Arbeitsgemeinschaft mit aktuellem Thema: POLYLOGARITHMS

Mathematisches Forschungsinstitut Oberwolfach

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Organizers:

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Introduction:

The k-th polylogarithm function is defined on $|z| < 1$ by

$$Li_k(z) = \sum_{n \geq 1} \frac{z^n}{n^k}.$$

In the past 25 years or so, polylogarithms have appeared in many different areas of Mathematics. The following list is taken, for the most part, from [Oe]: volumes of polytopes in spherical and hyperbolic geometry, volumes of hyperbolic manifolds of dimension three, combinatorial description of characteristic classes, special values of zeta functions, geometry of configurations of points in $\mathbb{P}^1$, cohomology of $GL_n(\mathbb{C})$, calculation of Green's functions associated to perturbation expansions in quantum field theory, Chen's iterated integrals, differential equations with nilpotent monodromy, nilpotent completion of $\pi_1(\mathbb{P}^1 - \{0, 1, \infty\})$, regulators in algebraic K-theory, Hilbert's problem on cutting and pasting, Bethe's Ansatz in thermodynamics, and combinatorial problems in quantum field theory.

Of course, these problems are not all unrelated. One common thread is that values of polylogarithms appear naturally as periods of certain mixed Hodge structures associated to mixed Tate motives over number fields. How these periods are related to special values of L-functions is a part of the Beilinson conjectures, which were discussed in a previous Arbeitsgemeinschaft. Since that time, the general picture has clarified. A number of lectures will be devoted to aspects of this more general philosophy (lectures 2–5, 10, 11, 14-19). The p-adic aspects of the theory have been studied and will be explained in lecture 12. In addition, a vast generalization, multiple polylogarithms of the form

$$Li_{s_1,\ldots,s_k} := \sum_{n_1>\ldots>n_k\geq 1} \frac{z_1^{n_1}\cdots z_k^{n_k}}{n_1^{s_1}\cdots n_k^{s_k}},$$

have come to play a role. These will be presented in lectures 4 and 13.

Polylogarithms play a role in physics. Lecture 8 will explain how zeta values, polylogarithms, and multiple polylogarithms appear in calculations of perturbative expansions in quantum field theory.

Vaguely speaking, Li_k is associated with a certain characteristic class with values in $\mathbb{C}/(2\pi i)^k\mathbb{Z}$. Much of the focus in the study of periods and related questions has been on the "noncompact" part of this class, i.e. the real part (resp. imaginary part) of $Li_k(z)$ for k odd (resp. k even). However, the "compact" part has its own fascination. Lecture 9 is devoted to the Rogers dilogarithm, which is closely related to the real part of $Li_2(z)$, and thus (via the above characteristic class) to the torsion in $K_3(\mathbb{R})$. The expansion

$$\sum_{k=1}^{\infty} \log(1-ue^{2\pi ik\tau}) = Li_2(u)/2\pi i\tau + \text{higher order terms in } \tau$$

is the key to a series of relations between these torsion classes, leading terms in character expansions in quantum field theory, crystal bases, and various partition formulas in combinatorics.

The final lectures 14-19 return to the relationship with special values of L-functions. Lectures 14-15 are devoted to the Zagier conjectures for number fields. Lectures 16-17 concern the elliptic polylogarithm sheaves and Zagier's conjecture for elliptic motives. Finally, lectures 18-19 and the elliptic polylogarithm concern how polylogarithms relate to Euler systems and the Bloch-Kato conjectures on special values of L-functions.

Talks:

1. ## Function theory of higher logarithms

   Purpose: To define the higher logarithms Li_k, certain single-valued variants P_k, and to develop those functional equations that will be used in later talks.

