Arbeitsgemeinschaft mit aktuellem Thema:
Calculus of Functors
Mathematisches Forschungsinstitut Oberwolfach
28.March – 3.April, 2004

Organizers:

Thomas G. Goodwillie	Randy McCarthy
Mathematics Department	University of Illinois at Urbana-Champaign
Box 1917	273 Altgeld Hall
Brown University	1409 W. Green St.
Providence, RI 02912	Urbana, IL 61801
tomg@math.brown.edu	randy@math.uiuc.edu

Introduction:

This workshop is about two related sets of ideas. Let us call them the homotopy calculus and the manifold calculus. Each of them is a method of describing spaces (or other objects) up to weak homotopy equivalence by making heavy use of categories, functors, and naturality. In a typical application of the method, one gains information about a space by viewing the space as a special value of a suitable functor, analyzes the functor using "calculus", and then specializes. Thus the principal objects of study become some rather broad category of functors. A constant theme is the systematic approximation of these functors by functors of much more special kinds.

The homotopy calculus deals with homotopy functors from, for example, the category of topological spaces to itself. Here "homotopy functor" means "functor that takes (weak) equivalences to (weak) equivalences". The main sources for the general theory are [4][5][6].

The manifold calculus deals with contravariant functors from the partially ordered set of open subsets of a fixed smooth manifold M to, for example, the category of spaces. Again the functors must satisfy a kind of homotopy invariance; roughly speaking, if $U \supseteq V$ is a collar then the map $F(U) \to F(V)$ is an equivalence. The main sources for the general theory are [17] [10].

For lack of time we have omitted a third theory, the orthogonal calculus, which deals with functors, continuous on morphisms, from the category of finite-dimensional real Hilbert spaces and isometric linear injections to the category of spaces. See [18].

Let us first discuss the homotopy calculus. The central idea here is approximation of functors by "linear" functors, just as in the ordinary differential calculus the central idea is the approximation of functions by linear functions. Linearity means the following. Call the homotopy functor F excisive if it takes homotopy pushout squares to homotopy pullback squares and call it reduced if the unique map $F(*) \to *$ is a weak equivalence. Call it linear if it is both excisive and reduced. A typical linear functor from based spaces to based spaces will, up to natural equivalence, have the form $L(X) = \Omega^\infty(C \wedge X)$, at least on finite CW complexes X. Here C is some spectrum, which can be called the coefficient of the linear functor.

There is a standard process, which is sometimes called stabilization and here is called linearization, for turning a reduced functor F into a linear functor L. Roughly speaking, there is a natural map from $F(X)$ to $\Omega F(\Sigma X)$ and one iterates this to make the stabilization, the homotopy colimit of $\Omega^k F(\Sigma^k X)$ as k goes to infinity. If F is linear then L is (equivalent to) F, and in general L is the universal example (in an appropriate up-to-homotopy sense) of a linear functor under L. The coefficient of L is called the derivative of F at the one point space.

More generally the derivative $\partial_y F(Y)$ of F at the space Y and basepoint y can be defined as the coefficient of the stabilization of the functor

$$Z \mapsto \mathrm{hofiber}(F(Y \vee_y Z) \to F(Y))$$

from based spaces to based spaces.

There is another useful generalization. The excision condition concerns the behavior of a functor on two-dimensional cubical diagrams. We call a functor n-excisive if it satisfies a cerain condition involving $(n+1)$-dimensional cubical diagrams, so that 1-excisive means excisive. It turns out that again for any F there is a universal n-excisive functor under F. We call it $P_n F$ and think of it as the nth Taylor polynomial of F. There are maps $P_n F \to P_{n-1} F$, and F maps into the limit of this "Taylor tower".

The nth layer of the tower, meaning the homotopy fiber of $P_n F \to P_{n-1} F$, is analogous to a homogeneous polynomial; it is an n-excisive functor whose $(n-1)$-excisive approximation is trivial. Such things turn out always to have the form $\Omega^\infty(C \wedge X^{\wedge n})_{h\Sigma_n}$, at least on finite CW complexes X. Here the coefficient C is a spectrum with an action of the symmetric group Σ_n, and it is called the nth derivative of F (at $*$).

