Arbeitsgemeinschaft mit aktuellem Thema:

Modern Foundations for Stable Homotopy Theory

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Introduction:

Stable homotopy theory started out as the study of generalized cohomology theories for topological spaces, in the incarnation of the stable homotopy category of spectra. In recent years, an important new direction became the *spectral algebra* or *stable homotopical algebra* over structured ring spectra. Homotopy theorists have come up with a whole new world of 'rings' which are invisible to the eyes of algebraists, since they cannot be defined or constructed without the use of topology; indeed, in these 'rings', the laws of associativity, commutativity or distributivity only hold up to an infinite sequence of coherence relations. The initial 'ring' is no longer the ring of integers, but the *sphere spectrum* of algebraic topology; the 'modules' over the sphere spectrum define the stable homotopy category. Although ring spectra go beyond algebra, the classical algebraic world is properly contained in stable homotopical algebra. Indeed, via Eilenberg-MacLane spectra, classical algebra embeds into stable homotopy theory, and ordinary rings form a full subcategory of the homotopy category of ring spectra. Topology interpolates algebra in various ways, and when rationalized, stable homotopy theory tends to become purely algebraic, but integrally it contains interesting torsion information.

There are plenty of applications of structured ring spectra and spectral algebra within homotopy theory, and in recent years, these concepts have started to appear in other areas of mathematics (we will sketch connections to algebraic K-theory and arithmetic, and to algebraic geometry via motivic homotopy theory and derived algebraic geometry).

After this general introduction, we now want to give some history and more background and point out some areas where structured ring spectra methods have been used for calculations, theorems or constructions. Several of the following topics will be taken up in the talks of this AG.

Some history. A crucial prerequisite for spectral algebra is an associative and commutative smash product on a good point-set level category of spectra, which lifts the well-known smash product pairing on the *homotopy category.* To illustrate the drastic simplification that occurred in the foundations in the mid-90s, let us draw an analogy with the algebraic context. Let R be a commutative ring and imagine for a moment that the notion of a chain complex (of R-modules) has not been been discovered, but nevertheless various complicated constructions of the unbounded derived category $\mathcal{D}(R)$ of the ring R exist. Moreover, constructions of the *derived* tensor product on the *derived* category exist, but they are complicated and the proof that the derived tensor product is associative and commutative occupies 30 pages. In this situation, you could talk about objects A in the derived category together with morphisms $A \otimes^L_R A \to A$, in the derived category, which are associative and unital, and possibly commutative, again in the derived category. This notion may be useful for some purposes, but it suffers from many defects – as one example, the category of modules (under derived tensor product in the derived category), does not in general form a triangulated category.

Now imagine that someone proposes the definition of a chain complex of R-modules and shows that by formally inverting the quasi-isomorphisms, one can construct the derived category. She also defines the tensor product of chain complexes and proves that tensoring with a bounded below (in homological terms) complex of projective modules preserves quasi-isomorphisms. It immediately follows that the tensor product descends to an associative and commutative product on the derived category. What is even better, now one can suddenly consider differential graded algebras, a 'rigidified' version of the crude multiplication 'up-to-chain homotopy'. We would quickly discover that this notion is much more powerful and that differential graded algebras arise all over the place (while chain complexes with a multiplication which is merely associative up to chain homotopy seldom come up in nature).

Fortunately, this is not the historical course of development in homological algebra, but the development in stable homotopy theory was, in several aspects, as indicated above. The first construction of what is now called 'the stable homotopy category', including its symmetric monoidal smash product, is due to Boardman (unpublished); accounts of Boardman's construction appear in [109] and [2, Part III] (Adams has to devote more than 30 pages to the construction and formal properties of the smash product). With this category, one could consider ring spectra 'up to homotopy', which are closely related to multiplicative cohomology theories.

However, the need and usefulness of ring spectra with rigidified multiplications soon became apparent, and topologists developed different ways of dealing with them. One line of approach used operads for the bookkeeping of the homotopies which encode all higher forms of associativity and commutativity, and this led to the notions of A_∞- respectively E_∞-ring spectra. Various notions of point-set level ring spectra had been used (which were only later recognized as the monoids in a symmetric monoidal model category). For example, the orthogonal ring spectra had appeared as *$\mathscr{I}_*$-prefunctors* in [64], the *functors with smash product* were introduced in [13] and symmetric ring spectra appeared as *strictly associative ring spectra* in [40, Def. 6.1] or as *FSPs defined on spheres* in [41, 2.7].

At this point it had become clear that many technicalities could be avoided if one had a smash product on a good point-set category of spectra which was associative and unital *before* passage to the homotopy category. For a long time no such category was known, and there was even evidence that it might not exist [53]. In retrospect, the modern spectra categories could maybe have been found earlier if Quillen's formalism of *model categories* [71] had been taken more seriously; from the model category perspective, one should not expect an intrinsically 'left adjoint' construction like a smash product to have a good homotopical behavior in general, and along with the search for a smash product, one should look for a compatible notion of cofibrations.