   Details: Definitions and basic properties of the Li_k [Oe, Section 1]. The monodromy of the Li_k [Oe, 2.1–2.3]: careful introduction of the notion of multi-valued functions; [Oe, Prop. 1]; the matrices L_n, A_n, and ρ_n from [Oe, 2.3]. The functions P_k [Oe, 3.1, 3.2]: definition as in [Oe, 3.1] — don't worry, a conceptual explanation will follow later (in talk 3); mention that P_2 equals $i \cdot D$, where D is the Bloch–Wigner dilogarithm (which will be treated in talk 2); mention the properties in [Oe, 3.2] without spending too much time on the proofs. Functional equations [Oe, Section 4]: deduce formulae (34) and (38) [Oe, p. 60] and mention [Bl, Thm. 7.4.5] (these results will be needed in talk 2); if time permits, mention [Oe, 4.2,4.3].

   Notations: follow the notations in [Oe].

   References: [Oe], the sources quoted in [Oe], in particular, [Bl] and [H2].

2. ## The Bloch–Wigner dilogarithm

   Purpose: To explain [Z1, Thm. 3].

   Details: State this result. Then prepare the proof: (a) Prove [H2, Lemma 4.1], using formula [Oe, (38)] (see talk 1). Sketch (without spending too much time on generalities about K-theory) how this induces a map

   $$K_3(\mathbb{C}) \longrightarrow \mathbb{R} \; ;$$

   we accept that this map equals half the Borel regulator [H2, Thm. 4.2]. Aliter: give the construction from [Bl, Sections 5, 6]. (b) Quote Borel's theorem [Bo, Thm. 6.2].

   References: [H2], the sources quoted in [H2, Section 4], [Bl], [Bo], [Z1].

3. Interpretation in terms of mixed variations of Hodge structure

Purpose: To use the language and concepts from Hodge theory to encode the function theory of the Li_k and of the P_k.

Details: Definition of variations of Hodge structure (on curves) [H2, p. 24], giving little emphasis on properties (v) and (vi); give the definition for $\mathbb{Q}$- and $\mathbb{R}$-coefficients. Tate objects $\mathbb{Q}(k)$ [BD2, 1.6]: definition; explicit description of $\mathrm{Ext}^1(\mathbb{Q}(0), \mathbb{Q}(k))$ over a point; the map

$$\mathcal{O}^*(S) \longrightarrow \mathrm{Ext}^1_{S(\mathbb{C})}(\mathbb{Q}(0), \mathbb{Q}(1)) \ , f \longmapsto [f] \ .$$

The projective system of variations $pol^{(N)}$ (not $\mathcal{M}^{(N)}$!) [BD2, 1.3] (by [H2, Thm. 7.1], $pol^{(N)}$ satisfies the axioms of a variation). This system $pol := (pol^{(N)})_N$ will be referred to as the "classical polylogarithm". Give the list of basic properties of pol [BD2, 1.9]; this list will be needed in talk 14. Explain [BD2, 1.5] how the functions P_k introduced in talk 1 ($= D_k$ in [BD2]) occur naturally when passing to $\mathbb{R}$-coefficients.

Notations: except for the symbol $\mathcal{M}^{(N)}$ (which we replace by $pol^{(N)}$), follow the notations in [BD2]; in particular, use lower (not upper) triangular matrices, respect the normalization for the description of the Ext^1, and introduce the notation $[f]$ as this will be used later on.

References: [BD2], [H2], [W1].

4. Mixed Hodge structure on π_1

Purpose: This talk concentrates on the mixed Hodge structure on (the completed group ring of) π_1. In order to do so, the notion of iterated integrals is needed. In particular, we want to see how multiple polylogs arise in the context of the fundamental group of $\mathbb{P}^1 - \{0, 1, \infty\}$.

Details: Introduce iterated integrals as in [H1, Section 1]. Example: higher logarithms, and (more generally) multiple polylogarithms [G3, Section 12]. Give the necessary ingredients for the statement of Chen's theorem [H1, Thm. (4.1), Thm. (4.4)], and sketch the proof. Describe how this result is used to put the Hodge and weight filtration on the truncated group ring of π_1 when the variety is smooth. State [H1, Thm. (5.1)] and give some indications on the proof. The shuffle product [H1, Lemma (2.11)] should be mentioned at some point, as this will be referred to in talk 13. The speaker of talk 5 may be willing to take over some of the material...