Most functors encountered in practice are not n-excisive for any n, but are stably n-excisive. F is called stably 1-excisive if for a homotopy pushout square

$$\begin{array}{ccc} X & \to & X_1 \\ \downarrow & & \downarrow \\ X_2 & \to & X_{12} \end{array}$$

the functor always yields a square

$$\begin{array}{ccc} F(X) & \to & F(X_1) \\ \downarrow & & \downarrow \\ F(X_2) & \to & F(X_{12}) \end{array}$$

such that the map from $F(X)$ to the homotopy pullback is $k_1 + k_2 - c_1$ connected, where k_i is the connectivity of the map $X \to X_i$ and c_1 is a constant depending only on F. F is callled stably n-excisive if it satisfies a similar condition involving $(n+1)$-dimensional cubes. If F is stably n-excisive for all n and the associated sequence of constants c_n has slope ρ, then the functor is called ρ-analytic.

If F is ρ-analytic then for ρ-connected spaces X the canonical map $F(X) \to P_nF(X)$ has a connectivity that tends to infinity with n. ("The Taylor series converges to the function" within a "radius" determined by ρ.)

If F is ρ-analytic and $\partial_y F(Y) \simeq *$ for all (Y, y) then F is locally constant: any $(\rho - 1)$-connected map $X \to Y$ of spaces, or at least of finite complexes, induces an equivalence $F(X) \to F(Y)$. This can be proved using Taylor towers. It was proved in [5] by a more direct method.

So much for the homotopy calculus. We now turn more briefly to the manifold calculus. The most important example is the functor $\mathrm{Emb}(-, N)$ which takes an open set U of M to the space of smooth embeddings of U in another manifold N.

Here again there is a notion of n-excisive functor, and there is a way of building a universal n-excisive functor T_nF under F. It can be defined in a few words: $(T_nF)(U)$ is the homotopy limit of $F(V)$ over all open sets V in U that are tubular neighborhoods of sets having at most n elements. Once again, if F satisfies a kind of analyticity (stable excision) condition then the resulting tower converges for a large class of objects. Again there is a classification theorem for homogeneous functors (n-excisive functors with trivial

($n-1$)-excisive part). We state the result briefly, assuming for simplicity that M is compact and without boundary: up to equivalence any homogeneous n-excisive functor is given by specifying some fibration over the space $C(n, M)$ of unordered configurations of n points in M and a section σ defined outside some compact set. The functor then assigns to each open U in M the space of sections of the fibration restricted to $C(n, U)$ that coincide with σ near infinity.

The functor $\mathrm{Emb}(-, N)$ is sufficiently analytic that these methods give very strong information about the space of embeddings of M in N if the codimension $\dim(N)$-$\dim(M)$ is at least three. In fact, in some useful but complicated sense the homotopy type of $\mathrm{Emb}(M, N)$ is determined by the family of spaces $\mathrm{Emb}(U, N)$, where U ranges through those open sets of M that are tubular neighborhoods of finite sets.

The talks at this workshop will deal mostly with the general results mentioned above and some generalizations. Of course important examples will be introduced, but we will not venture very far into serious applications of the theory, such as applications of homotopy calculus to algebraic K-theory and to classical homotopy theory.

The decision to occupy ourselves more with general theory than with applications was made partly because there is a lot of general theory to cover and partly to keep the talks accessible to a broad audience. We hope that there will also be informal sessions in the evenings on more specialized topics.

Homotopy calculus and manifold calculus can be presented as separate and parallel subjects, but in fact the former had its genesis in the latter and there is an ongoing interplay between the two. This will be the subject of the final talk.

Anyone who is contemplating giving a talk should feel free to ask the organizers to expand on the brief descriptions below.

Talks:

1. **Introduction**

   This talk, by Goodwillie, will broadly survey the field and the week ahead. It will go into detail about some things, including (1) the classes of functors to be studied in the two kinds of calculus and (2) the stabilization or linearization process which is the beginning of the subject.

2. Cubical diagrams and n-th order excision

This talk sets the stage by establishing some terminology and basic facts concerning n-dimensional cubical diagrams (or for brevity "cubes"). The content is essentially section 1 of [5].

A cube of spaces determines a map from the "initial" space to the homotopy limit of the rest of the diagram. The cube is homotopy-cartesian (or for brevity cartesian) if this map is a weak equivalence, and is k-cartesian if the map is k-connected. A cube of based spaces has a total homotopy fiber, which can be identified with the homotopy fiber of this map. There are also the dual notions.