In the mid-90s, several categories of spectra with nice smash products were discovered, and simultaneously, model categories experienced a major renaissance. Around 1993, Elmendorf, Kriz, Mandell and May introduced the *S-modules* [28] and Jeff Smith gave the first talks about *symmetric spectra*; the details of the model structure were later worked out and written up by Hovey, Shipley and Smith [46]. In 1995, Lydakis [55] independently discovered and studied the smash product for Γ-spaces (in the sense of Segal [97]), and a little later he developed model structures and smash product for *simplicial functors* [56]. Except for the *S-modules* of Elmendorf, Kriz, Mandell and May, all other known models for spectra with nice smash product have a very similar flavor; they all arise as categories of continuous, space-valued functors from a symmetric monoidal indexing category, and the smash product is a convolution product (defined as a left Kan extension), which had much earlier been studied by category theorist Day [20]. This unifying context was made explicit by Mandell, May, Schwede and Shipley in [61], where another example, the *orthogonal spectra* were first worked out in detail. The different approaches to spectra categories with smash product have been generalized and adapted to equivariant homotopy theory [22, 59, 60] and motivic homotopy theory [23, 47, 48]. In this AG we will present these various setups with an emphasis on symmetric spectra.

Algebra versus homotopy theory. Many constructions and invariants for classical rings have counterparts for structured ring spectra. These ring spectra have well behaved module categories; algebraic K-theory, Hochschild homology or André-Quillen homology admit refinements; classical constructions such as localization, group rings, matrix rings, Morita theory and Galois theory carry over, suitably adapted. This export of concepts from algebra to topology illuminates both fields. For example, Dwyer, Greenlees, and Iyengar [25] have shown that Gorenstein duality, Poincaré duality, and Gross-Hopkins duality become results of a single point of view. Morita theory for ring spectra [6, 96, 92] gives a new perspective at *tilting theory*. The extension of Galois theory to ring spectra [83] has genuinely new kinds of examples given by classical and higher forms of real topological K-theory. Certain algebraic extensions of commutative rings can be lifted to the sphere spectrum; for example, roots of unity can be adjoined to E_∞-ring spectra away from ramification [89]. Some algebraic notions, for example the units or the center of a ring, are more subtle, and their generalizations to ring spectra show richer features than the classical counterparts.

Power operations. Recognizing a multiplicative cohomology theory as an E_∞-ring spectrum leads to additional structure which can be a powerful theoretical and calculational tool. We want to illustrate this by the example of the Adams spectral sequence [2, III.15], [66, II.9], [103, Ch. 19], a tool which has been used extensively for calculations of stable homotopy groups. In its most classical instance, the Adams spectral sequence converges to the p-completed stable homotopy groups of spheres and takes the form

$$E_2^{s,t} \; = \; \mathrm{Ext}_{\mathcal{A}_p}^{s,t}(\mathbb{F}_p, \mathbb{F}_p) \implies (\pi_{t-s}^{\mathrm{stable}} S^0)_p^\wedge \quad .$$

The E_2-term is given by Ext-groups over the Steenrod algebra $\mathcal{A}_p$ of stable mod-p cohomology operations. This spectral sequence neatly separates the problem of calculating homotopy groups into an algebraic and a purely homotopy theoretic part.

The Steenrod algebra has various explicit descriptions and the E_2-term can be calculated mechanically. In fact, computer calculations have been pushed up to the range $t-s \leq 210$ (for $p = 2$) [19, 70]. The 1-line is given by the primitive elements in the dual Steenrod algebra, which for $p = 2$ are classes $h_i \in \mathrm{Ext}_{\mathcal{A}_2}^{1,2^i}(\mathbb{F}_2, \mathbb{F}_2)$ for $i \geq 0$. The first few classes h_0, h_1, h_2 and h_3 are infinite cycles and they detect the Hopf maps $2\iota, \eta, \nu$ and σ; the Hopf maps arise from the division algebra structures on $\mathbb{R}$, $\mathbb{C}$, the quaternions and the Cayley numbers.

On the other hand, identifying differentials and extensions in the Adams spectral sequence is a matter of stable homotopy theory. The known differentials can be derived by exploiting more and more subtle aspects of the homotopy-commutativity of the stable homotopy groups of spheres (or, in our jargon, the E_∞-structure of the sphere spectrum). The first d_2-differential in the mod-2 Adams spectral sequence is a consequence of the graded-commutativity $yx = (-1)^{|x||y|}xy$ in the ring of stable homotopy groups of spheres: like any element of odd dimension, the third Hopf map σ in the stable 7-stem has to satisfy $2\sigma^2 = 0$. The class $h_0h_3^2$ which represents $2\sigma^2$ in $E_2^{3,17}$ is non-zero, so it has to be in the image of some differential. In these dimensions, the only possible differential is $d_2(h_4) = h_0h_3^2$, which simultaneously proves that the class $h_4 \in E_2^{1,16}$ is not an infinite cycle and thus excludes the existence of a 16-dimensional real division algebra.

