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Von Neumann Algebras and Ergodic Theory of Group Actions

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Abstract. The theory of von Neumann algebras has seen some dramatic advances in the last few years. Von Neumann algebras are objects which can capture and analyze symmetries of mathematical or physical situations whenever these symmetries can be cast in terms of generalized morphisms of the algebra (Hilbert bimodules, or correspondences). Analyzing these symmetries led to an amazing wealth of new mathematics and the solution of several long-standing problems in the theory.

Popa's new deformation and rigidity theory has culminated in the discovery of new cocycle superrigidity results à la Zimmer, thus establishing a new link to orbit equivalence ergodic theory. The workshop brought together world-class researchers in von Neumann algebras and ergodic theory to focus on these recent developments.

Mathematics Subject Classification (2000): 46L10.

Introduction by the Organisers

The workshop *Von Neumann Algebras and Ergodic Theory of Group Actions* was organized by Dietmar Bisch (Vanderbilt University, Nashville), Damien Gaboriau (ENS Lyon), Vaughan Jones (UC Berkeley) and Sorin Popa (UC Los Angeles). It was held in Oberwolfach from October 26 to November 1, 2008.

This workshop was the first Oberwolfach meeting on von Neumann algebras and orbit equivalence ergodic theory. The organizers took special care to invite many young mathematicians and more than half of the 28 talks were given by them. The meeting was very well attended by over 40 participants, leading senior researchers and junior mathematicians in the field alike. Participants came from about a dozen different countries including Belgium, Canada, Denmark, France, Germany, Great Britain, Japan, Poland, Switzerland and the USA.

The first day of the workshop featured beautiful introductory talks to orbit equivalence and von Neumann algebras (Gaboriau), Popa's deformation/rigidity techniques and applications to rigidity in II_1 factors (Vaes), subfactors and planar algebras (Bisch), random matrices, free probability and subfactors (Shlyakhtenko), subfactor lattices and conformal field theory (Xu) and an open problem session (Popa). There were many excellent lectures during the subsequent days of the conference and many new results were presented, some for the first time during this meeting. A few of the highlights of the workshop were Vaes' report on a new cocycle superrigidity result for non-singular actions of lattices in $\mathrm{SL}(n,\mathbb{R})$ on $\mathbb{R}^n$ and on other homogeneous spaces (joint with Popa), Ioana's result showing that every sub-equivalence relation of the equivalence relation arising from the standard $\mathrm{SL}(2,\mathbb{Z})$-action on the 2-torus $\mathbb{T}^2$ is either hyperfinite, or has relative property (T), and Epstein's report on her result that every countable, non-amenable group admits continuum many non-orbit equivalent, free, measure preserving, ergodic actions on a standard probability space. Other talks discussed new results on fundamental groups of II_1 factors, L^2-rigidity in von Neumann algebras, II_1 factors with at most one Cartan subalgebra, subfactors from Hadamard matrices, a new construction of subfactors from a planar algebra and new results on topological rigidity and the Atiyah conjecture. Many interactions and stimulating discussions took place at this workshop, which is of course exactly what the organizers had intended.

The organizers would like to thank the Mathematisches Forschungsinstitut Oberwolfach for providing the splendid environnment for holding this conference. Special thanks go to the very helpful and competent staff of the institute.

Workshop: Von Neumann Algebras and Ergodic Theory of Group Actions

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Abstracts

Free products, Orbit Equivalence and Measure Equivalence Rigidity

Aurélien Alvarez
(joint work with Damien Gaboriau)

All the groups we consider are countable and measure spaces are standard Borel non-atomic probability spaces.

Consider two families of infinite countable groups $(\Gamma_i)_{i\in I}$ and $(\Lambda_j)_{j\in J}$, where I and J denote countable sets. Let α and β be two free probability measure preserving actions of the free products $\ast_{i\in I} \Gamma_i$ and $\ast_{j\in J} \Lambda_j$ on probability spaces whose restrictions to the factors $\alpha|\Gamma_i$ and $\beta|\Lambda_j$ are ergodic.

Both papers [IPP05], [CH08] establish rigidity phenomena in operator algebras and derive orbit equivalence results for the components of free products from an assumption of von Neumann stable equivalence on the actions. To this end, some strong algebraic constraints on the involved groups are imposed. More precisely in [IPP05, Cor. 0.5, Cor. 7.6, Cor. 7.6'], the analysis relies on the notion of relative property (T) in von Neumann algebras introduced by Popa in [Pop06], and thus the groups are required to admit a non virtually abelian subgroup with the relative property (T) and some ICC-like and normal-like properties (for instance, they may be ICC property (T) groups) (see [IPP05, Assumption 7.5.1]). In [CH08, Cor. 6.7], the operator algebraic notion involved is primality, so that the assumption on the groups Γ_p, Γ'_p is to be ICC non-amenable direct products of infinite groups. More generally, we can prove the following result (cf. [AG]):

Theorem 1. *If the groups Γ_i and Λ_j are non-amenable with vanishing first ℓ^2-Betti number ($\beta_1(\Gamma) = 0$) and if the actions α and β are stably orbit equivalent*

$$(\ast_{i\in I} \Gamma_i)\curvearrowright^{\alpha}(X,\mu) \overset{\mathrm{SOE}}{\sim} (\ast_{j\in J} \Lambda_j)\curvearrowright^{\beta}(Y,\nu),$$

then there is a bijection $\theta : I \to J$ (hence $Card(I) = Card(J)$) for which the restrictions are stably orbit equivalent

$$\alpha|\Gamma_i \overset{\mathrm{SOE}}{\sim} \beta|\Lambda_{\theta(i)}.$$

We would like to go further and extend the previous result in the context of *measure equivalence*. This notion was introduced by Gromov in 1993 as a measure theoretic counterpart of quasi-isometry (cf. [Gro93]). The connection with orbit equivalence was established by Furman in 1999 giving the following characterization.

Proposition 2 ([Fur99])**.** *Two groups Γ and Λ are measure equivalent if and only if Γ and Λ admit stably orbit equivalent actions.*

Let us recall that lattices in the same locally compact second countable groups are examples of measure equivalent groups, but also commensurable (up to finite kernels) groups. The class of the trivial group consists of all finite groups and the class of **Z** consists of all infinite amenable groups (which is a consequence of the invariance of this notion under amenability and Ornstein-Weiss theorem [OW80]). Let mention some well known invariants for this notion : amenability, Haagerup property, Kazdhan's property (T), finite non-zero cost, ℓ^2-Betti numbers up to scaling, the set $I_{ME}(\Gamma)$ of compression constants for a group Γ, etc. We introduce a new invariant, called *measurably freely indecomposability.*

In the class of groups, there is a obvious notion of *freely indecomposability.* A group Γ is *freely indecomposable* if it is not isomorphic with a free-product $\Gamma_1 * \Gamma_2$ with non-trivial Γ_i. In the context of measure equivalence relations, there is also a notion of *freely indecomposable* ($\mathcal{FI}$) equivalence relations but less obvious to define (cf. [AG]).

Definition 3. *A group Γ is measurably freely indecomposable ($\mathcal{MFI}$) if all its free probability measure preserving (p.m.p.) actions give rise to freely indecomposable measure equivalence relations.*

Of course, a free product of two infinite groups is not $\mathcal{MFI}$, and in fact none of its free p.m.p. actions is $\mathcal{FI}$ and the same holds for infinite amenable groups. On the other hand, freely indecomposable groups in the classical sense are not necessarily $\mathcal{MFI}$, for instance the fundamental group of a closed orientable surface of genus ≥ 2 which has just one end and which is measure equivalent to a free group (cf. [Gab05]).

Question 4. *Does there exist a countable group admitting an action giving rise to a $\mathcal{FI}$ equivalence relation and an action giving rise to a non-$\mathcal{FI}$ one ?*

We now give a quite large class of examples.

Theorem 5 ([AG]). *Every non-amenable countable group with vanishing first ℓ^2-Betti number is measurably freely indecomposable ($\mathcal{MFI}$).*

Recall that the ℓ^2-Betti numbers are a sequence of numbers $\beta_r(\Gamma)$ defined by Cheeger-Gromov (cf. [CG86]) attached to every countable group Γ and that they have a general tendency to concentrate in a single dimension r and to vanish in the other ones (see [BV97], [Lüc02]). The first ℓ^2-Betti number vanishes for many "usual" groups, for instance for amenable groups, direct products of infinite groups, lattices in $\mathrm{SO}(p,q)$ ($p.q \neq 2$), lattices in $\mathrm{SU}(p,q)$, groups with Kazhdan's property (T). It is worth noting that infinite property (T) groups follow $\mathcal{MFI}$ from Adams-Spatzier (cf. [AS90, Th 1.1], see also [Gab00, Ex. IV.12]). Other examples of vanishing β_1 are given by the groups with an infinite finitely generated normal subgroup of infinite index, groups with an infinite normal subgroup with relative property (T), amalgamated free products of groups with $\beta_1 = 0$ over an infinite subgroup, mapping class groups... On the other hand, for a free product of two (non trivial) groups we have $\beta_1(\Gamma_1 * \Gamma_2) > 0$ unless $\Gamma_1 = \Gamma_2 = \mathbf{Z}/2\mathbf{Z}$, in which case $\Gamma_1 * \Gamma_2$ is amenable. This fact is an immediate application of the following formulae: $\beta_1(\Gamma_1 * \Gamma_2) = \beta_1(\Gamma_1) + \beta_1(\Gamma_2) + 1 - (\beta_0(\Gamma_1) + \beta_0(\Gamma_2))$ and $0 \leq \beta_0(\Lambda) \leq 1/2$ for any countable group Λ.