Reference: [H1], [G3], [D].

5. ## Mixed structure on $\pi_1(\mathbb{P}^1 - \{0, 1, \infty\})$

Purpose: To see what the theory developed in the previous talk does for us when the variety equals $\mathbb{P}^1 - \{0, 1, \infty\}$, and to indicate analogues of this construction for other mixed theories.

Details: Specialize the geometric situation to $\pi_1(\mathbb{P}^1 - \{0, 1, \infty\})$ as in [D]. Give an overview of the more general program of associating motives to a fundamental group; these motives are mixed Tate motives under special hypotheses on the geometry.

References: [D], [H2, Sections 8, 10], [Wo], [DG].

6. ## Volumes of hyperbolic manifolds

Purpose: The connection between volumes of hyperbolic manifolds and the polylogarithm.

Details: Start with [Ra] 7.3. as an introduction. Then explain section 2 of [G4]. Give a sketch of the proof of theorem 1.1 and its consequences 1.2. and 1.3. in [G4]. The results in 2.3 in [G4] should be mentioned without proof. The main result follows from 2.17. Theorem 1.3 in [G4] is a consequence of the result in talk 14. This talk has a non-empty intersection with talk 2.

References: [G4], [Ra].

7. ## Analytic torsion

Purpose: Introduce higher torsion following [Bi-Lo] and compute it on S^1-bundles, give an conceptual explanation why polylogs occur in this setting.

Details: Start with the Chern-classes of flat bundles in [Bi-Lo], I g) and explain without proof the Riemann-Roch-Grothendieck for flat bundles [Bi-Lo], 3.17. Spend some time to introduce carefully the higher torsion forms and the transgression formula [Bi-Lo] 3.22 and 3.23. Without technical details, sketch the computation of the higher torsion for S^1-bundles in [Bi-Lo] 4.13. If time permits, explain the analogue of [Ig] theorem 2.8.4 for higher torsion. Here lemma 2.8.3 in [Ig] follows from the axiomatic description of the higher torsion forms in [Bi-Lo], app. I.

References: [Bi-Lo], [Ig].

8. Physics

Purpose: Explain to mathematical audience how zeta values, polylogarithms, and multiple polylogarithms arise in integrals associated to vertex diagrams in perturbative quantum field theory.

Details: Feynman showed that the perturbation expansion in QED was indexed by certain graphs called, appropriately enough, Feynman graphs. The coefficient associated to such a graph is given by an integral which, in a simple situation, has the form $\int_{\mathbb{R}^{4n}} \frac{d^{4n}x}{Q_1(x)\cdot \ldots \cdot Q_m(x)}$ where the $Q_i(x)$ are certain rank 4 positive semi-definite quadrics associated to the graph. Calculations of these integrals by physicists frequently yield zeta values, polylogarithms, and multiple polylogarithms. In the case when $2m \geq 4n$ these integrals can be interpreted as periods of limiting mixed Hodge structures. (Add to each quadric a small imaginary parameter and remark that the integral becomes a period for the complement of the perturbed union of quadrics in $\mathbb{P}^{4n}$.)

The physics literature is quite complicated, and it is difficult to see a pattern. The lecture should formulate a precise mathematical program and mention concrete examples relevant to the study of polylogs.

References: Papers of D. Kreimer and D. Broadhurst. A book by Todorov on analytic properties of matrix elements in perturbation theory.

9. Rogers Dilogarithm

Purpose: Explain applications of the real dilogarithm to torsion in K_3. Explicit relation with topics in representation theory such as characters of Kac-Moody algebras, crystal bases, conformal weights, and related combinatorial questions.