Graded-commutativity of the multiplications is only a faint shadow of the E_∞-structure on the sphere spectrum. A more detailed investigation reveals that an E_∞-structure on a ring spectrum yields *power operations* [18, Ch. I], [78, Sec. 7]. Various kinds of power operations can be constructed on the homotopy and (generalized) homology groups of the E_∞-ring spectrum and on the E_2-term of the Adams spectral sequence. These operations interact in certain ways with the differential, and this interaction is an effective tool for determining such differentials. For example, the operations propagate the first d_2-differential to an infinite family of differentials $d_2(h_{i+1}) = h_0h_i^2$, which proves that the classes h_i for $i = 0, 1, 2$ and 3 are the only infinite cycles on the 1-line and excludes the existence of other real division algebras.

Algebraic K-theory. Structured ring spectra and algebraic K-theory are closely related in many ways, and in both directions. Algebraic K-theory started out as the Grothendieck group of finitely generated projective modules over a ring. Quillen introduced the higher algebraic K-groups as the homotopy groups of a certain topological space, for which he gave two different constructions (the *plus-construction* and the *Q-construction* [73]). Nowadays, algebraic K-theory can accept various sorts of categorical data as input, and produces spectra as output (the spaces Quillen constructed are the underlying infinite loop spaces of these K-theory spectra). For example, the symmetric monoidal category (under direct sum) of finitely generated projective modules leads to the K-theory spectrum of a ring, and the category of finite sets (under disjoint union) produces the sphere spectrum. If the input category has a second symmetric monoidal product which suitably distributes over the 'sum', then algebraic K-theory produces commutative ring spectra as output.

It was Waldhausen who pioneered the algebraic K-theory of structured ring spectra, introduced the *algebraic K-theory of a topological space X* as the algebraic K-theory of the *spherical group ring* $S[\Omega X] = \Sigma^\infty \Omega X_+$ and established close links to geometric topology, see [110]. Waldhausen could deal with such ring spectra, or their modules, in an unstable, less technical fashion before the modern categories of spectra were found: his first definition of the algebraic K-theory of a ring spectrum mimics Quillen's plus-construction applied to the classifying space of the infinite general linear group. Similarly, Bökstedt [13] (in cooperation with Waldhausen) defined *topological Hochschild homology* using so-called *Functors with Smash Product*, which were only later recognized as the monoids with respect to the smash product of simplicial functors [56]. In any event, a good bit of the development of A_∞-ring spectra was motivated by the applications in algebraic K-theory.

Later, Waldhausen developed one of the most flexible and powerful K-theory machines, the $S_\bullet$*-construction* [112] which starts with a *category with cofibrations and weak equivalences* (nowadays often referred to as a *Waldhausen category*). Originally, Waldhausen associated to this data a sequential spectrum and indicated [112, p. 342] how a categorical pairing leads to a smash product pairing on K-theory spectra. The $S_\bullet$-construction can be applied to a category of cofibrant highly structured module spectra. Applying this to Eilenberg-MacLane ring spectra gives back Quillen K-theory, and applied to spherical group rings of the form $S[\Omega X]$ gives an interpretation of Waldhausen's $A(X)$.

Hesselholt eventually recognized that the iterated $S_\bullet$-construction gives K-theory as a symmetric spectrum and that it turns good products on the input category into symmetric ring spectra [33, Prop. 6.1.1]. Elmendorf and Mandell [31] have a different way of rigidifying the algebraic K-theory of a bipermutative category into an E_∞-symmetric ring spectrum.

Units of a ring spectrum. One of the more subtle generalizations of a classical construction is that of the units of a ring spectrum. The units of a structured ring spectrum form a loop space, and in the presence of enough commutativity (i.e., for E_∞- or strictly commutative ring spectra), the units even form an infinite loop space. These observation are due to May and are highly relevant to orientation theory and bordism. A recent application of these techniques is the construction by Ando, Hopkins and Rezk, of E_∞-maps $MO\langle 8\rangle \to \mathbf{tmf}$ from the string-cobordism spectrum to the spectrum of topological modular forms which realize the Witten genus on homotopy groups [43, Sec. 6] (this refines earlier work of Ando, Hopkins and Strickland on the $MU\langle 6\rangle$-orientation of elliptic spectra [42, 4, 5]). The details of this are not yet publicly available, but see [43, Sec. 6], [50], [78].

Waldhausen's first definition of algebraic K-theory of a ring spectrum is based on Quillen's plus-construction and uses the units of matrix ring spectra. While for a commutative ring in the classical sense, the units are always a direct summand in the first K-group, Waldhausen showed that the units of the sphere spectrum do not split off its K-theory spectrum, not even on the level of homotopy groups. This phenomenon has been studied systematically by Schlichtkrull [87], producing non-trivial classes in the K-theory of an E_∞-ring spectrum from homotopy classes which are not annihilated by the Hopf map η.