A map $X \to Y$ of n-cubes can be viewed as an $(n+1)$-cube, and there is an important and elementary family of statements such as "if Y and $(X \to Y)$ are k-cartesian then X is k-cartesian" and "if $(X \to Y)$ and $(Y \to Z)$ are k-cartesian then $(X \to Z)$ is k-cartesian". These should be explained in some detail.

The talk can be mainly about cubes of spaces, but the ideas are rather general and there should be some discussion of other cases, such as cubes of spectra and chain complexes. In particular there is the important point that for spectra or chain complexes homotopy cartesian is the same as homotopy cocartesian.

The second half of the talk is about n-excisive functors. In each of the two kinds of functor calculus (homotopy and manifold) some functors are called n-excisive. Give these definitions (from section 3 of [5] and from section 2 of [17] respectively. Weiss uses the term "polynomial of degree at most n".)

Give examples of such functors.

For example, the product with a fixed space gives a functor from spaces to spaces that preserves homotopy cocartesian squares, and a similar statement holds for smash products. Smashing with a fixed spectrum gives a 1-excisive functor from based spaces to spectra. If a functor $F(X_1, ..., X_n)$ of n variables is 1-excisive in each variable then $F(X, ..., X)$ is n-excisive (Prop 3.4 of [5]). Define the nth order crosseffect of a homotopy functor (see page 23 of [6]) and show that in the case of an n-excisive functor the crosseffect is 1-excisive in each variable.

In the manifold calculus, examples arise as follows: If $E \to M$ is a fibration over the manifold M then the functor $F(U) = \{\text{sections } U \to E\}$ is 1-excisive. One can mention (without proof) that up to natural weak equivalence this is the only sort of example of a 1-excisive functor F such that $F(\emptyset) \simeq *$. [One can also mention (again without proof) that the example $F(U) = \{\text{Immersions of } U \text{ in } N\}$ is 1-excisive; in fact, a statement of that kind is the key step in the proof of the Smale-Hirsch reduction of immersion theory to homotopy theory.]

There are similar ways of constructing n-excisive examples, using a fibration over the space of unordered configurations in M(see section 7 of [17]). These should be described, probably without proof.

3. **Analyticity and the higher order Blakers-Massey theorems**

In the homotopy calculus setting a functor is called "analytic" if it is sufficiently "stably n-excisive" for all n. The definition is in section 4 of [5].

The most important example is the identity functor from spaces to spaces. The Blakers-Massey theorem says that a cocartesian square of spaces

$$\begin{array}{ccc} A & \to & B \\ \downarrow & & \downarrow \\ C & \to & D \end{array}$$

is $(p+q-1)$-cartesian if $A \to B$ and $A \to C$ are p and q connected. An easier "dual" statement says that if the square is cartesian then it is $(p+q-1)$-cocartesian if $C \to D$ and $B \to D$ are p and q connected. There are important generalizations to n-cubes, which we call the higher Blakers-Massey theorems or HBM for short.

The first quarter of the talk can be devoted to stating these results ([5] 2.3, 2.4, 2.5, 2.6) and defining analyticity ([5] section 4).

The next part can be used to demonstrate how these results can be used to show that various other functors from spaces to spaces or spaces to spectra are analytic. Certainly do 4.4 and 4.5 from [5], and maybe 4.6.

The last part can sketch the proof of HBM. Of the four statements, 2.3 is the most important, 2.4 is a generalization, and 2.5 and 2.6 are their duals. The proof of 2.3 is by induction on n, and the proof requires that 2.4 is proved at the same time.

Suggestion: sketch the proof of 2.3 for 2-cubes, deduce 2.4 for 2-cubes, deduce 2.3 for 3-cubes, and maybe deduce 2.4 for 3-cubes. That at least gives the main idea without too much bookkeeping.

If the speaker wants to present a different proof of HBM instead, s/he is welcome to do so. The proofs in [5] use smooth general position arguments. The classical (n=2) case of 2.3 is usually proved using homology. [2] has a different proof of 2.3.