Details: Discuss regulator map $K_3(\mathbb{C}) \to \mathbb{C}/(2\pi i)^2\mathbb{Z}$ (coordinate with lecture 2. See [FS]). Relate resulting map $K_3(\mathbb{R}) \to \mathbb{R}/(2\pi)^2\mathbb{Z}$ to Rogers dilogarithm $L(x)$ defined for $0 < x < 1$ by

$$L(x) = \sum \frac{x^n}{n^2} + \frac{1}{2} \log x \cdot \log(1-x).$$

(cf. [Kv], 1.1). Next discuss expansion

$$\sum_{k=1}^{\infty} \log(1 - ue^{2\pi ik\tau}) = Li_2(u)/2\pi i\tau + \dots$$

and the calculation in terms of $L(x)$ of the asymptotic expansion in N of $\log^2 a_N$ in the q-expansion $((q)_n := (1-q)(1-q^2)\cdots(1-q^n))$

$$\sum_{n=(n_1,\dots,n_k)\in\mathbb{Z}^k} \frac{q^{nBn^t}}{(q)_{n_1}\cdots(q)_{n_k}} = \sum a_N q^N$$

where B is symmetric positive definite $k \times k$ matrix. (See lemma 4 in section 2.1.2 of [Kv]). Discuss applications to conformal weights, crystal bases, etc. For conformal weights, see [Na]. For crystal bases, try [FS].

References: [Kv], [Na], [FS].

10. Etale realization

Purpose: To describe ℓ-adic realizations of polylogs.

Details: This talk should include Soulé's construction of cyclotomic regulator elements [So], and the étale realizations of Deligne's torsors [D]. Mention the Euler system property of the Soulé elements [Hu-Ki] 3.1. Time permitting, some discussion of relations with Ihara's program [NW] would be worthwhile. The lecturer should be in contact with whoever is giving lectures 17 and 18 to be certain that requisite material for number theory applications is included.

References: [So], [D], [NW].

11. Motivic version

Purpose: Present polylogarithms as geometric objects.

Details: This lecture presents various constructions for the mixed Tate motive associated to the polylogarithm: the synthetic approach which views a motive as a "gluing together" of its realizations; or the approach of [BK] which follows [D] in defining mixed Tate motives to be representations of a Lie algebra in the category of graded vector spaces, and then constructs a candidate for this Lie algebra using algebraic cycles.

References: [BD2], [BK], [D].

12. ## p-adic version

Purpose: Explain p-adic polylogarithms and multiple polylogarithms.

Details: Lecture should discuss various definitions of p-adic polylogarithms. Key points should be relations with p-adic integration, relation with p-adic periods, and relation with p-adic L-functions. Time and interest permitting, some discussion of polylogarithms in characteristic p could be included.

References: [Br], work of Gros, Somekawa, Besser, Bannai, and Furusho.

13. ## Multiple polylogarithms

Purpose: To introduce the iterated zeta functions, and to study structural properties of multiple polylogarithms.

Details: Basic definitions and function theory. Shuffle and stuffle products, standard relations, Hopf algebra structure. As far as possible, these properties should be deduced from the general formalism of iterated integrals as introduced in talk 4; a certain coordination with the speaker of that talk therefore is desirable. Work of Goncharov [G3], [G2]. Conjecture of Waldschmidt [Wa].

References: [G2], [G3], [Wa].

14. ## Zagier's conjecture: statement and motivic motivation

Purpose: To state Zagier's conjecture, and to motivate it using the sheaf theoretic philosophy à la Deligne–Grothendieck.

Details: State the weak version of Zagier's conjecture as in [BD2, 1.7]. Terminology: "surjectivity conjecture" := the *conjecture additionnelle* concerning surjectivity of the φ_k from [BD2, p. 105]. Using Borel's theorem [Bo, Thm. 6.2], show how the weak version of Zagier's conjecture, together with the surjectivity conjecture implies the general form of [Z1, Thm. 3] (for $\zeta_F(k)$ and D_k instead of $\zeta_F(2)$ and $D = D_2$; F a number field); this is referred to as "Zagier's conjecture" [Z2]. State conjecture [BD2, 1.10] and show [BD2, Section 2] that it implies the weak version of Zagier's conjecture. Note that there are two unconditional proofs of the weak version of Zagier's conjecture for number fields: [BD1] and [J, Section 5].

References: [BD2], [BD1], [Bo], [J], [Z1], [Z2].