Homotopical algebraic geometry. In this Arbeitsgemeinschaft, we are trying to convey the idea that the foundations of multiplicative stable homotopy theory are now in good shape, and ready to use. In fact, the machinery allows to 'glue' commutative ring spectra into more general algebro-geometric objects, making them the affine pieces of 'schemes' or even 'stacks'. This area is becoming known as *homotopical algebraic geometry*, and one set of foundations has been pioneered by Toën and Vezzosi in a series of papers [104, 105, 106, 107].

Another promising line of research is to investigate small (e.g., Deligne-Mumford) stacks which come equipped with a flat morphism to the moduli stack of formal groups; one might hope to lift the graded structure sheaf of such a stack to a sheaf of ring spectra and capture a snapshot of stable homotopy theory. In this context, one may think of the generalized algebro-geometric objects as an ordinary scheme or stack, together with a sheaf of E_∞-ring spectra, which locally looks like 'Spec' of an E_∞-ring spectrum. The structure sheaf of the underlying ordinary stack can be recovered by taking π_0 of the sheaf of ring spectra. It is essentially by this program that Hopkins and his coworkers produced *topological modular forms* (the 'universal' version of elliptic cohomology) and the recent work of Lurie shows that these ideas can be extended to almost any situation where the Serre-Tate theorem on deformations applies. Here the flow of information goes both ways: the number theory informs homotopy theoretic calculations, but surprising algebraic phenomena, such as the Borcherds congruence in modular forms, have natural homotopy theoretic explanations [43, Thm. 5.10].

While this is a 'hot' area in algebraic topology, the organizers decided not to have talks about it in this AG because of the lack of publicly available literature (but see [11, 36, 42, 43, 44, 50, 77]).

Rigidity theorem. After having discussed various models for the stable homotopy category, the last day of the AG will be devoted to its 'rigidity' property [93]; this says that the stable homotopy category admits essentially only one model. More precisely, any model category whose homotopy category is equivalent, as a triangulated category, to the homotopy category of spectra is already *Quillen equivalent* to the model category of spectra. Loosely speaking, this says that all higher order homotopy theory is determined by the homotopy category, a property which is very special. Examples of triangulated categories which have inequivalent models are given in [91, 2.1, 2.2] or [21, Rem. 6.8] (which is based on [86]).

In algebra, a 'rigidity theorem' for unbounded derived categories of rings is provided by *tilting theory*; it is usually stated in the form that if two rings are derived equivalent, then, under a flatness assumption, there is a complex of bimodules X such that derived tensor product with X is an equivalence of triangulated categories [79] (a reworking of this result in model category terms, which also removes the flatness assumption, can be found in [21, Thm. 4.2]). Rigidity fails for categories of dg-modules over differential graded algebras [21, Rem. 6.8]. Incidentally, neither for derived categories of rings nor for the stable homotopy category is it known whether every derived equivalence lifts to a Quillen equivalence, or equivalently, whether there are exotic self-equivalences of these triangulated categories.

Questions about rigidity and exotic models are related to the problem of whether algebraic K-theory is an invariant of triangulated categories. Quillen equivalent model categories have equivalent K-theory spectra [21, Cor. 3.10], [85], and in special 'rigid' situations, the triangulated category determines the model, and thus the algebraic K-theory.

Talks:

1. The stable homotopy category
2. Model categories I
3. Model categories II
4. Symmetric spectra
5. Stable model structure for symmetric spectra
6. Symmetric ring and module spectra
7. S-modules
8. Other kinds of spectra
9. Algebraic K-theory
10.* Units, Thom spectra and orientations
11. Power operations and the Adams spectral sequence
12.* Complex cobordism, formal groups, and MU-algebras
13. Obstruction theories
14. Galois theory
15. Rigidity for the stable homotopy category
16. Reduction to Adams filtration 1
17. Proof of the rigidity theorem

Below we give detailed instructions and references of what should be covered in the talks of this AG. The talks fall into three groups: during the first two days (talks 1-8) we develop the foundations for structured ring spectra. The next two days (talks 9-14) are reserved for applications where the methods of spectral algebra have been useful; these talks are generally more challenging than the earlier ones, give fewer details and have more of a survey character. The organizers are aware that the plans for talks 9-14 are very ambitious, but since these talks are largely independent of each other, it is not a problem if in some cases, the material cannot all be fit into a 60 minute talk. The final day (talks 15-17) is devoted to the proof of the rigidity theorem, stating that the stable homotopy category has only one model, up to Quillen equivalence.

The points listed as 'required' should be covered in any case, since they are either fundamental or needed in later talks. The points listed as 'desirable' are not strictly necessary, but should be covered if time allows.

