Arbeitsgemeinschaft mit aktuellem Thema:

Higher Torsion Invariants in Differential Topology and Algebraic K-Theory

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Introduction:

The classical Franz-Reidemeister torsion and its cousins, the Whitehead torsion and Ray-Singer analytic torsion, are topological invariants of manifolds with local coefficient systems (or flat vector bundles) that can distinguish homotopy equivalent spaces that are not homeomorphic. In this Arbeitsgemeinschaft, we want to sketch several natural generalisations of these classical invariants to families of manifolds.

Regard a family $p\colon E \to B$ of compact manifolds M, equipped with a flat vector bundle $F \to M$. Then the fibrewise cohomology groups $H^\bullet(E/B; F)$ form flat vector bundles over the base B. The starting point for our investigations are analogues of the Atiyah-Singer family index theorem that relate F to $H^\bullet(E/B; F)$. To a flat vector bundle $F \to M$, one associates

Kamber-Tondeur characteristic classes $c_\bullet(F)$ in $H^{\mathrm{odd}}(M;\mathbb{R})$, which vanish if F carries a parallel metric. By Bismut-Lott [8], one has

$$\sum_i (-1)^i c_\bullet\big(H^i(E/B;F)\big) = \int_{E/B} e(TM)\, c_\bullet(F) \quad \in H^*(B;\mathbb{R})\ ,$$

where $e(TM)$ is the Euler class of the vertical tangent bundle, and the right hand side is the Becker-Gottlieb transfer in de Rham cohomology. If one specifies some additional geometric data, then all classes above are naturally represented by specific differential forms. On the level of differential forms, the equation above only holds up a correction term $d\mathcal{T}$. Here $\mathcal{T}$ is the higher analytic torsion, which depends naturally on the fibration and the geometric data. If both $H^\bullet(E/B;F)$ and F admit parallel metrics, then $\mathcal{T}$ gives rise to a secondary characteristic class $\mathcal{T}(E/B;F) \in H^{\mathrm{even},\geq 2}(B;\mathbb{R})$. An introduction to this approach is given in [29].

Dwyer-Weiss-Williams [24] construct Reidemeister torsion for a smooth fiber bundle $p\colon E \to B$ as a byproduct of a family index theory. If p is any fiber bundle with fibers compact topological manifolds and base a CW complex, then the family index theory states that $\chi(p)$, the A-theory Euler characteristic of p is determined by the A-theory Euler class of $\tau_{fib}(p)$, the tangent bundle along the fiber. Here A-theory is algebraic K-theory of spaces in the sense of Waldhausen. More precisely, by applying fiberwise Poincare duality, and then an assembly map to the A-theory Euler class, one gets the A-theory Euler characteristic. If p is a smooth bundle, then one gets a stronger smooth index theorem where the A-theory Euler class is replaced by the Becker-Euler class, which lives in the (twisted) stable cohomotopy of E. When B is a point this result is equivalent to the classical Poincare-Hopf theorem.

The third approach is due to Igusa-Klein [33] (see [34] for an introduction), and is somewhat different in nature. Here, one regards a generalised fibrewise Morse function on $M \to B$. Together with a flat vector bundle $F \to M$, this gives rise to a classifying map from B to a Whitehead space, and the higher Franz-Reidemeister torsion is the pullback of a universal class on the Whitehead space. There are conjectural relations between all three definitions of higher torsion.

It turns out that higher torsion invariants are somewhat finer than classical FR torsion, since they detect higher homotopy classes of the diffeomorphism group of high-dimensional manifolds that vanish under the forgetful map to the homeomorphism group. In particular, these invariants distinguish differentiable structures on a given topological fibre bundle $M \to B$, where one may even fix differentiable structures on M, B and the typical fibre. There are also applications of higher torsions to problems in graph theory and moduli spaces of compact surfaces. Some of these will be sketched throughout this Arbeitsgemeinschaft.

We have grouped the talks as follows.

1. The first talk gives a short introduction to classical torsion invariants.

2. In talks 2–7, we discuss the Dwyer-Weiss-Williams homotopy theoretical approach.

3. Parametrized Morse theory, Kamber-Tondeur classes and Igusa-Klein torsion are discussed in talks 8–16, and some applications are given.