15. **Zagier's conjecture: the case $k = 3$**

Purpose: To sketch Goncharov's proof of Zagier's conjecture on $\zeta_F(3)$.

Details: This talk is largely independent of the others, but necessitates a speaker willing to invest some energy...

References: [G1].

16. **The elliptic polylogarithm I**

Purpose: To give an overview of the variant for elliptic curves of the theory developed so far, in particular, in talks 1, 3, 14, and 15.

Details: The speakers of talks 16 and 17 have to cooperate closely. They are expected to make choices. The analogues of the higher logarithms Li_k are defined in [W2, p. 289]; the analogues of the P_k are the Kronecker double series $D_{a,b}$ [Z1, Section 2] (this would correspond to talk 1). For the Hodge theoretic interpretation (corresponding to talk 3), see [BL, Section 4] or [W2, Sections 3, 4]. We suggest to collect enough material to be able to give the elliptic variant of talk 14: statement of the weak version of Zagier's conjecture for elliptic curves [W3, 1.6], and (sketch of the) sheaf theoretic motivation of this conjecture [W3, Section 4]. This material would typically be sufficient to understand [W3, 3.1, 3.2], and could form the content of the first of these two talks. Surjectivity of the φ_k, together with Beilinson's conjecture for elliptic curves and their symmetric powers, would imply Zagier's conjecture on $L(\mathrm{Sym}^{k-2} E, k-1)$. State what is known in the cases $k = 2$ and $k = 3$ [RS, W3], [GL].

References: [W3], [BL], [GL], [RS], [W2], [Z1].

17. **The elliptic polylogarithm II**

See the description of the previous talk.

18. **Application to special values of L-functions I**

Purpose: Statement of the (equivariant) Bloch-Kato conjecture on special values of L-functions in the case of Artin and Dirichlet motives. The Euler system property of the etale realization of the polylog.

Details: Start with the explanation of conjecture 1.2.8 in [Hu-Ki] and comment on also on the equivariant case. Then formulate the theorem 1.3.1 in [Hu-Ki] and the equivariant theorem in [Bu-Gr]. Mention as a corollary theorem 1.4.1 in [Hu-Ki]. For other proven cases of the Bloch-Kato conjecture see the overview article [Ki]. Then an overview of the proof following 2.1 in [Hu-Ki] should be given. Recall the Euler system of the etale realization of the polylog (3.1. in [Hu-Ki]) and the relation of the polylog functions with the Dirichlet L-functions (3.2. in loc. cit.). Finally sketch the class number formula case as explained in 2.3. [Hu-Ki].

References: [Bu-Gr], [Hu-Ki].

19. **Application to special values of L-functions II**

    Purpose: Give an overview of the proof of the Bloch-Kato conjecture for Dirichlet motives.

    Details: Follow the steps 1)-4) in the plan of the proof given in 2.1 [Hu-Ki]. Use the results from the theory of Euler systems and the reciprocity law only as a "black box". Concentrate on the main argument, which proves the main conjecture of Iwasawa theory and reduces the theorem to the class number formula. Here the "twist-invariance" of the polylog is decisive.

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Participation:

The idea of the Arbeitsgemeinschaft is to learn by giving one of the lectures in the program. All speakers should follow closely the instructions given above, and use the notations as indicated. The 19 talks are designed to last 60 minutes. Please avoid taking more time. If you have questions, don't hesitate to contact one of the organizers by email.

If you intend to participate, please send your full name and full postal address to

`guido.kings@mathematik.uni-regensburg.de`

by AUGUST 15th 2004 at the latest.

You should also indicate which talk you are willing to give:
First choice: talk no. ...
Second choice: talk no. ...
Third choice: talk no. ...

You will be informed shortly after the deadline if your participation is possible and whether you have been chosen to give one of the lectures.

The Arbeitsgemeinschaft will take place at Mathematisches Forschungsinstitut Oberwolfach, Lorenzenhof, 77709 Oberwolfach-Walke, Germany. The institute offers accomodation free of charge to the participants. Travel expenses cannot be covered. Further information will be given to the participants after the deadline.