1. **The stable homotopy category**
This talk should provide background and motivation for the stable homotopy category, as well as a first rigorous definition. The motivational part should at least mention the Spanier-Whitehead category, Thom's theorem classifying cobordism classes of manifolds in terms of the homotopy groups of Thom spectra, and the representability theorems for generalized (co-)homology theories. The speaker may consult [2, Part III, Sec. 1], [62, Part I] and [65] for further motivation.
Required: The Freudenthal suspension theorem and the Spanier-Whitehead category as a first attempt [62, Ch. 1], with its triangulated structure and smash product. Generalized homology and cohomology theories and representability theorems ([75, A.3, A.4], [103, Ch. 7, 9], [62, Ch. 4]; in particular [103, Thm. 9.27 and 9.28]). Bordism and Thom spectra [68, §18], [75, B.1, B.2], [103, Thm. 12.30], [84]. Definition of the stable homotopy category as the cofibrant Ω-spectra in the sense of [17, Sec. 2] with morphisms the homotopy classes of spectrum morphisms. Examples of spectra and (co-)homology theories: sphere spectrum, suspension spectra, Eilenberg-MacLane spectra, Thom spectra.
Desirable: Additional examples of spectra, e.g. topological K-theory (very end of Ch. 11 in [103]), or algebraic K-theory spectra.

2. **Model categories I**
An introduction to the basic concept around model categories; besides elementary category theory and basic topology, hardly any prerequisites are needed. The main references are Quillen's original book [71], the modern introduction by Dwyer and Spalinski [24] and Hovey's monograph [45].
Required: Axioms for a model category as in [72, II.1] or [24, Sec. 3]. Construction of the homotopy category and characterization as a localization ([71, I.1 Thm. 1'], [24, Thm. 6.2] or [45, Thm. 1.2.10]). Quillen adjoint pairs and their derived functors; Quillen equivalences ([71, I.4 Thm. 3], [24, Thm. 9.7] or [45, Thm. 1.3.10 and 1.3.13]). First examples: model structures

for spaces and chain complexes [24, 71, 45]
Desirable: Additional examples, e.g. the stable category of modules of a Frobenius ring [45, Sec. 2.2].

3. **Model categories II**
This talk should present more examples of model category structures and revisit the definition of the stable homotopy category given in the first talk from the new perspective. In each example, the definitions of cofibrations, fibrations and weak equivalences have to be explained carefully. The *small object argument* is the most frequently used construction to provide the factorizations in a model category.
Required: Model structure for simplicial sets ([71, II.3], [35, I.11]). Sequential spectra with the stable model structure of Bousfield and Friedlander [17, Sec. 2]. The *small object argument* [71, II, p. 3.4], [24, 7.12], [45, Thm. 2.1.14].
Desirable: The seven model categories and six Quillen equivalences of rational homotopy theory [72].

4. **Symmetric spectra**
This first talk about symmetric spectra should focus on the categorical aspects, i.e., make us familiar with the category of symmetric spectra and the smash product. The original paper [46] by Hovey, Shipley and Smith is very readable and essentially self-contained, and its sections 1 and 2 can serve as the main source. Complementary points of view and additional examples can be found in [92, Sec. 4.2].
Required: Define the category of symmetric spectra and their smash product; re-interpret symmetric spectra as S-modules in symmetric sequences [46, Prop. 2.2.1]; explain and illustrate the universal property of the smash product [92, Sec. 4.2]. Revisit the earlier examples as symmetric spectra and explain the multiplications, when available: sphere spectrum, suspension spectra, Eilenberg-MacLane spectra [92, Ex. 4.14], Thom spectra [92, Ex. 4.15], monoid ring spectra [92, Ex. 4.12], cobordism spectra such as MO and MU [92, Ex. 4.15].
Desirable: (Re-)visit topological K-theory [49] as a symmetric spectrum.

5. **Stable model structure for symmetric spectra**
The stable model structure for symmetric spectra and its compatibility with the smash product are the focus of this talk. These are the prerequisites for model structures for ring and module spectra.

Required: Define stable equivalences (Def. 3.1.3) and show that level equivalences and π_*-isomorphisms are stable equivalences (Cor. 3.1.8, Thm. 3.1.11). Illustrate the difference between stable equivalences and π_*-isomorphisms with Example 3.1.10; explain that $\pi_0 F_1 S^1$ is indeed an infinite direct sum of copies of $\mathbb{Z}$. State Theorem 3.4.4 on the stable model structures (without proof). Mention the Quillen equivalence between symmetric and sequential spectra (Thm. 4.2.5). Explain the compatibility between cofibrations and stable equivalences in Section 5.3, in particular Corollaries 5.3.8 and 5.3.10 (the references all refer to [46]).
Desirable: The material from [98] which sheds more light on the stable equivalences.

