matematica Pura e
Aplicada - IMPA
Jardin Botanico
Estrada Dona Castorina, 110
22460 320 Rio de Janeiro, RJ –BRAZIL

Prof. Dr. Paul H. Rabinowitz
rabinowi@math.wisc.edu
Department of Mathematics
University of Wisconsin-Madison
480 Lincoln Drive
Madison, WI 53706-1388 – USA

Dr. Federico J. Rodriguez Hertz
fede@impa.br
frhertz@fing.edu.uy
IMERL
Facultad de Ingenieria
Julio Herera y Reissig 565
Montevideo CP 11300 – Uruquay

Dr. David Sauzin
sauzin@imcce.fr
CNRS-IMCCE
77, avenue Denfert-Rochereau
F-75014 Paris

Dr. Felix Schlenk
schlenk@math.ethz.ch
Departement Mathematik
ETH-Zentrum
Rämistr. 101
CH-8092 Zürich

Prof. Dr. Matthias Schwarz
mschwarz@mathematik.uni-leipzig.de
Fakultät für Mathematik/Informatik
Universität Leipzig
Augustusplatz 10/11
D–04109 Leipzig

Dr. Karl Friedrich Siburg
siburg@math.ruhr-uni-bochum.de
Mathematisches Institut
Ruhr Universität Bochum
Universitätsstraße 150
D–44780 Bochum

Dr. Andrea Venturelli
venturel@bretagne.ens-cachan.fr
venturel@imcce.fr
Laboratoire d'Analyse non linéaire
et Géometrie
Université d'Avignon
33, rue Louis Pasteur
F-84000 Avignon

Prof. Dr. Clarence Eugene Wayne
cew@math.bu.edu
Dept. of Mathematics
Boston University
111 Cummington Street
Boston, MA 02215-2411 – USA

Prof. Dr. Krzysztof Wysocki
wysocki@ms.unimelb.edu.au
Department of Mathematics
The University of Melbourne
Parkville, Victoria 3052 – Australia

Prof. Dr. Zhihong Jeff Xia
xia@math.northwestern.edu
Dept. of Mathematics
Lunt Hall
Northwestern University
2033 Sheridan Road
Evanston, IL 60208-2730 – USA

Prof. Dr. Jean-Christophe Yoccoz
Jean-C.Yoccoz@college-de-france.fr
Mathématiques
Collège de France
(Annexe)
3, rue d'Ulm
F-75005 Paris Cedex 05

Prof. Dr. Lai-Sang Young
lsy@cims.nyu.edu
Courant Institute
of Math Sciences
251 Mercer Str.,
New York, NY 10012 – USA

Prof. Dr. Eduard Zehnder
zehnder@math.ethz.ch
Departement Mathematik
ETH-Zentrum
Rämistr. 101
CH-8092 Zürich