4. Finally, based on talks 10 and 11, we introduce analytic torsion in the talks 17–19.

Talks:

1. **Classical torsion invariants (overview).**
   Recall the definition of Franz-Reidemeister torsion, together with its geometric motivation and application to lense spaces in [48], [26]. Mention simple homotopy theory and Whitehead torsion [14]. Define Ray-Singer analytic torsion [47] and state the Cheeger-Müller theorem, which relates Ray-Singer torsion to Franz-Reidemeister torsion, see [13], [45]. If time permits, sketch Bismut-Zhang's proof by Witten deformation following [9] (the equivariant aspects should be neglected).

2. **Waldhausen K-Theory.**
   This talk should summarize the definition of Waldhausen K-theory, state the main properties: additivity and fibration theorem and give the three main examples relevant to pseudoisotopies:

   (a) finite sets, giving QS^0;

   (b) finitely generated projective complexes, giving the Quillen K- theory of a ring;

   (c) finite cell complexes over X, giving $A(X)$.

Focus should be on the third example, which is fundamental for some of the following talks.

References: [49] and [50], sections II.9 and IV.6.

3. **Homotopy limits as spaces of sections and construction of $\chi(p)$, I.**

   Regard a family $p\colon E \to B$ of smooth compact manifolds M. When B is not a point, then $\chi(p)$ is not just an integer. Instead it is a section of a certain associated fibration, and the construction of $\chi(p)$ uses the machinery of homotopy limits and colimits.

   Define the homotopy limit and colimit of a functor $\mathcal{F}\colon \mathcal{C} \to \mathit{Spaces}$, where Spaces is the category with objects spaces homotopy equivalent to CW complexes and maps homotopy equivalences. Discuss the special case when $\mathcal{F}$ factors thru a functor $\bar{\mathcal{F}}\colon \mathcal{C} \to \mathit{Cat}$, the category of small categories. In that case a point in the homotopy limit of $\mathcal{F}$ is determined by a rule which assigns to each object c in $\mathcal{C}$ an object $\chi(c)$ in $\bar{\mathcal{F}}(c)$ and which assigns to any map $\alpha\colon c_1 \to c_2$ a map $\chi(\alpha)\colon \bar{\mathcal{F}}(\alpha)_*(\chi(c_1)) \to \chi(c_2)$. (assuming the rule satisfies a certain 1-cocylcle condition, see 1.3 in [24].) We call such a rule χ, a *characteristic*. A key example for us is when $\mathcal{C}$ is the category of finitely dominated spaces with maps homotopy equivalences, $\bar{\mathcal{F}}(X)$ is the category of retractive spaces over X, and $\chi(X)$ is the retractive space $S^0 \times X$(see p. 40-41 in [24]).

   Suppose $\mathcal{C} = \mathit{Simp}(B)$, the category of maps of standard simplicies into the connected CW complex B. Then the homotopy colimit is equivalent to the total space of a fibration over B with fibers homotopy equivalent to $\mathcal{F}(\Delta^0 \to B)$, and the homotopy limit is equivalent to the space of sections of that fibration.

   References: see next talk.

4. **Homotopy limits as spaces of sections and construction of $\chi(p)$, II.**

   Recall the description of $A(X)$ as the K-theory of finitely dominated retractive spaces over X, cf. talk 2. Give the construction of $\chi(p)$ as a section of the fibration $A_B(E) \to B$, where the fiber over $b \in B$ is $A(p^{-1}(b))$. Here p is any fibration with finitely dominated fibers and B a CW complex. If $B = *$, then $\chi(p)$ can be identified with the classical Euler characteristic of E plus Wall's finiteness obstruction. In the special case where $p\colon E \to B$ is the universal fibration with fibers homotopy equivalent to F, then $\chi(p)$ can be viewed as a "crossed homomorphism" from $G(F)$ to $A(F)$ which becomes the Whitehead torsion map when composed with $A(F) \to Wh_1(\pi_1 F)$. (Here $G(F)$ is the topological monoid of self homotopy equivalences of F.)

The map from B to a point induces a map $A_B(E) \to A(E)$. Linearization gives us a map from $A(E) \to K(Z\pi)$, where π is the fundamental group of E. A representation ρ of π on a f.g. projective module P over a ring R then induces a map $K(Z\pi) \to K(R)$. Let $\chi_\rho(p) : B \to K(R)$ be the map gotten by composing $\chi(p)$ and these maps. Explain that when R is a regular ring, then $\chi_\rho(p)$ has a very simple description in terms of $\pi_1(B)$ acting on the homology of the fibers of p with coefficients in P.