6. **Symmetric ring and module spectra**
With the symmetric monoidal smash product and a compatible model structure in place, we are ready to explore ring and module spectra. Model structures for modules over a fixed symmetric ring spectrum and for the associative (but not necessarily commutative) symmetric ring spectra are directly inherited from the stable model structure of symmetric spectra [46, Cor. 5.4.2, 5.4.3]; this method is quite general, as explained in [94]. The story for commutative ring spectra is more subtle, and one has to tweak the stable model structure and work with the *positive* (co-)fibrations [61, Sec. 15]. This talk should also provide the link between commutative symmetric ring spectra and the older notion of E_∞-ring spectra.
Required: Stable model structures for modules over a symmetric ring spectrum [46, Cor. 5.4.2], for symmetric ring spectra [46, Cor. 5.4.3]. The *positive* stable model structure for symmetric spectra [61, Sec. 14]; explain that the permutation action of the symmetric group Σ_n on a smash power $X^{\wedge n}$ is free whenever X is positively cofibrant (compare [61, Lemma 15.5]). The positive stable model structure for *commutative* symmetric ring spectra [61, Thm. 15.1]. It should be explained in what sense strictly commutative and E_∞-ring spectra have the same homotopy theory, and why this does not depend on the choice of E_∞-operad. The Quillen equivalence between commutative and E_∞-ring spectra is a special case of Quillen equivalences associated to weak equivalences of suitable operads, see [38, Thm. 1.2.4] and the remark immediately thereafter, or [31, Thm. 1.4].
Desirable: It would be good if this lecture could give some hints about what the various model structures (stable [46, Sec. 3], S- [99, Thm. 4.2] and their positive variants [61, Sec. 14]) are good for, i.e., when it is appropriate to use one or the other. [30] and [94, Rem. 4.5] may be helpful in explaining

why the 'positive' aspect is needed for a stable structure on *commutative* symmetric ring spectra; these arguments are a reincarnation in model category terms of Lewis' observation [53] that no category of spectra can have all desirable properties.

7. **S-modules**
   The S-modules of Elmendorf, Kriz, Mandell and May are the most widely used modern category of spectra, and this talk should give an idea of what they are and how they work. One reason for the widespread use of S-modules is probably that [28] treats many applications of structured ring spectra in this context, and thus provides a comprehensive reference. On the other hand, the book [28] is not self-contained, and preparing this talk can be a challenge without previous exposure to the material.
   Required: Before digging into [28] and [54], Elmendorf's introduction [29] and the article [27] are worth reading. The construction of the category of S-modules should be sketched, as well as the smash product and the model structure. Sketch Chapter VII of [28], explaining how modules over a fixed S-algebra and the categories of S-algebras and commutative S-algebras inherit model structures.
   Desirable: Explain the concept of Bousfield localization for spectra [15, 16], and how this is implemented via E-local model structures in Chapter VIII of [28].

8. **Other kinds of spectra**
   We have chosen symmetric spectra as the main example in the AG, and we devote one talk to S-modules in the sense of [28]. However, many other varieties of spectra with good smash products exist, and each has its own virtues and disadvantages. This talk should present the general 'diagram spectra' method [61] and discuss generalizations to equivariant and motivic homotopy theory; it will have a survey character, concentrating on concepts, and with few or no proofs.
   Required: Explain how categories of continuous, space-valued functors from a suitable symmetric monoidal indexing category give rise to model categories of 'diagram' spectra with smash product [61]; point out how symmetric spectra, orthogonal spectra [61, Ex. 4.4], and simplicial functors [56] are examples. Point out that for orthogonal spectra and simplicial functors the stable equivalences agree with the π_*-isomorphisms.
   Desirable: Sketch how this approach can be adapted to equivariant homo-

topy theory [60, 59, 22, 39] and motivic homotopy theory [48, 23].

9. **Algebraic K-theory**
A good deal of the development of A_∞-ring spectra was motivated by the application of algebraic K-theory to high-dimensional geometric topology. This talk should introduce the algebraic K-theory of a ring spectrum and give some motivation for studying it.
Required: Waldhausen's $S_\bullet$-construction [112], [111, Sec. 1] and Hesselholt's recognition of the iterated $S_\bullet$-construction as a symmetric spectrum [33, Sec. 6]. The algebraic K-theory of a structured ring spectrum via the $S_\bullet$-construction [28, VI]. Algebraic K-theory is invariant under Quillen-equivalences [21, Cor. 3.10], [85]. Quillen's K-theory of a ring and algebraic K-theory of Eilenberg-MacLane spectra [28, VI.4]. Reserve some time for algebraic K-theory of the sphere spectrum and spherical monoid rings $S[\Omega X]$, its relation to Waldhausen's $A(X)$ and the spaces of h-cobordisms or concordances/pseudoisotopies on X when X is a manifold; possible references for this part are [110, Sec. 3], [113] and [26, Sec. 9].
Desirable: Alternative definitions of the algebraic K-theory of a structured ring spectrum in the spirit of Quillen's plus-construction [110, Sec. 2], [14, Def. 5.4], [88, A.2]. Rational calculations of $A(*)$ [110, Thm. 3.2]. The construction due to Elmendorf and Mandell [31] of E_∞-symmetric ring spectra starting from bipermutative categories.