4. Disjunction and excision for spaces of embeddings

In the manifold calculus there is an analogous notion of "analytic functor". The definition is in [10], 2.1. The main reason why functor calculus is useful in the study of spaces of (codimension > 2) embeddings is that the functor $F(U) = \mathrm{Emb}(U, N)$ is analytic. This talk is about that analyticity The strong version by Klein and Goodwillie, quoted in [10] 1.1, has unfortunately still not been written up properly. A large part of the proof is contained in the pair of preprints [7] [8], which yield a slightly weaker version. A much weaker (in most cases) but still useful version can be obtained rather easily from HBM.

This talk will contain (1) the definition of "analytic" in the manifold calculus, (2) the statement of Goodwillie and Klein, (3) exploration of the geometric content of the statement ("disjunction" results are equivalent to excision results), (4) proof of the much weaker statement mentioned above, and (5) maybe some discussion of the proof of the stronger version.

The introduction to [8] should be a good source for (2), (3), and (4).

The ingredients in the proof of (5) are HBM, Goodwillie's thesis [3] and the Browder-Casson-Sullivan-Wall theorem from surgery theory. Goodwillie's thesis is an analyticity statement for the functor $\mathrm{CEmb}(U, N)$, concordance embeddings. (It is much more analytic than $\mathrm{Emb}(U, N)$.)

There will probably not be time for any but the briefest outline of (5), even treating the surgery and [3] as black boxes. The speaker should probably confer with Goodwillie about how to handle this.

5. n-excisive functors

This talk presents a strategy, which we may call Mayer-Vietoris induction, for exploiting nth order excision.

For example, in the manifold setting an n-excisive functor of open sets is in some sense determined by its behavior on those objects which are disjoint unions of at most n balls; namely if F and G are two such functors and $T : F \to G$ is a natural map then in order for $T : F(U) \to G(U)$ to be a weak equivalence for every U it is sufficient if this is so when U is of that special kind. This is 5.1 of [17].

Similar statements hold in the homotopy setting: an n-excisive functor from spaces (or from spaces over a fixed space) to spectra is determined on finite complexes by its behavior on sets with at most n elements. See for example 5.8 of [6] for a proof when $n = 1$, and it is not hard to generalize. It is important to have spectra rather than spaces here. A useful observation is that these statements are true for space-valued functors if "n-excisive" is replaced by the dual condition saying that strongly cocartesian $(n+1)$-cubes go to COcartesian cubes. This "dual" variant is also true in the spectrum-valued case; in fact in that case the dual statement is the same as the original because cartesian equals cocartesian.

There should be time at the end of the talk to discuss a family of stronger statements, which assert that in the homotopy setting an n-excisive functor is determined in a much stronger sense by its values on sets with at most n elements. For example, an n-excisive functor from finite CW complexes to spectra is naturally equivalent to the homotopical left Kan extension of its restriction to the full subcategory of sets with at most n elements (and the homotopical left Kan extension of any functor on that subcategory is n-excisive). This yields a nice classification theorem for such n-excisive functors. A simple result in the same family says that a reduced 1-excisive homotopy functor F from based finite complexes to based spaces is naturally equivalent to the functor "$- \wedge F(S^0)$".

Again all of this is true for functors to spaces as long as "n-excisive" is replaced by the dual notion. From an expository point of view this dual space case may be a good way of approaching the spectrum case.

The proofs of these assembly statements (reconstructing a functor from its restriction to a small class of objects in the presence of an excision hypothesis) make use of the weaker statements in the first part of the talk.

Perhaps the speaker can supply the proofs, with hints from the organizers if needed.

6. **Weiss' Taylor tower in the manifold case, part I**

For a good functor F from open subsets of M to spaces Weiss makes a functor T_nF and shows that it is the universal example of an n-excisive functor with a map from F. It may be thought of as the nth Taylor polynomial of F. The functors form a tower (there is a map $T_nF \to T_{n-1}F$) and the whole tower may be thought of as a Taylor series.

This talk will give (1) the definition of T_nF, (2) a reasonable sketch of the proof that T_nF is n-excisive and universal, (3) a proof that the tower converges to F when the functor is analytic. It will also include (4) the statement of the theorem classifying the "homogeneous degree n" functors – those which can occur as the nth layer (homotopy fiber of $T_nF \to T_{n-1}F$) in a Taylor tower.