References:

Homotopy limits and colimits: [10], [23] (especially 3.12), Sec. 1 of [24]

A-theory: [49]

Construction of $\chi(p)$: Sections 1 and 6 of [24]

Computation of $\chi_\rho(p)$: p. 43-46 of [24]

5. **Controlled topology and disassembly of $\chi(p)$.**
Give a controlled interpretation of the natural transformation $w : A \to Wh$, where the A-theory assembly map is the homotopy fiber of w (see 7.5 and 7.6 in [24]). Suppose $\mathcal{C}$ from the last lecture is a category of compact topological manifolds and homeomorphisms. We want to use the controlled description of w plus an Eilenberg swindle argument to trivialize the construction of χ composed with w. In fact, to give us more flexibility we want to enlarge $\mathcal{C}$ to the category of euclidean neighborhood retracts where a map from X to Y is given by a continuous, proper cell-like continuous map from an open subset of X to Y (see p.51 -54 in [24]). The goal is to get a characteristic $\chi^{\%}$ where $A(X)$ is replaced by the locally finite homology of X represented by $A(*)$.

Suppose F is a compact topological manifold. This characteristic determines a lifting $\chi^{\%}(p)$ of $\chi(p)$ thru the fiberwise A-theory assembly map for fiber bundles p with structure group $Top^{\delta}(F)$, the topological group of homeomorphisms equipped with the discrete topology. The construction of $\chi^{\%}(p)$ for all fiber bundles with fibers homeomorphic to F is gotten by using McDuff's result that the map $BTop^{\delta}(F) \to BTop(F)$ is a homology equivalence(see [24] and [43]) . In the special case when p is the universal fiber bundle with fibers homeomorphic to F, the trivialization of $w(\chi(p)$ can be viewed as a parametrized version of topological invariance of Whitehead torsion.

In the next lecture we'll use $\chi^{\%}$ to construct A-theory Euler classes for Eucildean bundles.

References:

Controlled topology: [1], [46], [51]

McDuff's Theorem: [43]

Construction of $\chi^{\%}$: Sections 2 and 7 of [24]

6. **Homology and cohomology with coefficients in a parametrized spectrum.**
   Give definition of homology and cohomology with coefficients in a parametrized spectrum. Give construction of the Becker-Euler class of a vector bundle (see p. 29-30 in [24]). Give "discrete model" for the classifying map of the euclidean tangent bundle of a manifold(see Theorem 3.2). Describe Poincare duality and inverse Poincare duality via "scanning" (p.20-29 in [24]).

   Notice that after applying Poincare duality the Becker-Euler class of the tangent bundle of a smooth compact manifold M lives in 0-dim. homology of M with coefficients in the sphere spectrum. Show that the fiberwise Poincare dual of the tangent bundle along the fiber of a smooth bundle is the Becker-Gottlieb-Dold transfer (see Theorem 5.4 and [20]). Notice the analogy between this result and the index theorem which is our main goal.

   Give construction of the A-theory Euler class of a Euclidean bundle (see p. 15-18 in [24]). The unit map from $QS^0 \to A(*)$ induces a map which sends the Becker-Euler class of a vector bundle to the A-theory Euler class of the underlying Euclidean bundle (see Thm 4.10 in [24]).

References

Parametrized spectrum: [3], Remark 2.12 in [24], [42]

Becker-Gottlieb-Dold transfer: [4], [5], [16], [17], [21]

Becker-Euler class: [3], [15], [40]

7. **Proof of index theorem and construction of Reidemeister torsion.**
   Give proof of index theorem when B is a point using scanning description of Poincare duality (see p 25-26 of [24]). Proof of index theorem for families (see p.26-28 of [24]). Proof of smooth index theorem for families (see p. 39 of [24]). Construction of Reidemeister torsion (see p.64-65).

   References: [20], [18], [19], [2]

8. **Generalized Morse functions.**
   This talk should give the definition of generalized Morse functions and framed functions and state the main theorem. Eliashberg's proof using wrinkled maps [25] should be outlined.

   A *generalized Morse function (GMF)* on a smooth manifold M is a smooth function having only nondegenerate and cubic (A_3) singularities. Eliashberg showed that this space satisfies an h-principle: it has the homotopy type of the corresponding space of sections of the jet bundle. [30] shows that this has the $\dim M$-homotopy type of $Q(M_+ \wedge BO)$.