We are now in position to state our general Measure Equivalence result (cf. [AG]):

Theorem 6 (ME Bass-Serre rigidity)**.** *Consider two families of infinite countable $\mathcal{MFI}$ groups (for instance non-amenable with vanishing first ℓ^2-Betti number) $(\Gamma_i)_{i\in I}$ and $(\Lambda_j)_{j\in J}$. If their free products are measure equivalent*

$$\mathop{*}_{i\in I} \Gamma_i \overset{\text{ME}}{\sim} \mathop{*}_{j\in J} \Lambda_j,$$

then there are two maps $\theta : I \to J$ and $\theta' : J \to I$ such that

$$\Gamma_i \overset{\text{ME}}{\sim} \Lambda_{\theta(i)} \quad \textit{and} \quad \Lambda_j \overset{\text{ME}}{\sim} \Gamma_{\theta'(j)}.$$

Moreover, if Γ_0, Λ_0 are two groups in the ME classes of some free groups, then the same conclusion holds under the assumption:

$$\mathop{*}_{i\in I} \Gamma_i * \Gamma_0 \overset{\text{ME}}{\sim} \mathop{*}_{j\in J} \Lambda_j * \Lambda_0.$$

Observe that we do not assume the generalized index $\kappa = 1$. Would we do so, we would not get $\kappa = 1$ in the conclusion. Also observe that the groups Γ_0, Λ_0 do not appear in the conclusion.

Recall that free p.m.p. group actions $\Gamma \curvearrowright^{\sigma} (X, \mu)$ define finite von Neumann algebras by the so called group-measure space construction of Murray-von Neumann or von Neumann crossed product $L^\infty(X,\mu) \rtimes_\sigma \Gamma$. Stably orbit equivalent actions define stably isomorphic crossed-products, but the converse does not hold in general, and this leads to the following definition. Two free p.m.p. actions $\Gamma \curvearrowright^{\sigma} (X, \mu)$ and $\Gamma' \curvearrowright^{\sigma'} (X', \mu')$ are called *von Neumann stably equivalent* if there is $\kappa \in (0, \infty)$ such that $L^\infty(X,\mu) \rtimes_\sigma \Gamma \simeq (L^\infty(X',\mu') \rtimes_{\sigma'} \Gamma')^\kappa$. It follows from [IPP05] that von Neumann stable equivalence entails stable orbit equivalence among the free p.m.p. actions of free products of (at least two) infinite groups, as soon as one of the two actions has the relative property (T). Meanwhile, Theorem 1.2 of [Gab08] establishes that any free product of at least two infinite groups admits a continuum of relative property (T) von Neumann stably inequivalent ergodic free p.m.p. actions, whose restriction to each free product component is conjugate with any prescribed (possibly non-ergodic) action. When injected in our context, this gives further classifications results for II_1 factors. For instance:

Theorem 7 ([AG])**.** *Let Γ_1, Γ_2 be non-ME, non-amenable groups with $\beta_1 = 0$ and $\beta_q(\Gamma_1) \neq 0, \infty$ for some $q > 1$. The crossed-product II_1 factors $M_1 *_A M_2$ associated with the various ergodic relative property (T) free p.m.p. actions $\Gamma_1 * \Gamma_2 \curvearrowright^{\sigma}$ (X, μ) are classified by the pairs $A \subset M_1$, and in particular by the isomorphism class of the centers $Z(M_1)$ of the crossed-product associated with the restriction of the action to Γ_1, equipped with the induced trace.*

The main reference for the proofs of the results presented in this talk is [AG]. The purpose of this article is connected with the uniqueness condition in free product decompositions in the measurable context. To this end, we take full advantage of a Bass-Serre theory developped in this context in [Alv08a] and [Alv08b].

References

[Alv08a] A. Alvarez. Théorème de Kurosh pour les relations d'équivalence boréliennes. *Preprint*, 2008.

[Alv08b] A. Alvarez. Une théorie de Bass-Serre pour les relations d'équivalence et les groupoïdes boréliens. *Preprint*, 2008.

[AG] A. Alvarez and D. Gaboriau Free products, Orbit Equivalence and Measure Equivalence Rigidity. *Preprint*, 2008.

[AS90] S. Adams and R. Spatzier. Kazhdan groups, cocycles and trees. *Amer. J. Math.*, **112** (1990), 271-287.

[BV97] M. E. B. Bekka and A. Valette. Group cohomology, harmonic functions and the first L^2-Betti number. *Potential Anal.*, **6** (1997), 313-326.

[CG86] J. Cheeger and M. Gromov. L_2-cohomology and group cohomology. *Topology*, **25** (1986), 189-215.

[CH08] I. Chifan and C. Houdayer. Prime factors and amalgamated free products. *Preprint*, 2008.

[Fur99] A. Furman. Orbit equivalence rigidity. *Ann. of Math. (2)*, **150** (1999), 1083-1108.

[Gab00] D. Gaboriau. Coût des relations d'équivalence et des groupes. *Invent. Math.*, **139** (2000), 41-98.

[Gab02] D. Gaboriau. Invariants L^2 de relations d'équivalence et de groupes. *Publ. Math. Inst. Hautes Études Sci.*, **95** (2002), 93-150.

[Gab05] D. Gaboriau. Examples of groups that are measure equivalent to the free group. *Ergodic Theory Dynam. Systems*, **25** (2005), 1809-1827.

[Gab08] D. Gaboriau. Relative property (T) actions and trivial outer automorphism groups. *Preprint*, 2008.

[Gro93] M. Gromov. Asymptotic invariants of infinite groups. *Geometric group theory, 2.* London Math. Soc. Lect. Note Ser. (1993).

[IPP05] A. Ioana, J. Peterson, and S. Popa. Amalgamated free product of w-rigid factors and calculation of their symmetry groups. *Acta Mathematica, to appear*, 2005.

[Lüc02] W. Lück. L^2-invariants: theory and applications to geometry and K-theory. **44**, Springer-Verlag, Berlin (2002).

[OW80] D. Ornstein and B. Weiss. Ergodic theory of amenable group actions I. The Rohlin lemma. *Bull. Amer. Math. Soc.*, **2** (1980), 161-164.

[Pop06] S. Popa. On a class of type II_1 factors with Betti numbers invariants. *Ann. of Math. (2)*, **163** (2006), 809-899.

Subfactors and Planar Algebras

Dietmar Bisch
(joint work with Vaughan Jones)

Let $N \subset M$ be an inclusion of II_1 factor with finite Jones index ([12]). The noncommutative L^2-space $L^2(M)$ is in a natural way an N-M bimodule via left and right multiplication of N resp. M. The contragredient bimodule is $L^2(M)$ as an M-N bimodule, and we can construct new bimodules by tensoring and completing these appropriately (see e.g. [2]). The task is then to find the irreducible N-N, N-M, M-M, M-N sub-bimodules of the bimodules constructed in this way. It turns out that the intertwiner spaces of these bimodules are given by the higher relative commutants associated to the subfactor ([16]). These higher relative commutants are centralizer algebras of N resp. M in the k-th II_1 factor in the Jones tower associated to $N \subset M$ ([12]), and the collection of these finite dimensional C*-algebras is called the *standard invariant* of $N \subset M$. It is given by

$$\begin{array}{ccccccccc} \mathbb{C} = N' \cap N & \subset & N' \cap M & \subset & N' \cap M_1 & \subset & N' \cap M_2 & \subset & \cdots \\ & & \cup & & \cup & & \cup & & \\ & & \mathbb{C} = M' \cap M & \subset & M' \cap M_1 & \subset & M' \cap M_2 & \subset & \cdots \end{array}$$

and can be axiomatized as a *planar algebra* ([13], [18]). Planar algebra technology has led to several interesting developments in subfactor theory in recent years, including classification results for subfactors ([9], [10], the discovery of new rigidity phenomena for intermediate subfactors ([8], [5], [6]), skein theoretic or statistical mechanical descriptions of explicit planar algebras associated to certain classes of subfactors (see e.g. [3], [4], [15]) etc. Due to Popa's reconstruction theorem [18], one can analyze the structure of subfactors by analyzing their standard invariants/planar algebras. Planar algebras are a powerful device that allows one to keep track of the complicated structures inherent in subfactors in an extremely efficient way. Note that the standard invariant is a complete invariant for *amenable* subfactors of the hyperfinite II_1 factor ([17]).