The references are [17] for (1), (2), and (4), and [10] for (3). The proof involves Mayer-Vietoris induction and a great deal of work with homotopy limits.

7. **Weiss' Taylor tower in the manifold case, part II**

This will be the proof of the classification theorem for homogeneous functors.

The speakers for 6 and 7 should probably communicate with each other, and they should feel free to reapportion the material among the two talks if this looks advisable.

8. **Weiss' Taylor tower, part III**

There are other ways of looking at the Weiss-Taylor tower. In the case when M is one-dimensional, there is something like a cosimplicial construction [16]. In the general case there is a configuration-space construction [9]. The quadratic part, $T_2\mathrm{Emb}(-, N)$, can be interpreted as a reformulation of work of Haefliger [11].

The speaker is free to present any or all of this.

9. **Taylor tower in the homotopy case, part I**

   For a homotopy functor F from, for example, spaces to spaces, Goodwillie makes a functor P_nF and shows that it is the universal example of an n-excisive functor with a map from F. It may be thought of as the nth Taylor polynomial of F. The functors form a tower (there is a map $P_nF \to P_{n-1}F$) and the whole tower may be thought of as a Taylor series.

   The case $n = 1$, already covered in the first talk, is the stabilization or linearization process mentioned in the introduction.

   This talk will give (1) the construction of P_nF, (2) the proof that it is n-excisive if F is analytic, and (3) the proof that the tower $(P_nF)(\mathrm{X})$ converges to $F(X)$ if F is analytic and X is within the "radius of convergence", more precisely if F is ρ-analytic and X is ρ-connected. It will also begin the treatment of homogeneous functors. These are the functors that can be the nth layer of a Taylor tower; they are also the n-excisive functors F for which $P_{n-1}F \simeq *$. The reference for all of this is section 1 of [6], but omitting the difficult 1.8 and 1.9.

   It was shown in talk 2 that if $F(X_1, ..., X_n)$ is 1-excisive in each variable then $F(X, ..., X)$ is n-excisive. It is now shown that if in addition $F(X_1, ..., X_n)$ is reduced in each variable then $F(X, ..., X)$ is homogeneous, at least in the presence of some connectivity assumptions. It is also shown that, in the spectrum-valued case, if the multilinear functor $F(X_1, ..., X_n)$ is symmetric with respect to permutation of its variables then the homotopy orbit spectrum for the resulting action of the symmetric group on $F(X, ..., X)$ is still n-excisive and homogeneous.

   State the converse: every homogeneous functor to spectra comes from a symmetric multilinear functor in this way.

10. **Taylor tower in the homotopy case, part II**

Prove the last statement in talk 9. The symmetric multilinear functor is the nth crosseffect of the corresponding homogeneous functor. It is called the nth differential of F. Reference: section 3 of [6]

Also prove that a homogeneous functor F to based spaces is canonically and uniquely of the form $\Omega^\infty G$, where G is a homogeneous functor to spectra. Reference: section 2 of [6].

Also deduce from the assembly results in talk 5 that on finite complexes a symmetric multilinear functor from based finite complexes to spectra has the form $F(X_1, ..., X_n) = C \wedge X_1 \wedge \ldots \wedge X_n$, where C is a spectrum with action of the symmetric group Σ_n.

The speakers for 10 and 11 should probably communicate with each other, and they should feel free to reapportion the material among the two talks if this looks advisable.

11. **How to identify the layers of a Taylor tower**

According to the results of talks 10 and 11 an analytic functor is the limit of a tower whose nth layer has the form

$$\Omega^\infty (C_n \wedge X \wedge \ldots \wedge X)_{h\Sigma_n},$$

where the "coefficient" C_n is a spectrum with Σ_n action. C_n is called the nth derivative of F. The question arises, for particular functors F, what is that spectrum C_n and what is the Σ_n action on it?

The key to answering this question is the fact that the nth cross-effect of the nth layer is the multilinearization of the nth cross effect of F itself. In a little more detail: The nth crosseffect of F is a symmetric reduced functor of n variables. The canonical map from F to P_nF induces a map from the crosseffect of F to the crosseffect of P_nF, which is equivalent to the crosseffect of the nth homogeneous layer of F. This is the universal example of a natural and symmetry-preserving map from the crosseffect of F to something symmetric and multilinear.