   A *framed function* on M is a GMF $f\colon M \to \mathbb{R}$ together with a framing of the nonpositive eigenspace of the second derivative of f so that the last vector lies in the null space of the second derivative and points in the positive cubic direction. The space of framed functions is $\dim M$-connected [31]. This implies that, for a smooth fibration $F \to M \to B$ with $dimB \leq dimF$, there exists a fiberwise framed function $f\colon M \to \mathbb{R}$ which is unique up to framed homotopy if $dimB < dimF$.

9. **A-infinity functors and fiberwise Morse functions.**
   This talk should explain the basic definition of an A-infinity functor. When restricted to an ordinary category we obtain a twisting cochain on the nerve of the category. This construction is basically due to Ed

Brown. One elementary example is singular homology (and cohomology) with coefficients in a field, see [37], [39], and other papers by B. Keller.

When we have a fiberwise Morse function, i.e., a smooth function on the total space of a bundle which is Morse on every fiber, then we can triangulate the base B and obtain a cellular chain complex over every simplex. The mapping which sends each simplex to the corresponding cellular chain complex is an A_∞ functor on the category of small simplices of B, or equivalently a twisting cochain on B, see chapter 4 of [33] and lecture 3 in [34] (This will be smoothly interpolated to produce a finite dimensional flat superconnection to give the higher FR torsion in another talk).

The de Rham analogue of a twisting cochain is a flat superconnection of total degree 1 as in section 1.a,b of [8]. If time permits, state theorem 1.66 and proposition 1.70 in [27], and theorem 6.20 in [28].

10. **Kamber-Tondeur classes.**
Recall equivalent definitions of flat vector bundles as representations of the fundamental group, as locally constant sheafs, and as vector bundles with a flat connection. Briefly recall Chern classes of vector bundles, and the construction of Chern-Weil forms for vector bundles with connection.

There is a parallel construction of Kamber-Tondeur classes $c_{2k+1}(F)$ for flat vector bundles, and of Kamber-Tondeur forms $c_{2k+1}(F, g^F)$ for flat vector bundles with Hermitian metrics g^F, see [22]. These classes are related to the Borel regulator map, see the introductions to [8] and cite Ibook.

The Kamber-Tondeur forms of one flat bundle with two different metrics g_0^F, g_1^F differ by an exact form $d\tilde{c}_{2k}(F, g_0^F, g_1^F)$, where $\tilde{c}_{2k}(F, g_0^F, g_1^F)$ is naturally defined up to exact forms. There is a natural generalisation of this construction to vector bundles with flat superconnections as in section 1 of [8].

If time permits, give the equivalent construction of ch^o in [6], and note that the higher Kamber-Tondeur classes are invariant under smooth deformations of the flat connection, see [6], sections 2.6, 2.7.

11. **Finite-dimensional torsion classes.**
Let $V \to M$ be a $\mathbb{Z}$-graded vector bundle with a flat connection ∇^V. Flat superconnections A' of total degree 1 on V as in section 1, 2 of [8] are analogous to Igusa's A_∞-functors in talk 9. The fibrewise cohomology becomes a bundle $H \to M$ with a natural flat Gauß-Manin connection and an induced metric g^H.

If $A' - \nabla^V$ takes values in a nilpotent subalgebra of $\mathrm{End}(V)$, one can construct a torsion form $T(A', g^V) \in \Omega^{\mathrm{even}}(M)$ such that

$$c_\bullet(V, g^V) - c_\bullet(H, g^H) = dT(A', g^V) .$$

The easiest construction of this type can be found in section 2 of [8], where $A' - \nabla^V$ is a parallel fibrewise differential. The "finite-dimensional index theorems" 2.22 and 2.24 should be stated. Remark that for families of acyclic complexes, the class $T(A', g^V)$ can be defined axiomatically as in [8], appendix I.

For the construction of higher Franz-Reidemeister torsion and for a comparison with higher analytic torsion, we need a slightly more involved construction where $A' - \nabla^V$ is now upper triangular with respect to a fibrewise Morse function. Two such constructions are described in section of 2.4 of [33] and in sections 2.b–f of [27], at least one of them should be described in some detail. By the results of section 8 in [28], both give the same universal class, which can be regarded as a Borel regulator in Volodin K-theory by section 1 of [33].