Abstractly, a planar algebra consists of a collection of vector spaces $\mathfrak{P} = (P_i)_{i\geq 0}$ and an action of Jones' planar operad on $\mathfrak{P}$. This operad consists of certain isotopy classes of planar tangles. To each tangle T one associates a multi-linear map Z_T on a finite collection of P_i's with image in a P_k. There is a natural operadic composition obtained by gluing tangles, and one requires that this operadic composition be compatible with the composition of the associated multi-linear maps (*naturality* of composition). It follows immediately that each P_i is an associative algebra, and, if $\dim P_0 = 1$, these algebras come with natural traces. Closed loops in a tangle account for a parameter δ associated to $\mathfrak{P}$, and the Temperley-Lieb algebra with parameter δ will be a planar subalgebra of $\mathfrak{P}$. Jones discovered planar algebras since this structure arises naturally whenever $N \subset M$ is an extremal subfactor ([13]). The vector spaces are $P_i = N' \cap M_{i-1}$, $\delta = [M : N]^{\frac{1}{2}}$, and the action of planar tangles can be defined using the description of the higher relative commutants as central vectors in the N-N bimodules $L^2(M_k)$. Conditional expectations, Jones projections and inclusions are represented by simple elementary tangles. Clearly, in the subfactor context the planar algebra will satisfy several additional conditions, including the existence of a $*$-structure on each P_i, finite dimensionality of the P_i's, and positivity of the partition function (i.e. the sesquilinear form coming from the trace). Extremality of the subfactor is equivalent to equality of left and right traces. See [13] for the details on all of this.

The planar algebra approach to subfactors makes it possible to study subfactors via generators and relations. From this perspective the simplest subfactors are those generated by the smallest number of generators satisfying the simplest relations. This idea was successfully pursued in [9], [10], where a strong rigidity result for the Fuss-Catalan planar algebras of [8] was obtained. It turned out that the Fuss-Catalan algebras can be constructed as a *free product* of two Temperley-Lieb planar algebras. The notion of *free product of planar algebras* was formally introduced in [11] (it is called *free composition* of subfactors in [8] , see also [7]), and our construction gives rise to new planar algebras and subfactors. Free products in the context of planar algebras are quite different from the usual algebraic free product of algebras. For instance, the underlying vector spaces are all finite dimensional. As an application of our notion of free product, we obtain the following theorem.

Theorem. Let $N \subset P \subset M$ be an inclusion of (extremal) subfactors with finite index. Assume $N' \cap P = \mathbb{C}$ and $P' \cap M = \mathbb{C}$. Then $\mathfrak{P}_{N\subset P} * \mathfrak{P}_{P\subset M} \subset \mathfrak{P}_{N\subset M}$.

Thus, in general, the planar algebra of $N \subset M$ will contain more structure then just the Fuss-Catalan planar algebra whenever an intermediate subfactor is present.

An important problem in the planar algebra approach to subfactors is to understand what kind of relations can be imposed on the generators to obtain a *subfactor* planar algebra (i.e. one arising as the standard invariant of a subfactor). For instance, finite dimensionality of the P_i's and positivity of the partition function need to follow from the relations imposed. This problem is not well understood, although a plethora of very interesting examples and constructions is known. These contain the exchange relation of [1] (see also [14]), the examples of [3], [4], classification of quadrilaterals ([5], [6]), which can be viewed as imposing a condition on the canonical bimodule associated to a subfactor etc. The analysis of lattices of intermediate subfactors can be viewed in this context as well, as well as the analysis of generalized Bisch-Haagerup subfactors. It would be interesting to determine the planar algebras associated to biprojections with certain angles. This in turn may help to understand better what "planar relations" should be.

References

[1] D. Bisch, *A note on intermediate subfactors*, Pacific J. Math. **163**, (1994), 201–216.

[2] D. Bisch, *Bimodules, higher relative commutants and the fusion algebra associated to a subfactor*, Operator algebras and their applications, 13-63, Fields Inst. Commun., 13, Amer. Math. Soc., Providence, RI, 1997.

[3] D. Bisch, P. Das, S. Ghosh, *The planar algebra of group-type subfactors*, preprint, arXiv:0807.4134.

[4] D. Bisch, P. Das, S. Ghosh, *The planar algebra of diagonal subfactors*, preprint.

[5] P. Grossman, V.F.R. Jones, *Intermediate subfactors with no extra structure*, J. Amer. Math. Soc. **20** (2007), no. 1, 219–265.

[6] P. Grossman, M. Izumi, *Classification of noncommuting quadrilaterals of factors*, Internat. J. Math. **19** (2008), no. 5, 557–643.

[7] D. Bisch, U. Haagerup, *Composition of subfactors: new examples of infinite depth subfactors*, Ann. Scient. Éc. Norm. Sup. **29** (1996), 329–383.

[8] D. Bisch, V.F.R. Jones, *Algebras associated to intermediate subfactors*, Invent. Math. **128** (1997), 89–157.

[9] D. Bisch, V.F.R. Jones, *Singly generated planar algebras of small dimension*, Duke Math. Journal **101** (2000), no. 1, 41–75.
[10] D. Bisch, V.F.R. Jones, *Singly generated planar algebras of small dimension, Part II*, Adv. Math. **175** (2003), no. 2, 297–318.
[11] D. Bisch, V.F.R. Jones, *The free product of planar algebras, and subfactors*, in preparation.
[12] V.F.R. Jones, *Index for subfactors*, Invent. Math. **72** (1983), 1–25.
[13] V.F.R. Jones, *Planar algebras I*, preprint, available at `http://www.math.berkeley.edu/~ vfr/`.
[14] Z. Landau, *Exchange relation planar algebras*, Geom. Dedicata **95** (2002), 183–214.
[15] S. Morrison, E. Peters, N. Snyder, *Skein theory for the D_{2n} planar algebras*, preprint, arXiv:0808.0764.
[16] A. Ocneanu, *Quantized groups, string algebras and Galois theory for algebras*, Operator algebras and applications, Vol. 2, 119–172, London Math. Soc. Lecture Note Ser., 136, Cambridge Univ. Press, Cambridge, 1988.
[17] S. Popa, *Classification of amenable subfactors of type II*, Acta Math. **172** (1994), 352–445.
[18] S. Popa S., *An axiomatizaton of the lattice of higher relative commutants*, Invent. Math. **120** (1995), 427–445.

Hadamard matrices and Bisch-Haagerup subfactors

Richard Burstein

Let $\begin{array}{ccc} A_{01} & \subset & A_{11} \\ \cup & & \cup \\ A_{00} & \subset & A_{01} \end{array}$ be a quadrilateral of finite-dimensional C^*-algebras, with trace. This quadrilateral is a commuting square if the conditional expectations onto A_{01} and A_{01} commute as operators on $L^2(A_{11})$. A commuting square is specified by the four algebra inclusions and additionally the biunitary connection, which is an element of $A_{00}' \cap A_{11}$.

A commuting square with certain additional properties (it should be symmetric and connected, and the trace should be Markov) induces a hyperfinite II_1 factor via iteration of the basic construction. Iterating the basic construction on the inclusion $A_{01} \subset A_{11}$ produces an infinite tower of C^* algebras

$$A_{01} \subset A_{11} \subset A_{21} \subset A_{31} ...$$

We label the Jones projections by $A_{i1} = \{A_{i-1,1}, e_i\}''$. Applying the GNS construction to this tower gives a hyperfinite II_1 factor $A_{\infty 1}$. We may also construct the C^* algebras A_{i0}, defined inductively by $A_{i0} = \{A_{i-1,0}, e_i\}''$. Then the A_{i0}'s form a Jones tower as well, and $A_{\infty 0} = \overline{\cup_i A_{i0}}^{st}$ is a finite-index subfactor of $A_{1\infty}$. These commuting-square subfactors are described in [JS].

Every finite level of the standard invariant of a commuting-square subfactor may be computed. Iterating the basic construction vertically as well as horizontally produces a grid of finite-dimensional C^* algebras A_{ij}, and the horizontal limit factors $A_{\infty 0} \subset A_{\infty 1} \subset A_{\infty 2} \subset ...$ are the tower of the basic construction for $A_{\infty 0} \subset A_{\infty 1}$. Then by Ocneanu's compactness argument, $A_{\infty 0}' \cap A_{\infty n} = A_{10}' \cap A_{0n}$ (see [JS]). This is computable in finite time, but the amount of time taken grows exponentially with n, so these subfactors are generally intractable.

A real Hadamard matrix is an n by n a matrix H with $HH^T = n\mathrm{id}$, and $H_{ij} = \pm 1$; these are used in many areas of mathematics. Complex Hadamard matrices are a generalization of this idea: these are unitary n by n matrices u with $|u_{ij}| = n^{-1/2}$.