This fact is easy to see when F is analytic. (It is also true without that hypothesis, and this will be shown in talk 15.)

One interpretation of this fact is that the nth derivative (the coefficient spectrum of the multilinear functor that encodes the nth homogeneous layer of F) is in fact equivalent to what you get if you differentiate F n times in the sense of talk 1. The reference for all of this is the beginning of section 6 of [6].

All of the above could take less than half a lecture. Using this as a tool, one should now work out some examples, including the nth derivative of the identity functor ([12]).

12. **The Taylor tower of the suspension spectrum of $\mathrm{Map}(K, -)$, and of $\Sigma^\infty\Omega^\infty$.**

If K is a finite complex, then (as seen in talk 3) the suspension spectrum of the functor $\mathrm{Map}(K, -)$ is an analytic functor. The nth derivative, and therefore the nth layer, is described in [6] using the method of talk 11, but in fact the whole tower was described in Gregory Arone's thesis. In particular the Taylor tower of $\Sigma^\infty\Omega^n X$ is very thoroughly understood.

As a sort of limiting case of this there is the tower of the functor $\Sigma^\infty\Omega^\infty X$. (This is our first encounter with functors from spectra to spectra.)

The reference for the former is [1]. One reference for the latter is [14].

13. **Homotopy calculus via cotriples**

A useful property of the nth crosseffect functor (introduced in talk 2) is that its diagonal is a cotriple (or comonad) on the category of homotopy functors. The purpose of this talk is to discuss the fact that the fiber of $F \to P_{n-1}F$ tends to be well approximated by the standard resolution of F with respect to this cotriple within the radius of convergence of F (that is, for ρ–connected spaces when F is ρ–analytic).

A homotopy functor F commutes with realizations if it is weakly equivalent to its homotopy left Kan extension over all finite sets. One should first establish that if F takes values in spectra and commutes with realizations then the the homotopy fiber of the standard resolution of F with respect to the nth crosseffect is $(n-1)$-excisive and is hence the loop of P_nF (easy). Then show that if F is ρ-analytic and X is ρ-connected then the functor $F(X \wedge -)$ commutes with realizations and deduce that the fiber of $F(X) \to P_{n-1}F(X)$ is as claimed.

The general case for homotopy functors to spaces should now be sketched. Perhaps this is a good time to also mention the associated spectral sequence for computing $P_{n-1}F(X)$ in this case whose E_2 page is given by $\pi_p P_{n-1}[\pi_q F(X \wedge -)]$. As an application indicate how the Taylor tower of the identity can be viewed as the cotriple derived theory of Curtis' filtration via lower central series.

The primary reference for the case of homotopy functors to spaces is [15] sections 9 and 10. This can also be used to deduce the case of functors to spectra but a more direct method is available if one recalls that realizations commute with finite pullbacks for spectra.

14. (Multi-)linear functors of spaces over a space

The emphasis so far has been primarily on functors of spaces, but the results apply also to functors of spaces over a fixed space B. In that context a linear functor is given not by smashing with a fixed spectrum C but by a sort of fiberwise smash product with a twisted family of spectra C_b. In the case where the linear functor is the differential of (the restriction to spaces over B of) a homotopy functor of spaces, then the spectrum C_b for a given point b in B is the derivative of F at (B, b). The reference for this is the end of section 5 of [6].

There is a chain rule. It has been written down by Klein and Rognes [13] in one way, but there is another way which is in some ways preferable. In particular it is more general. Here is a sketch of the statement:

If F is a homotopy functor from spaces to spaces then for a space X and points x in X and y in $Y = F(X)$ there is a derivative spectrum $J_x^y(F)$, the coefficient of the linearization of the functor

$$Z \mapsto \text{homotopy fiber over } y \text{ of } F(X \vee_x Z) \to F(X) = Y$$

from based spaces to based spaces.

If F and G are functors from spaces to spaces and $W = G(Y)$ and $Y = F(X)$ then the derivative $J_x^w(GF)$ of the composition may be obtained by smashing $J_y^w(G)$ with $J_x^y(F)$ for all y in Y and in some sense summing or integrating over all y. This is unpublished but should be understandable with hints from Goodwillie.