12. **Polylogarithms.**
Polylogarithms arise in the calculation of higher FR torsion because a GMF on a manifold M gives rise to a GMF on any covering space of M and the resulting cellular chain complex is related to the chain complex of M by a transfer formula. In Milnor's paper [44] on Kubert functions, he shows that any invariant which satisfies this transfer formula must either be a polynomial or a polylogarithm depending on parity. This talk should explain this characterization of polylogarithms and how the observation about covering spaces in chapter 2 of [33] forces the higher torsion to satisfy this characterization.

13. **Definition of higher Franz-Reidemeister torsion, I.**
John Klein was the first to define higher FR torsion in the special case

of a smooth bundle with a fiberwise Morse function (a function on the total space of a smooth bundle which is Morse on each fiber). His idea was to use a linearized version of Waldhausen's $A(X)$. A variation of this construction was given in an unpublished manuscript of Igusa-Klein [38]. This is the *filtered chain complex* model for algebraic K-theory. (See [33], [37], [32].)

The theorem of Igusa-Klein is that, for any coefficient system F of f.g. free R modules over a space X, there is a homotopy fiber sequence

$$FC^h(X, F) \to \Omega^\infty S^\infty(X_+) \to Z \times BGL(R)^+$$

where $FC^h(X, F)$ is the space of acyclic filtered chain complexes which are direct sums of finitely many stalks of the coefficient sheaf F. If we take $R = \mathbb{C}$ (the complex numbers) and $X = K(\pi, 1)$ where π is a finite group then the Borel regulator classes which are odd degree cohomology classes on $BGL(\mathbb{C})^+$ will transgress to even degree cohomology classes on $FC^h(X, F)$. These are universal higher Franz-Reidemeister torsion classes.

This first talk should explain how a fiberwise Morse function on the total space of a smooth bundle $E \to B$ should give rise to a mapping $B \to FC(E, F)$ for any coefficient system F over E. In certain cases this map can be modified to land in $FC^h(E, F)$. The definition of $FC(E, F)$ is designed to work for a fiberwise generalized Morse function. The framed function theorem [31] is used to give a canonical choise of the fiberwise GMF.

14. **Definition of higher Franz-Reidemeister torsion, II**
The definition of the higher FR-torsion becomes more concrete with the following:

    (a) the Kamber-Tondeur form gives one formula for the Borel regulator

    (b) we replace the category of fitered chain complex by the A_∞ category of f.g. cellular chain complex: $FC^h(BG, R) \simeq Wh^h(R, G)$.

    (c) we use the two index theorem to replace $Wh^h(R, G)$ with a space of invertible matrices.

(d) the Kamber-Tondeur form transgresses to this two index version of $Wh^h(R, G)$ when R is a matrix ring over $\mathbb{C}$ and G is the unitary group $U(n)$.

The purpose of this talk is to put together the topics of the (finite dimensional) Kamber-Tondeur form and the A_∞ structure associated to a fibration and the category of filtered chain complexes. The second chapter of the memoires paper [37] is best for this explanation since detailed proofs in [33] should not be explained.

15. **Axioms for higher torsion.**
Higher (nonequivariant) torsion can be characterized by two axioms: additivity and transfer. Any characteristic class of unipotent smooth bundles satisfying these two axioms has an even and odd part each of which is unique up to a scalar multiple in each degree $4k$. The unique even torsion is the higher tautological (or Miller-Morita- Mumford) class. The unique odd torsion is the higher FR torsion. This shows in particular that the even part of the higher FR torsion is proportional to the MMM class in the same degree.

One of the key points in both the proof and applications of this axiomatic approach is the calculation of the higher torsion of the exotic smooth bundles constructed by Allen Hatcher. This talk should use the axioms to compute the odd torsion for this example. The existence and uniqueness of odd and even torsion theories should be clearly explained. See the lecture notes [35] and arXiv notes [36].

16. **Computation of higher torsion.**
The main point of this talk is the Framing Principle which is the main tool for computing the higher FR-torsion. The statement is the following.