Let u be a unitary matrix in $M_n(\mathbb{C})$. The quadrilateral $\begin{array}{ccc} \mathbb{C}^n & \subset & M_n(\mathbb{C}) \\ \cup & & \cup \\ \mathbb{C} & \subset & u^*\mathbb{C}^n u \end{array}$ is a commuting square if and only if u is a complex Hadamard matrix. The induced commuting-square subfactor is then a Hadamard subfactor.

Hadamard subfactors are described in [Jon1], including all depth-2 examples, and a few more examples are classified in [CN]. However for nearly all Hadamard matrices nothing is known about the standard invariant beyond the first few levels.

In [Bur1] (see also [Bur2]) I examined a twisted tensor product of depth-2 Hadamard matrices (suggested to me by Jones). If H_1 and H_2 are two complex Hadamard matrices, of respective sizes m and n, then a twist Λ is a set of mn complex scalars of modulus 1. The tensor product H of H_1 and H_2 with twist Λ is an mn by mn Hadamard matrix, with $H_{ij,kl} = (H_1)_{ik}(H_2)_{jl}\Lambda_{jk}$. The commuting square induced by this matrix contains certain intermediate algebras of the form

$$\begin{array}{ccc} \mathbb{C}^n & \subset & M_n(\mathbb{C}) \\ \cup & & \cup \\ B_0 & \subset & B_1 \\ \cup & & \cup \\ \mathbb{C} & \subset & u^*\mathbb{C}^n u \end{array}$$

Horizontally iterating the basic construction on such a commuting square produces an intermediate subfactor $M_0 \subset P \subset M_1$, where $P = \{B_1, e_2, e_3, ...\}''$. If the two original Hadamard matrices H_1 and H_2 yielded depth-2 subfactors, then in fact there are finite abelian groups H and K with outer actions on P such that $M_0 = P^H$, $M_1 = P \rtimes K$. These group-type subfactors were studied in detail in [BH], and their principal graphs may be determined by computing the image of the free product $H * K$ in the outer automorphism group of P.

The actions of H and K are compatible with the tower of B_i's in the following sense: they fix all of the Jones projections e_i, and leave the B_i's invariant. Such compatible automorphisms are determined by their restriction to B_1. A compatible automorphism α is inner if and only if $\alpha|_{B_1} = Adu|_{B_1}$, for some unitary $u \in B_0$. So determining the outerness of a given word w in the free product $H * K$ requires only examining the restriction of w to B_1. These compatible automorphisms are a finite-dimensional version of the $\mathrm{Aut}(M, N)$ invariant of Loi [Loi], and are also discussed in [Sve].

For any particular twisted tensor product of depth-2 Hadamard matrices, the group $H * K/IntP$ may be readily computed, providing the principal graph of the induced Hadamard subfactor. Many interesting examples can be constructed with index as small as 6, with both finite and infinite index.

To fully classify the standard invariant of a Bisch-Haagerup subfactor, it is necessary to examine the 3-cohomological obstruction associated with the action of

$H * K$. Because compatible automorphisms are determined by their restriction to B_1, this data may be described more easily than in the general case. The characteristic invariant of the action (see [Jon2]) can readily be determined by examination of the twist Λ and the two groups. From this it is sometimes possible to classify twisted tensor product subfactors up to isomorphism. All index-4 Hadamard subfactors may be so classified.

However, determining the obstruction (or even whether $H * K/IntP$ has a lift into $AutP$) tends to become difficult as $H * K/Int$ becomes even moderately large. For the 16 by 16 real Hadamard matrix coming from twisted tensor product, $H * K/Int$ is order 256 non-abelian, and the obstruction is not known. Some computations suggest that the associated element of H^3 is trivial, but this is not proven.

For most Hadamard subfactors, the first few levels of the principal graph can be computed using Ocneanu compactness, but nothing else is known about them. Furthermore, all the twisted tensor product examples described above are amenable. No non-amenable Hadamard subfactors are known, despite the fact that most low-index Hadamard subfactors seem to have standard invariant A_n up to the limits of computation. For Hadamard subfactors which are not depth 2 or constructed via twisted tensor product, the classification problem remains almost completely open.

References

[BH] D. Bisch and U. Haagerup. Composition of subfactors: new examples of infinite depth subfactors. *Ann. Sci. École Norm. Sup. (4)*, 29(3)329–383, 1996.

[Bur1] R. Burstein. *Hadamard subfactors of Bisch-Haagerup type.* PhD Dissertation. University of California, Berkeley, Department of Mathematics, 2008.

[Bur2] R. Burstein. *Group-type subfactors and Hadamard matrices.* Preprint. *arxiv.org.* arxiv:0811.1275 [math.OA], 51 pp., 2008.

[CN] W. Camp and R. Nicoara. Subfactors and Hadamard matrices. Preprint. *arxiv.org*, arXiv:0704.1128 [math.OA]:18 pp, 2007.

[Jon1] V. Jones. Planar algebras, I. Preprint. *arxiv.org*, arxiv:math/9909027 [math.QA]:122 pp, 1999

[Jon2] V. Jones. Actions of finite groups on the hyperfinite type II_1 factor. *Mem. Amer. Math. Soc.*, 28(237):v+70, 1980.

[JS] V. Jones and K. Sunder. *Introduction to subfactors*, volume 234 of *London Mathematical Society Lecture Notes Series.* Cambridge University Press, Cambridge, 1997.

[Loi] P Loi. On automorphisms of subfactors. *J. Funct. Anal.*, 141(2):275–293, 1996.

[Sve] Anne Svendsen. Automorphisms of subfactors from commuting squares. *Trans. Am. Math. Soc*, 356(6):2515-2543, 2004.

Orbit inequivalent actions of non-amenable groups

Inessa Epstein

Let us consider a standard probability space (X, μ) where μ is non-atomic with a countable group Γ acting on (X, μ) in a Borel measure preserving manner. This gives rise to the orbit equivalence relation $E_\Gamma = \{(\gamma \cdot x, x) \mid x \in X\}$. Two such actions $\Gamma \curvearrowright^a (X, \mu), \Delta \curvearrowright^b (Y, \nu)$ are *orbit equivalent* if there exist conull subsets

$A \subset X$, $B \subset Y$ and a measurable, measure preserving bijection $f\colon A \to B$ such that for any $x, y \in A$, we have $xE_\Gamma y$ if and only if $f(x)E_\Delta f(y)$.

The theory of orbit equivalence was originally motivated by its connections to operator algebras. Via the "group measure space" construction of Murray-von Neumann [MvN36], one may from a measure preserving free ergodic action of an infinite countable group obtain a type II_1 von Neumann factor with an abelian Cartan subalgebra. Furthermore, by Feldman-Moore [FM77], two von Neumann algebras obtained in this fashion are isomorphic via an isomorphism preserving the Cartan subalgebras if and only if the corresponding actions are orbit equivalent.

The first orbit equivalence result is due to Dye [Dye59], [Dye63], who showed that all ergodic measure preserving actions of $\mathbb{Z}$ are orbit equivalent and that any such action of an abelian group is orbit equivalent to a $\mathbb{Z}$-action. Later, Ornstein- Weiss [OW80] (and, more generally, Connes-Feldman-Weiss [CFW81] in the context of amenable equivalence relations) provided a classification of all ergodic measure preserving actions of amenable groups by showing that these are all orbit equivalent to actions of $\mathbb{Z}$ as well. Consequently, the orbit equivalence relation remembers only that the group is amenable.

For non-amenable groups, the situation is quite different. Connes and Weiss [CW80] used strong ergodicity, an orbit equivalence invariant introduced by Schmidt [Sch80], to establish that all non-amenable groups without Kazhdan's property (T) admit at least two orbit inequivalent free, measure preserving ergodic actions. With a construction based on the work of McDuff [McD69], Bezuglyĭ and Golodets [BG81] showed that there exists a non-amenable group with continuum many orbit inequivalent such actions. Results concerning classes of groups exhibiting this phenomenon of continuum many actions gradually increased throughout the years. Zimmer's [Zim84] cocycle superrigidity gave rise to results by Gefter and Golodets [GG88] showing that this holds for certain groups with property (T), among them $\mathrm{SL}_3(\mathbb{Z})$.