10. **Units, Thom spectra and orientations**
The notion of 'units' of a structured ring spectrum is more subtle then the classical counterpart, and is related to orientation theory, Thom spectra and algebraic K-theory. The units of a structured ring spectrum form a loop space and in the presence of enough commutativity (i.e., for E_∞- or strictly commutative ring spectra), the units even form an infinite loop space.
Required: Present a construction of the units $FE = GL_1(E)$ of a structured ring spectrum E as either a topological group, group-like topological monoid or loop space (which are three faces of the same coin); show that for strictly commutative/E_∞-ring spectra, the units are an infinite loop space, i.e., the underlying space of an Ω-spectrum [64, Ch. I–IV], [87, Sec. 2 and 5]. Explain how the homotopy fiber of $BG = BGL_1(S) \to BGL_1(E)$ classifies E-oriented spherical fibrations [63, Sec. 14], [64, Ch. III]. Illustrate with (i) Adams' analysis of the J-homomorphism [63, Sec. 18], [64, Ch. V],

[1], using the Atiyah–Bott–Shapiro ko-orientation for Spin bundles and the cannibalistic characteristic class, or (ii) the Sullivan $ko[1/2]$-orientation for STop-bundles, as relevant to the surgery classification of topological manifolds [63, Sec. 18], [64, Ch. V], [100].

Desirable: Show how A_∞-maps to BO or BG give rise to Thom spectra that are associative ring spectra and E_∞-maps give rise to commutative ring spectra [54, Ch. IX]. Sketch Mahowald's recognition of several familiar spectra as such Thom spectra [58]. Indicate how null-homotopies through A_∞- or E_∞-maps of composites to $BGL_1(E)$ give associative or commutative ring spectrum maps from these Thom spectra to E. Illustrate with $MSpin \to ko$ (or $MO\langle 8\rangle \to tmf$ if the Ando–Hopkins–Rezk preprint becomes available).

11. **Power operations and the Adams spectral sequence**

This talk serves two purposes: to introduce power operations and acquaint us with the Adams spectral sequence. An E_∞-multiplication on a spectrum, or equivalently a model which is a commutative symmetric ring spectrum, can be used to construct *power operations*, which are powerful, calculational tools. Power operations come in various shapes and sizes, for example as cohomology operations, as operations in the homotopy or homology groups of a ring spectrum, or as operations in Ext-groups of a commutative Hopf algebra. Part of the work in preparing this talk should be to scan the literature and make up a coherent presentation of the material. One main reference is [18], which however uses H_∞-ring spectra; every E_∞-ring spectrum is also H_∞, and in order to minimize confusion, the speaker should prepare the talk for E_∞-ring spectra only and avoid the term 'H_∞'. General background on power operations is in [18, I.4], see also [78, Sec. 7]. The other aim of the talk is to introduce the Steenrod algebra and the Adams spectral sequence and use power operations to make calculations therein. The speaker should not be afraid of naming explicit elements and chasing them around in the Adams spectral sequence by operations and differentials.

Required: Start by explaining the 'cup-1-construction' Sq_1 [8, § 6-7] and its properties listed in [8, § 6]. Prove that $Sq_1(2\iota) = \eta$ [18, V Prop. 1.12] and deduce Toda's relation $\eta x \in \langle 2, x, 2\rangle$. Introduce the mod-$p$ Steenrod algebra and the Adams spectral sequence [2, III.15], [66, II.9], [103, Ch. 19]; discuss the 1-line and the 'Hopf-invariant one' problem [3]. Illustrate how power operations can be used to calculate differentials in the Adams spectral sequence [18, VI], [67]; give examples (such as Cor. 1.5, Prop 1.6 and others from [18, VI] or [67, Sec. 6]), rather then spending too much time on the general theory.

Desirable: Explain how Steenrod operations and Dyer-Lashof operations are power operations. Discuss 'geometric' constructions of power operations in theories with 'effective cycles', such as topological K-theory (by powers of vector bundles) or bordism (by powers of manifolds).

12. **Complex cobordism, formal groups, and MU-algebras**
Complex oriented cohomology theories are an important and much studied class of generalized cohomology theories; 'complex oriented' refers to the fact that such theories come with characteristic classes for complex vector bundles. Complex oriented cohomology theories are intimately related to formal group laws (here always one-dimensional and commutative) by a celebrated theorem of Quillen which says that the Thom spectrum MU carries the universal formal group law. But MU is also a commutative structured ring spectrum, and there is a systematic way to construct many important complex oriented theories as MU-algebra spectra.
Required: Introduce MU as a commutative symmetric ring spectrum (a slight twist is needed here since MU is naturally indexed on smash powers of the 2-dimensional sphere). Explain the homotopy ring of MU via the Adams spectral sequence [103, Thm. 20.27]; point out the special role of the v_n-classes as elements of Adams filtration 1. Complex orientations for cohomology theories, formal group laws and Quillen's theorem [2, Part II], [75, Ch. 3]. Constructing MU-algebra spectra by killing a regular sequence [28, Ch. V], [102], [34]. Discuss examples related to the height filtration of formal group laws, at least BP, $E(n)$ and $K(n)$.
Desirable:

13. **Obstruction theories**
This talk should give an overview about the obstruction theories available for constructing A_∞-structures and E_∞-structures (or equivalently: showing that a given symmetric spectrum is stably equivalent to a symmetric ring spectrum respectively commutative symmetric ring spectrum). Examples where these methods have been used successfully should be discussed. Robinson's survey article [81] is a good place to start, and the article [10] by Basterra and Richter should also be helpful; both contain many further references. The emphasize should be on E_∞-/commutative structures. This is another challenging talk since the speaker should draw from various different sources and mathematical areas.
Required: Explain the main ideas behind one or both of the obstruction theories for E_∞-structures by Goerss-Hopkins [37, 38] and by Robinson-

Whitehouse [82, 80]. Point out that these two obstruction theories have very different origins, but lead to isomorphic obstruction groups, see [10, Thm. 2.6]. Use the obstruction theory to outline a proof of the Hopkins-Miller theorem on the rigidification of Lubin-Tate spectra [76], [37]; the main results are Corollaries 7.6 and 7.7 of [37].
Desirable: Outline the obstruction theory approach by refining the k-invariants in a Postnikov tower to derivations [51]; details can be found in [52] for the A_∞-case and in [9] for the E_∞-case.

14. **Galois theory**
This talk should give a survey of Galois theory for structured ring spectra, following [83]; the paper [7] may also be useful. Supposedly, we all know the classical Galois theory for fields, but the talk should start by reviewing the algebraic Galois theory for commutative rings. Then Galois theory for ring spectra should be discussed with a particular focus on new phenomena and examples which have no counterparts in algebra. The speaker should avoid getting caught in formalities and make sure he/she gets to the interesting topological examples in Section 5 of [83]; here a significant amount of background in stable homotopy theory is needed.
Required: Galois theory for classical commutative rings with examples. Galois extensions of commutative ring spectra [83, Sec. 4]. Examples, in particular 5.2, 5.3, 5.4.5 of [83]. Galois theory for ring spectra [83, Sec.7, 11]

15. **Rigidity for the stable homotopy category**
The final three talks are devoted to a proof of the rigidity theorem which says that any two models for the stable homotopy category are Quillen equivalent. The various parts of the proof are contained in three papers [95, 91, 93]. The first talk should set the stage and provide the universal property of the model category of sequential spectra (of Bousfield-Friedlander type).
Required: Statement of the rigidity theorem [93]; examples of non-rigid triangulated categories [91, Ex. 2.1, 2.2], [21, Rem. 6.8]; universal property of the model category of spectra [95, Thm. 5.1]; the proof of the universal property should be given in the (easier) case of *simplicial* model categories [95, Constr. 6.2].
Desirable: Prove the (weak form of the) rigidity theorem for π_*^s-linear equivalences ('Uniqueness Theorem' of [95]). Interpret the rigidity theorem from the perspective of 'Morita theory for ring spectra' [96, Ex. 3.2 (i)] [95, Rk. 5.4].

16. **Reduction to Adams filtration 1**
This talk should describe the argument from [91] which reduces the proof of the rigidity theorem to the verification of the behavior of the Hopf maps and the p-local stable homotopy class α_1 for every odd prime p.
Required: Sketch the construction of the 'Greek letter families' in the stable homotopy groups of spheres, in particular the α- and β-families [75, Sec. 2.4], [74]. Explain how α_1 is detected by the Steenrod operation P^1 and how any extension of β_1 to the mod-p Moore spectrum is detected by the Steenrod operation P^p. Explain the reduction argument, Prop. 3.1 and 3.2 of [91], and the 2-local case of the rigidity theorem.
Desirable: Provide a proof of the relations $\eta^3 = 4\nu$ and $8\sigma \in \langle \nu, 8, \nu \rangle$ between the Hopf maps, for example using the Adams spectral sequence.

17. **Proof of the rigidity theorem**
In this last talk, the different pieces should be assembled to a proof of the rigidity theorem, following [93].
Required: Coherent modules over a Moore space [93, Sec. 2], and finally, the proof.
Desirable: Elucidate the relationship between the 'coherent modules' used in the proof and the A_n-modules over A_n-spaces in the sense of Stasheff [101] or in the operadic sense.

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Participation:

The idea of the Arbeitsgemeinschaft is to learn by giving one of the lectures in the program.

If you intend to participate, please send your full name and full postal address to

`schwede@math.uni-bonn.de`

by August 15, 2005 at the latest.

You should also indicate which talk you are willing to give:
First choice: talk no. ...
Second choice: talk no. ...
Third choice: talk no. ...

You will be informed shortly after the deadline if your participation is possible and whether you have been chosen to give one of the lectures.

The Arbeitsgemeinschaft will take place at Mathematisches Forschungsinstitut Oberwolfach, Lorenzenhof, 77709 Oberwolfach-Walke, Germany. The institute offers accomodation free of charge to the participants. Travel expenses cannot be covered. Further information will be given to the participants after the deadline.