Suppose $E \to B$ is a smooth bundle and F is a 1-dimensional hermitian coefficient system on E with respect to which the bundle is unipotent (i.e., $\pi_1 B$ acts unipotently on the homology of the fiber with coefficients in F). Suppose that $f : E \to \mathbb{R}$ is a fiberwise GMF. Then the higher FR torsion of the family of chain complexes over B given by f differs from the higher FR-torsion given by the canonical fiberwise framed function, say g, by a multiple of the push-down of the chern character of the negative eigenspace bundle of f:

$$\tau_{2k}(C(g)) = \tau_{2k}(C(f)) + (-1)^k \zeta(2k+1) p_*(ch_{2k}(\gamma_f) \otimes \mathbb{C})$$

where ζ is the Riemann zeta function.

This is proved in a special case in [33] and in general in [37]. The analogous statement for higher analytic torsion is shown in [6] and [27]. Combining the Framing Principle with the basic properties of higher FR torsion, we can compute the higher FR-torsion in many cases.

The numerous formulae are: involution, suspension, splitting (additivity), stability, relative case, transfer. At the end of the arxiv paper [36], the Framing Principle is used to show that higher FR torsion satisfies the transfer axiom (additivity is clear).

17. **The Bismut-Lott index theorem and higher analytic torsion.** Using the Kamber-Tondeur forms of talk 10, define higher analytic torsion as in Definition 3.22 of [8], and state the Bismut-Lott index theorem 0.1 and its geometric refinement 0.2. If time permits, sketch a proof of theorem 0.2 along the lines of chapter 3 in [8] (it helps to also sketch the proof of theorems 2.22 and 2.24). Remark that theorem 0.2 allows to define a cohomological torsion invariant if both the flat bundle on the total space and its fibrewise cohomology carry parallel metrics; see [27], Definition 2.88.

18. (optional) **The Witten deformation and the analytic framing principle.**
    Conjecturally, the higher analytic torsion of the previous talk is related to Igusa-Klein's higher Franz-Reidemeister torsion. Note that the axioms stated in talk 15 have not been checked completely for the higher analytic torsion.

    In the special case that $E \to B$ admits a fibrewise Morse function, we can prove this conjecture using the Witten deformation. In the process, we analytically recover Igusa's framing principle stated in talk 16. The main results in chapter 7 of [6] and theorem 0.3 in [28] should be stated.

    The main steps in the proof are

    (a) the Witten deformation as in chapter 8 of [6],

    (b) several convergence results stated in chapter 9,

(c) a computation near the critical points in chapter 4 that gives the "analytic framing principle",

(d) the identification of the contribution from the "small eigenvalues" with the finite-dimensional torsion form of talk 11 in chapters 9, 10 of [6] and chapter 4 of [27],

(e) a comparison with Igusa's constructions in chapters 6 and 8 of [28].

There will not be time to explain any details.

As an application, theorem 0.2 of [27], [28] should be stated (this is based on Hatcher's example of talk 15, and is only slightly more general than the applications given there).

19. (optional) **Higher Equivariant torsion and some computations.** For fibre bundles with compact structure group, the higher analytic torsion can be understood and computed using ideas from equivariant index theory.

    Define equivariant Franz-Reidemeister torsion for actions of compact Lie groups G acting isometrically on a compact manifold and parallelly on a flat vector bundle $F \to M$, see [11]. Recall Köhler's computations for compact symmetric spaces [41]. Similarly define the infinitesimal equivariant analytic torsion following [8], chapter 4, [12] and [7], which is the universal higher analytic torsion for fibre bundles with compact structure group.

    Explain Bunke's results in [11] and [12] that the non-constant part of both equivariant torsions are functions of the G-Euler characteristic of M. Finally, state theorem 0.1 of [7], which relates both equivariant torsions, and explain the class V.

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Participation:

The idea of the Arbeitsgemeinschaft is to learn by giving one of the lectures in the program.

If you intend to participate, please send your full name and full postal address to

`sebastian.goette@mathematik.uni-regensburg.de`

by **Friday, 10th of February, 2006** at the latest.

You should also indicate which talk you are willing to give:
First choice: talk no. . . .
Second choice: talk no. . . .
Third choice: talk no. . . .

You will be informed shortly after the deadline if your participation is possible and whether you have been chosen to give one of the lectures.

The Arbeitsgemeinschaft will take place at Mathematisches Forschungsinstitut Oberwolfach, Lorenzenhof, 77709 Oberwolfach-Walke, Germany. The institute offers accomodation free of charge to the participants. Travel expenses cannot be covered. Further information will be given to the participants after the deadline.