Recently, using a separability argument as well as continuum many non - isomorphic irreducible representations of the group, Hjorth [Hjo05] proved that actually all infinite groups with property (T) admit continuum many orbit inequivalent free, measure preserving, ergodic actions. On the other hand, if one considers the semidirect product $\mathbb{F}_n \ltimes \mathbb{Z}^2$ formed by letting $\mathrm{SL}_2(\mathbb{Z})$ act by matrix multiplication on $\mathbb{Z}^2$ and restricting this action to $\mathbb{F}_n$ when viewed as a finite index subgroup of $\mathrm{SL}_2(\mathbb{Z})$, then the pair $(\mathbb{F}_n \ltimes \mathbb{Z}^2, \mathbb{Z}^2)$ has relative property (T). In Popa [Pop06a], the rigid notion of relative property (T) was defined for actions of groups with the natural action of $\mathbb{F}_n$ on $\mathbb{T}^2$ possessing this property. Gaboriau and Popa [GP05] used this action of $\mathbb{F}_n$ on $\mathbb{T}^2$ to give rise to the existence of continuum many orbit inequivalent actions for non-cyclic free groups. Ioana [Ioa06] then gave an actual constructive proof of continuum many actions for such groups.

In [Ioa07a], Ioana considered groups Γ such that $\mathbb{F}_2 \leq \Gamma$. Given $\Delta \leq \Gamma$ and an action of Δ, there is a way to co-induce from this an action of Γ so that the resulting action of Γ restricted to Δ has the original action by Δ as a factor. He used the previously mentioned action of $\mathbb{F}_2$ on $\mathbb{T}^2$ as well as continuum many actions of $\mathbb{F}_2$ obtained from irreducible non-isomorphic representations of $\mathbb{F}_2$ and showed that co-inducing actions of Γ from these actions yields continuum many orbit inequivalent actions for groups containing $\mathbb{F}_2$ as a subgroup.

Existence of continuum many actions has also been exhibited for weakly rigid groups by Popa [Pop06b], direct products of groups satisfying a certain cohomological property, including hyperbolic groups and non-amenable free products, by Monod-Shalom [MS06], products of non-amenable with infinite amenable groups by Ioana [Ioa07b], and mapping class groups by Kida [Kid07].

The main goal is to present the proof of the following theorem:

Theorem 1. *Let Γ be a countable, non-amenable group. Suppose that there are free, measure preserving actions $\Gamma \curvearrowright (X,\mu)$, $\mathbb{F}_2 \curvearrowright (X,\mu)$ on a standard probability space (X,μ) such that Γ acts ergodically and $E_{\mathbb{F}_2} \subseteq E_\Gamma$. Then Γ admits continuum many orbit inequivalent free, measure preserving, ergodic actions.*

Gaboriau and Lyons [GL07] showed that every countable, non-amenable group admits a free, measure preserving, ergodic action on a standard probability space (X,μ) so that the orbit equivalence relation induced by the action contains the orbit equivalence relation induced by a free, measure preserving action of $\mathbb{F}_2$ on (X,μ). It was shown by Olʹšanskiĭ [Olʹ80] that not all non-amenable groups admit $\mathbb{F}_2$ as an actual subgroup. From Gaboriau and Lyons' result and Theorem 1, we obtain the following corollary:

Corollary 2. *Suppose that Γ is a countable, non-amenable group. Then Γ induces continuum many orbit inequivalent free, measure preserving, ergodic actions.*

We give a construction of co-induced actions that uses subequivalence relations instead of subgroups and generalizes the previous notion of co-induction. In particular, given free measure preserving actions $\Gamma, \Delta \curvearrowright (X,\mu)$ such that $E_\Delta \subset E_\Gamma$ and a measure preserving action $\Delta \curvearrowright^a (Z,\nu)$, we show how to construct actions $\Gamma \curvearrowright^c (Y,m), \Delta \curvearrowright^d (Y,m)$ so that $E^d_\Delta \subset E^c_\Gamma$ and d has a as a factor. In the special case when Δ is a subgroup of Γ, this essentially reduces to the standard co-induction. We finally use our co-induced actions from and combine them with a theorem of Ioana to provide the proof of Theorem 1.

References

[BG81] Sergey I. Bezuglyi and Valentin Ya. Golodets. Hyperfinite and II_1 actions for nonamenable groups. *J. Funct. Anal.*, 40(1):30–44, 1981.

[CFW81] Alain Connes, Jacob Feldman, and Benjamin Weiss. An amenable equivalence relation is generated by a single transformation. *Ergodic Theory and Dynamical Systems*, 1(4):431–450 (1982), 1981.

[CW80] Alain Connes and Benjamin Weiss. Property (T) and asymptotically invariant sequences. *Israel J. Math.*, 37(3):209–210, 1980.

[Dye59] Henry A. Dye. On groups of measure preserving transformations. I. *Amer. J. Math.*, 81:119–159, 1959.

[Dye63] Henry A. Dye. On groups of measure preserving transformations. II. *Amer. J. Math.*, 85:551–576, 1963.

[FM77] Jacob Feldman and Calvin C. Moore. Ergodic equivalence relations, cohomology, and von Neumann algebras. I. *Trans. Amer. Math. Soc.*, 234(2):289–324, 1977.
[GG88] Sergey L. Gefter and Valentin Ya. Golodets. Fundamental groups for ergodic actions and actions with unit fundamental groups. *Publ. Res. Inst. Math. Sci.*, 24(6):821–847 (1989), 1988.
[GL07] Damien Gaboriau and Russell Lyons. A measurable-group-theoretic solution to von Neumann's problem. preprint, available at `arXiv:0711.1643`, 2007.
[GP05] Damien Gaboriau and Sorin Popa. An uncountable family of nonorbit equivalent actions of $\mathbb{F}_n$. *J. Amer. Math. Soc.*, 18(3):547–559 (electronic), 2005.
[Hjo05] Greg Hjorth. A converse to Dye's theorem. *Trans. Amer. Math. Soc.*, 357(8):3083–3103 (electronic), 2005.
[Ioa06] Adrian Ioana. A construction of non-orbit equivalent actions for $\mathbb{F}_n$. preprint, available at `arXiv:math/0610452`, 2006.
[Ioa07a] Adrian Ioana. Orbit inequivalent actions for groups containing a copy of $\mathbb{F}_2$. preprint, available at `arXiv:math.GR/0701027`, 2007.
[Ioa07b] Adrian Ioana. A relative version of Connes' $\chi(M)$ invariant and existence of orbit inequivalent actions. *Ergodic Theory Dynam. Systems*, 27(4):1199–1213, 2007.
[Kid07] Yoshikata Kida. Orbit equivalence rigidity and certain generalized bernoulli shifts of the mapping class group. 2007. preprint, available at `arXiv:math/0607601`.
[McD69] Dusa McDuff. Uncountably many II_1 factors. *Ann. of Math. (2)*, 90:372–377, 1969.
[MS06] Nicolas Monod and Yehuda Shalom. Orbit equivalence rigidity and bounded cohomology. *Ann. of Math. (2)*, 164(3):825–878, 2006.
[MvN36] Francis J. Murray and John von Neumann. On rings of operators. *Ann. of Math. (2)*, 37(1):116–229, 1936.
[Ol′80] Alexander Ju. Ol′šanskiĭ. On the question of the existence of an invariant mean on a group. *Uspekhi Mat. Nauk*, 35(4(214)):199–200, 1980.
[OW80] Donald S. Ornstein and Benjamin Weiss. Ergodic theory of amenable group actions. I. The Rohlin lemma. *Bull. Amer. Math. Soc. (N.S.)*, 2(1):161–164, 1980.
[Pop06a] Sorin Popa. On a class of type II_1 factors with betti numbers invariants. *Annals of Mathematics*, 163:809–899, 2006.
[Pop06b] Sorin Popa. Strong rigidity of II_1 factors arising from malleable actions of w-rigid groups. I. *Invent. Math.*, 165(2):369–408, 2006.
[Sch80] Klaus Schmidt. Asymptotically invariant sequences and an action of SL(2, $\mathbb{Z}$) on the 2-sphere. *Israel J. Math.*, 37(3):193–208, 1980.
[Zim84] Robert J. Zimmer. *Ergodic theory and semisimple groups*, volume 81 of *Monographs in Mathematics*. Birkhäuser Verlag, Basel, 1984.

Orbit equivalence and von Neumann Algebras

Damien Gaboriau

I gave an introductory overview of the central theme of our conference : the connections between von Neumann algebras and orbit equivalence. The central object is the **group measure space construction** of Murray and von Neumann (1936). Starting from an ergodic free probability measure preserving (p.m.p.) action α of a **countable** group Γ on the standard Borel space (X, μ), they produce the von Neumann algebra

$$M_\alpha := \Gamma \ltimes \alpha L^\infty(X, \mu)$$

containing a copy of the group and a copy of the **Cartan sub-algebra** $A := L^\infty(X, \mu)$, and reflecting certain properties of the action.

In fact, this von Neumann algebra depends on the action only throu the partition of the space into orbits, or equivalently throu the orbit-equivalence relation

$$\mathcal{R}_\alpha = \{(x, \alpha(\gamma).x) : x \in X,\ \gamma \in \Gamma\}$$

and moreover $\mathcal{R}_\alpha$ can be recover from M_α and the additionnal data of the Cartan subalgebra $A \subset M_\alpha$.