15. **Calculus without estimates**

This talk gives the general proofs of several results which we have seen proved for analytic functors:

• P_nF is n-excisive and universal (talk 9).

• $F(X, ..., X)$ is homogeneous if $F(X_1, ..., X_n)$ is multilinear (talk 9).

• The multilinearized nth cross effect of F is the nth crosseffect of P_nF (talk 12).

The first of these should be proved last. References for these are [6] 1.8, 3.1, 6.1.

16. **The intersection of manifold calculus and homotopy calculus**

Goodwillie will give this final talk, exploring the interplay between the manifold calculus (as applied to spaces of embeddings) and the homotopy calculus (as applied to the identity functor).

References

[1] G. Arone, *A generalization of Snaith-type filtration* , Trans. Amer. Math. Soc. 351 (1999), 1123-1150.

[2] G. Ellis, R. Steiner, *Higher-dimensional crossed modules and the homotopy groups of* $(n+1)$*-ads*, J. Pure Appl. Algebra 46 (1987), no. 2-3, 117-136.

[3] T. Goodwillie, *A multiple disjunction lemma for smooth concordance embeddings*, Mem. Amer. Math. Soc. 86 (1990), no. 431, 1-317.

[4] T. Goodwillie, *Calculus I: The first derivative of pseudoisotopy theory*, K-Theory, 4 (1990), 1-27.

[5] T. Goodwillie, *Calculus II: Analytic Functors*, K-Theory, 5 (1992), 295-332.

[6] T. Goodwillie, *Calculus III: Taylor series*, Geometry and Topology, 7 (2003), 645-711.

[7] T. Goodwillie, *Excision estimates for spaces of homotopy equivalences*, preprint at: http://www.math.brown.edu/faculty/goodwillie.html.

[8] T. Goodwillie, *Excision estimates for spaces of diffeomorphisms* ,draft version at: http://www.math.brown.edu/faculty/goodwillie.html.

[9] T. Goodwillie, J. R. Klein, M. Weiss, *A Haefliger style description of the embedding calculus tower*, Topology 42 (2003), no. 3, 509–524.

[10] T. Goodwillie, M. Weiss, *Embeddings for the point of view of immersion theory: Part II*, Geometry and Topology, 3 (1999) 103-118.

[11] A. Haefliger, *Plongements différentiables dans le domaine stable*, Commentarii Math. Helv. 37 (1962/1963) 155-167.

[12] B. Johnson, *The derivatives of homotopy theory*, Trans. Amer. Math. Soc. 347 (1995), no. 4, 1295–1321.

[13] J. R. Klein, J. Rognes, *A chain rule in the calculus of homotopy functors*, Geom. Topol. 6 (2002), 853–887.

[14] N, Kuhn, *Stable splittings and the diagonal*, Homotopy methods in algebraic topology (Boulder, CO, 1999), 169-181, Contemp. Math., 271, Amer. Math. Soc., Providence, RI, 2001.

[15] A. Mauer-Oats, *Goodwillie calculi*, preprint at: http://www.math.purdue.edu/ amauer/thesis/.

[16] D. P. Sinha, *The topology of spaces of knots*, preprint at: http://front.math.ucdavis.edu/author/Sinha-D*.

[17] M. Weiss, *Embeddings for the point of view of immersion theory: Part I*, Geometry and Topology, 3 (1999) 76-101.

[18] M. Weiss, *Orthogonal Calculus*, Trans. Amer. Math. Soc. 347 (1995) 3743-3796, *Erratum*, Trans. Amer. Math. Soc. 350 (1998) 851-855.

Participation:

The idea of the Arbeitsgemeinschaft is to learn by giving one of the lectures in the program.

If you intend to participate, please send your full name and full postal address to

`randy@math.uiuc.edu`

by February 10, 2004 (if possible).

You should also indicate which talk you are willing to give:
First choice: talk no. ...
Second choice: talk no. ...
Third choice: talk no. ...

You will be informed shortly after the deadline if your participation is possible and whether you have been chosen to give one of the lectures.

The Arbeitsgemeinschaft will take place at Mathematisches Forschungsinstitut Oberwolfach, Lorenzenhof, 77709 Oberwolfach-Walke, Germany. The institute offers accomodation free of charge to the participants. Travel expenses cannot be covered. Further information will be given to the participants after the deadline.