This leads to weaker and weaker notions of equivalence for group actions $i = 1, 2$

$$\Gamma_i \overset{\alpha_i}{\curvearrowright} (X_i, \mu_i)$$

obtained by successively forgetting about the orbit parametrization, and the Cartan subalgebra:

- **Conjugacy:** $\Gamma_1 \sim \Gamma_2$ and an equivariant map $\Phi : X_1 \to X_2$
- **Orbit equivalence:** $\mathcal{R}_{\alpha_1} \sim \mathcal{R}_{\alpha_2}$
- **von Neumann Equivalence:** $M_{\alpha_1} \sim M_{\alpha_2}$

Moreover "Orbit Equivalent Actions" $\iff$ "Isomorphic Pairs", i.e. $\mathcal{R}_{\alpha_1} \sim \mathcal{R}_{\alpha_2} \iff (A_1 \subset M_1) \simeq (A_2 \subset M_2)$.

"How far is the general theory of II_1 factors from that of orbit equivalence ?" Results of Connes-Jones (1982), Voiculescu (1996), Ozawa-Popa (2007) give an idea of the variety of situations: there are II_1 factors M
– with at least two Cartan A_1, A_2 leading to non-isomorphic pairs $A_1 \subset M \not\simeq A_2 \subset M$ and thus two non-isomorphic equivalence relations,
– without any Cartan subalgebra (thus not produced by any ergodic group action)
– with exactly one (up to conjugacy) Cartan subalgebra.

A *rigidity* phenomenon (HT-factors) discovered by S. Popa ensures the uniqueness of the Cartan sub-algebra among those satisfying a certain Kazhdan-type condition.

The fundamental group = {scales of self-similarities} $\subset$ R+*

The Murray-von Neumann fundamental group (1943) of a a II_1 factor M is

$$\mathcal{F}(M) := \{\tau(p)/\tau(q) : p, q \text{ projections } pMp \simeq qMq\}$$

In all their examples $\mathcal{F}(M) = \mathrm{R}_+^*$ and the first restrictions on some $\mathcal{F}(M)$ appeared with Connes' result (1980), for the von Neumann algebra of ICC Kazhdan property (T) groups: it has to be countable. settings. The first constraints on the fundamental were discovered by A. Connes (1980)

The Dynamical fundamental group of an ergodic p.m.p. equivalence relation $\mathcal{R}$ is:

$$\mathcal{F}(\mathcal{R}) := \{\mu(U)/\mu(V) : U, V \text{ Borel subsets of } X \text{ s.t. } \mathcal{R}_{|U} \simeq \mathcal{R}_{|V}\}$$

or equivalently in orperator algebras terms:

$$\mathcal{F}(A \subset M) = \{\tau(p)/\tau(q) : pMp \overset{\Theta}{\simeq} qMq \text{ and } pAp \overset{\Theta}{\simeq} qAq\}$$

S. Popa (1986) proved that is countable for ICC Kazhdan property (T) group erg. free p.m.p. actions. And Gefter-Golodets (1988) the triviality of the dynamical fundamental group for lattices in higher rank (connected simple with trivial center) Lie groups (ex: $G = \mathrm{SL}(n, \mathrm{Z})$, $n \geq 3$).

Some Orbit Equivalence invariants are very usefull for this problem, namely ℓ^2-Betti numbers (or else cost of equivalence relations), since their non-triviality ensures triviality of the dynamical fundamental group.

ℓ^2 **Betti numbers** are a sequence of numbers associated to various "dynamical situations" $\rightsquigarrow \beta_0,\ \beta_1,\ \beta_2, \cdots$

- [Atiyah 1976] for free cocompact actions on manifolds
- [Connes 1979] for measured foliations on compact manifolds
- [Cheeger-Gromov 1986] for countable discrete groups
- [G. 2002] for countable standard p.m.p. equivalence relation. $\rightsquigarrow$ the ℓ^2-Betti numbers of groups turns out to be (numerical) Orbit Equivalence invariants: *Let $\mathcal{R}$ be a countable standard p.m.p. equivalence relation given by a free p.m.p. action α of a group Γ, then*

$$\beta_n(\mathcal{R}_\alpha, \mu) = \beta_n(\Gamma)$$

Moreover, they enjoy the following Compression Formula (G. 2002): *A countable standard p.m.p. equiv. rel. $\mathcal{R}$ and $U_1, U_2 \subset X$ Borel subsets that meet μ-a.e. equivalence class of $\mathcal{R}$, then the restrictions $\mathcal{R}_{U_1}, \mathcal{R}_{U_2}$ satisfy:*

$$\mu(U_1)\ \beta_n(\mathcal{R}_{U_1}, \overline{\mu}_{U_1}) = \mu(U_2)\ \beta_n(\mathcal{R}_{U_2}, \overline{\mu}_{U_2})$$

This in turn implies that if one of the ℓ^2-Betti numbers of Γ is $\neq 0$ nor ∞, then the dynamical fundamental of any free p.m.p. action of Γ is trivial. This is satisfied in particular for the standard action of $\mathrm{SL}(2, \mathrm{Z})$ on the 2-dimensional torus T^2 (since $\beta_1(\mathrm{SL}(2,\mathrm{Z})) = 1 + \frac{1}{12}$).

S. Popa's rigidity phenomenon (HT-factors) alluded to above thus led him to exhibit the first II_1 factor with trivial fundamental group, the group-measure space factor produced by the standard action of $\mathrm{SL}(2, \mathrm{Z})$ on the 2-dimensional torus.

How Many Actions for a Group ? Results of Dye and Ornstein-Weiss show that infinite amenable groups produce a single action up to Orbit Equivalence. For non-amenable groups, the situation is drastically different. A long sequence of works led eventually to a "complete" solution very recently, via various techniques ranging from "higher rank-like" to "Kazhdan-like" techniques (Connes-Weiss 1980, Bezuglyĭ-Golodets 1981, Gefter-Golodets 1988, Hjorth 2004, Gaboriau-Popa 2005, Monod-Shalom 2006, Popa 2006). Every non-amenable group produces uncountably many non Orbit Equivalent p.m.p. ergodic free actions [Ioana 2006] for groups containing a copy of the free group F_2 (co-induction method), and [Epstein 2007] for the general case (generalized co-induction).

In the course of her proof I. Epstein uses a "measurable version" of the von Neumann's Problem...

von Neumann's Problem

It is known since Ol'shanskiĭ(80's) that not every non-amenable group contains a copy of F_2. However, this becomes true whenever one relaxes the notion of "containing" and replacing it by a measurable version. *Every non-amenable group Γ contains* F_2, *measurably* [G.-Lyons, 2007]: for the Bernoulli action $\Gamma \overset{\text{shift}}{\curvearrowright} X =$

$(\mathrm{S}^1)^\Gamma$ w. Lebesgue measure $\exists$ a free p.m.p. ergodic action $\mathrm{F}_2 \overset{\alpha}{\curvearrowright} X$, s.t. F_2-orbits $\subset$ Γ-orbits.

$\rightsquigarrow$ This result gives as a by-product a **new way to embed free group factor** in some II_1 factors. Consider the restricted wreath product $H \wr \Gamma$ for H infinite and Γ non-amenable, then *The von Neumann factor* $L(H \wr \Gamma)$ *contains a copy of the von Neumann factor* $L(\mathrm{F}_2)$ *of the free group.*

Bass-Serre rigidity, free products

A result of Tornquist (2006) allows to "put groups actions in general position": *There is an action of the free product* $*_{i\in I}\Gamma_i \overset{\sigma}{\curvearrowright} (X,\mu)$ *whose restriction to the components is conjugated with any prescribed action* $\Gamma_i \overset{\alpha_i}{\curvearrowright} (X,\mu)$.

The resulting actions may even be assumed to have the relative property (T) of Popa, and one may produce uncountably many of them (up to von Neumann equivalence) [G. 2008].

Several Bass-Serre rigidity results appeared during the last years:
Consider two families of groups (Γ_p) *and* $({\Gamma_p}')$ *and two free p.m.p. OE actions*

$$(*_{p\in P}\Gamma_p) \overset{\alpha}{\curvearrowright} (X,\mu) \quad OE \quad (*_{p\in P}{\Gamma_p}') \overset{\alpha'}{\curvearrowright} (X',\mu')$$

with ergodic restrictions $\alpha_{|\Gamma_p}$ *and* ${\alpha'}_{|{\Gamma_p}'}$.

Then up to reordering: $\Gamma_p \overset{\alpha_{|\Gamma_p}}{\curvearrowright} (X,\mu) \quad OE \quad {\Gamma_p}' \overset{{\alpha'}_{|{\Gamma_p}'}}{\curvearrowright} (X',\mu')$ ***under the assumption*** *the* $\Gamma_p, {\Gamma_p}'$ *are all either*

- *Kazhdan-like + ICC-like [Ioana-Peterson-Popa 2005]*
- *ICC non-amenable direct products [Chifan-Houdayer 2008]*
- *with trivial first* ℓ^2*-Betti number* $\beta_1 = 0$ *[Alvarez-G. 2008]*

The third essentially covers the other two... But, for the first two, in fact vNE $\Rightarrow$ OE For the third, in fact the ergodicity hypothesis may be removed
$\rightsquigarrow$ this is necessary for application to **Measure Equivalence** (another story...).

The planar algebra of group-type subfactors

Shamindra Ghosh

(joint work with Dietmar Bisch, Paramita Das)

We consider two types of subfactors:
(i) *Bisch-Haagerup subfactor*: $P^H \subset P \rtimes K$ arising from two finite groups H and K acting outerly on a II_1-factor P,
(ii) *Diagonal subfactors*: $N \subset M_I(N)$ arising from a finite subset $\{\alpha_i : i \in I\} \subset Aut(N)$ for a II_1 factor N where an element $x \in N$ sits in $M_I(N)$ diagonally with the i-th diagonal element being given by $\alpha_i(x)$.

Bisch and Haagerup in [1] characterized properties of (i), such as, strong amenability and amenability in terms of the group $G = \langle H, K\rangle_{Out(P)}$. (i) is also a simple mechanism of creating an infinite depth subfactor composing two finite depth ones if and only if G is infinite. B-H subfactors were also used to produce a continuum of non-isomorphic hyperfinite subfactors with the isomorphic standard invariant.

On the other hand, Popa (in [8]) characterized amenability, property (T) of the diagonal subfactors in terms of the group $G = \langle \theta_i | i \in I \rangle_{Out(N)}$. Consequently, Popa and Bisch (in [2]) proved that property (T) of B-H subfactors is equivalent to that of G. My broader goal is to understand various properties of subfactors using planar algebras. For this, the B-H and the diagonal subfactors serve as an ideal testing ground.

In [3] and [5], we describe the planar algebra P^{BH} of $P^H \subset P \rtimes K$ abstractly. We started with a group G generated by two of its finite subgroups H and K and a scalar 3-cocycle $\omega \in H^3(G, S^1)$. The vector space P_n^{BH} is generated by sequences of length $2n$ with elements coming from K and H alternately such that the product of the elements is 1. The action of tangles heavily depends on ω and has a surprising similarity with IRF models arising in Statistical Mechanics. We then proved that:
(i) The planar algebra of $P^H \subset P \rtimes K$ is given by P^{BH} corresponding to the group $G = \langle H, K \rangle_{Out(P)}$ and ω being the obstruction of lifting G to $Aut(P)$.
(ii) Every P^{BH} is a planar algebra associated to a B-H subfactor.
(iii) Any subfactor whose planar algebra is given by P^{BH} must necessarily be a B-H subfactor.

For the diagonal subfactor, in [4], we define a planar algebra P^Δ starting with a group G generated by a finite subset $\{\theta_i : i \in I\}$ and a cocycle $\omega \in H^3(G, S^1)$. The vector space P_n^Δ is generated by multi-indices of length $2n$ with elements from I such that product of the corresponding group elements in G (with alternate elements being inversed) is 1. The action of tangle is dependent on ω and consistent labelling of strings with indices. When ω is trivial, i.e., a coboundary, then P^Δ matches with Jones's example of planar algebra associated to finitely generated group. Finally, we proved:
(i) The planar algebra of the diagonal subfactor $N \subset M_I(N)$ arising from a $\{\alpha_i\}_{i \in I} \subset Aut(N)$ is given by P^Δ corresponding to $G = \langle [\alpha_i]_{Out(N)} : i \in I \rangle$ and ω being the obstruction of G.
(ii) Every P^Δ is a planar algebra associated to a diagonal subfactor.

In the computation of the action of certain tangles for P^Δ, namely, *multiplication*, *inclusion*, *Jones projection* and *right-conditional expectation* tangles, the 3-cocycle ω does not feature at all. Thus, the salient feature of this planar algebra is that our distinguished basis matches with the 'loop-basis' of the filtered $*$-algebra arising out of random walk on the principal graph. In fact, the main difficulty was to choose the right basis so that this feature holds. However, this feature is missing for P^{BH}.

References

1. D. Bisch, U. Haagerup, *Composition of subfactors: New examples of infinite depth subfactors*, Ann. Sci. Ec. Norm. Sup., 29, (1996), 329-383
2. D. Bisch, S. Popa, *Examples of subfactors with Property (T) standard invariant*, Geom. Func. Anal. 9, (1999), 215-225
3. D. Bisch, P. Das, S. Ghosh, *Planar algebra of the group-type subfactors*, Submitted, arXiv:0807.4134v1

4. D. Bisch, P. Das, S. Ghosh, *Planar algebra of the diagonal subfactors*, Submitted
5. D. Bisch, P. Das, S. Ghosh, *Planar algebra of the group-type subfactors with cocycle-obstruction*, In preparation
6. V. Jones, *Planar Algebras*, arxiv:math/9909027v1
7. V. Jones, D. Shlyakhtenko, K. Walker, *An orthogonal approach to the subfactor of a planar algebra*, arXiv:0807.4146
8. S. Popa, *Sousfacteurs, actions des groupes et cohomologie*, C.R. Acad. Sci. Paris 309 (I) (1989) 771-776

Strong singularity for subfactors

Pinhas Grossman
(joint work with Alan Wiggins)

A subalgebra of a II_1 factor $B \subseteq M$ is singular if B has no non-trivial normalizer in M. The notion of α-strong singularity, where $0 < \alpha \leq 1$, was introduced by Sinclair and Smith as an analytic property which (for any α) implies singularity [3]. Specifically, $B \subseteq M$ is α-strongly singular if the inequality $\alpha\|u - \mathbb{E}_B(u)\|_2 \leq \|\mathbb{E}_B - \mathbb{E}_{uBu^*}\|_{\infty,2}$ holds for every unitary $u \in M$. Sinclair, Smith, White, and Wiggins showed that for a maximal Abelian subalgebra, singularity is in fact equivalent to strong singularity with constant $\alpha = 1$ [4]. A natural question is whether this is also true for other classes of von Neumann subalgebras, for example subfactors.

Many examples of singular subfactors are given by 2-supertransitive subfactors, which means that the first relative commutant is generated by the Jones projection. This property is enjoyed by any subfactor whose Jones index is strictly between 3 and 4, and there also exist examples for every integer index of at least 3. Any 2-supertransitive subfactor is singular.

It turns out that there exist singular subfactors which are not 1-strongly singular. The unique subfactor of the hyperfinite II_1 factor with index $2 + \sqrt{2}$ is singular (by 2-supertransitivity), and we produce an upper bound on α for which it is α-strongly singular. The proof proceeds by embedding the subfactor in a quadrilateral of factors (which was studied in [1]), and using the angle of the quadrilateral to bound α. The same method of proof should work for many other subfactors.

However, any 2-supertransitive subfactor is actually strongly singular with a constant related to the index, so we obtain a proper interval for the maximal α for which the subfactor is α-strongly singular.

References

[1] Grossman, P. and Jones, V. F. R. (2007). Intermediate subfactors with no extra structure. *Journal of the American Mathematical Society*, **20**, 219–265
[2] Grossman, P. and Wiggins, A. (preprint). Strong Singularity for Subfactors.
[3] Sinclair, A. M. and Smith R. R. (2002). Strongly singular masas in type II_1 factors. *Journal of Geometric and Functional Analysis*, **12**, 199–216
[4] Sinclair, A. M., Smith, R. R., White, S., and Wiggins, A. (preprint). Strong singularity of singular masas. To appear in *Illinois Journal of Mathematics*.

Prime factors and amalgamated free products

Cyril Houdayer
(joint work with Ionut Chifan)

A von Neumann algebra $\mathcal{M}$ is said to be *prime* if it cannot be written as a tensor product $\mathcal{P}_1 \otimes \mathcal{P}_2$ of diffuse von Neumann algebras. We first review some well-known results on primality for finite von Neumann algebras. Using free probability theory, Ge in [2] proved that free group factors are prime. Subsequently, in [16] Stefan generalized Ge's results to subfactors of finite index of the interpolated free group factors. A major breakthrough came when, using C*-algebraic methods, Ozawa discovered [5] a class $\mathcal{S}$ of groups Γ such that the von Neumann algebra $L(\Gamma)$ is *solid*, i.e. the relative commutant of any diffuse von Neumann subalgebra is amenable. In particular, this entails that if $\Gamma \in \mathcal{S}$ is an infinite conjugacy class (ICC) group then every nonamenable subfactor of $L(\Gamma)$ is prime. In [5] it was shown that the class $\mathcal{S}$ contains the hyperbolic groups, the Lie groups of rank one, etc. An interesting result obtained recently [7] by Ozawa is that $\mathbf{Z}^2 \rtimes \mathrm{SL}(2, \mathbf{Z}) \in \mathcal{S}$. Ozawa was also the first one to prove in [6], an analogue of the Kurosh theorem for type II_1 factors. He showed that any free product of weakly exact type II_1 factors, i.e. factors that contain a weakly dense exact C*-algebra, is prime. From a completely different prospective, using L^2-derivations techniques, Peterson was able to generalize in [8] some of Ozawa's results. For instance, he showed that the free product of diffuse finite von Neumann algebras is prime as well as the group von Neumann algebra $L(\Gamma)$ of a countable discrete group Γ with a non-vanishing first L^2-Betti number ($\beta_1^{(2)}(\Gamma) > 0$). Finally, using the *deformation/spectral gap* rigidity principle, Popa proved in [9] that for any non-amenable group Γ acting by Bernoulli shift on the AFD II_1 factor R, the crossed product von Neumann algebra $(\bigotimes_\Gamma R) \rtimes \Gamma$ is prime. This is precisely the result which inspired the authors for the present work.

In the type III setting, there are fewer results. Using Stefan's results, Shlyakhtenko showed in [14] that the unique free Araki-Woods factor of type III_λ (see [15]) is prime. Vaes & Vergnioux [21] constructed type III factors associated with discrete quantum groups for which they proved generalized solidity, and thus primality. Gao & Junge, proved in [1] that any free product of amenable von Neumann algebras w.r.t. faithful normal states is generalized solid and thus prime. Note that from [13], such a free product necessarily has the complete metric approximation property.

In this work, we provide new indecomposability results for amalgamated free products of (not necessarily tracial) von Neumann algebras $\mathcal{M} = \mathcal{M}_1 *_B \mathcal{M}_2$ over a common amenable von Neumann algebra B. Proofs are based on Popa's *deformation/rigidity* argument. We refer to [3, 4, 8, 9, 10, 11, 12, 22] for some applications of this theory.

For the purpose of this work, we will extend the intertwining techniques from [12] as well as some results of [4] in the context of *semifinite* von Neumann algebras. Given a type III factor $\mathcal{M}$, these techniques allow us to work with its *core* $\mathcal{M} \rtimes_{\sigma^\varphi} \mathbf{R}$

(which is of type II_∞) rather than $\mathcal{M}$ itself. The main technical result roughly says that in a semifinite amalgamated free product, one can locate the position of the relative commutant of a non-amenable subfactor. The strategy to prove this result follows the *deformation/spectral gap* rigidity principle developed by Popa in [9, 10]. We briefly remind below the two concepts that we will play against each other to prove our main result:

(1) The first ingredient we will use is the "malleable deformation" by automorphisms (α_t, β) defined on $N *_B (B \otimes L(\mathbf{F}_2))$. This deformation was introduced for the first time in [4] and it represented one of the key tools that lead to the computation of the symmetry groups of amalgamated free products of weakly rigid factors. It was shown in [9] that this deformation automatically features a certain "transversality property" (see Lemma 2.1 in [9]) which will be of essential use in our proof.
(2) The second ingredient we will use is the following property proved by Popa in [10], for free products of finite von Neumann algebras. Roughly, with B a finite amenable von Neumann algebra, any von Neumann subalgebra $Q \subset N$ with no amenable direct summand has "spectral gap" with respect to the orthogonal complement of N in $N *_B (B \otimes L(\mathbf{F}_2))$. In other words, there exists a finite subset $F \subset \mathcal{U}(Q)$ such that if $x \in N *_B (B \otimes L(\mathbf{F}_2))$ almost commutes with all the unitaries $u \in F$, then x is almost contained in N.

We obtain the following theorem that generalizes many previous results on primality, and moreover gives new examples of prime factors (of type II_1 and of type III):

Theorem 1. *For $i = 1, 2$, let $\mathcal{M}_i$ be a von Neumann algebra. Let $B \subset \mathcal{M}_i$ be a common von Neumann subalgebra, with $B \neq \mathcal{M}_i$, such that there exists a faithful normal conditional expectation $E_i : \mathcal{M}_i \to B$. Assume that B is a finite von Neumann algebra of type* I*, e.g. B is finite dimensional or B is abelian. Denote by $\mathcal{M} = \mathcal{M}_1 *_B \mathcal{M}_2$ the amalgamated free product. If $\mathcal{M}$ is a non-amenable factor, then $\mathcal{M}$ is prime.*

Using some of Ueda's results on factoriality and non-amenability of plain free products and of amalgamated free products over a common Cartan subalgebra (see [17, 18, 19, 20]), we obtain the following corollaries:

Corollary 2. *For $i = 1, 2$, let $(\mathcal{M}_i, \varphi_i)$ be any von Neumann algebra endowed with a f.n. state. Assume that the centralizer $\mathcal{M}_1^{\varphi_1}$ is diffuse and $\mathcal{M}_2 \neq \mathbf{C}$. Then the free product $(\mathcal{M}, \varphi) = (\mathcal{M}_1, \varphi_1) * (\mathcal{M}_2, \varphi_2)$ is a prime factor.*

Corollary 3. *For $i = 1, 2$, let $\mathcal{M}_i$ be a non-type* I *factor, and $B \subset \mathcal{M}_i$ be a common Cartan subalgebra. Then the amalgamated free product $\mathcal{M} = \mathcal{M}_1 *_B \mathcal{M}_2$ is a prime factor.*

In particular, let $\Gamma = \Gamma_1 * \Gamma_2$ be a free product of countable infinite groups. Let $\sigma : \Gamma \curvearrowright (X, \mu)$ be a free action such that the measure μ is quasi-invariant under σ, and such that the restricted action $\sigma_{|\Gamma_i}$ is ergodic and non-transitive for $i = 1, 2$. Then the crossed product $L^\infty(X, \mu) \rtimes \Gamma$ is a prime factor. In the type II_1 case, we get a more general result:

Theorem 4. *For $i = 1, 2$, let M_i be a* II_1 *factor and $B \subset M_i$ be a common abelian von Neumann subalgebra such that $\tau_{1|B} = \tau_{2|B}$. Then the amalgamated free product $M = M_1 *_B M_2$ is a non-amenable* II_1 *factor. Thus, M is prime.*

References

[1] M. Gao & M. Junge, *Examples of prime von Neumann algebras.* Int. Math. Res. Notices. Vol. 2007 : article ID rnm042, 34 pages.

[2] L. Ge, *Applications of free entropy to finite von Neumann algebras,* II. Ann. of Math. **147** (1998), 143–157.

[3] C. Houdayer, *Construction of type* II_1 *factors with prescribed countable fundamental group.* J. reine angew Math., to appear. `arXiv:0704.3502`

[4] A. Ioana, J. Peterson & S. Popa, *Amalgamated free products of w-rigid factors and calculation of their symmetry groups.* Acta Math. **200** (2008), 85–153.

[5] N. Ozawa, *Solid von Neumann algebras.* Acta Math. **192** (2004), 111–117.

[6] N. Ozawa *A Kurosh-type theorem for type* II_1 *factors.* Int. Math. Res. Notices. Vol. 2006 : article ID 97560, 21 pages.

[7] N. Ozawa, *An example of a solid von Neumann algebra.* `arXiv:0804.0288`

[8] J. Peterson, *L^2-rigidity in von Neumann algebras.* Invent. Math., to appear.

[9] S. Popa, *On the superrigidity of malleable actions with spectral gap.* J. Amer. Math. Soc. **21** (2008), 981–1000.

[10] S. Popa, *On Ozawa's property for free group factors.* Int. Math. Res. Notices. Vol. 2007 : article ID rnm036, 10 pages.

[11] S. Popa, *Strong rigidity of* II_1 *factors arising from malleable actions of w-rigid groups* I. Invent. Math. **165** (2006), 369-408.

[12] S. Popa, *On a class of type* II_1 *factors with Betti numbers invariants.* Ann. of Math. **163** (2006), 809–899.

[13] É. Ricard & Q. Xu, *Khintchine type inequalities for reduced free products and applications.* J. reine angew. Math. **599** (2006), 27–59.

[14] D. Shlyakhtenko, *Prime type* III *factors.* Proc. Nat. Acad. Sci., **97** (2000), 12439–12441.

[15] D. Shlyakhtenko, *Free quasi-free states.* Pacific J. Math. **177** (1997), 329–368.

[16] M. B. Stefan, *The primality of subfactors of finite index in the interpolated free group factors.* Proc. Amer. Math. Soc. **126** (1998), 2299–2307.

[17] Y. Ueda, *Amalgamated free products over Cartan subalgebra.* Pacific J. Math. bf 191 (1999), 359–392.

[18] Y. Ueda, *Remarks on free products with respect to non-tracial states.* Math. Scand. **88** (2001), 111–125.

[19] Y. Ueda, *Fullness, Connes' χ-groups, and ultra-products of amalgamated free products over Cartan subalgebras.* Trans. Amer. Math. Soc. **355** (2003), 349–371.

[20] Y. Ueda, *Amalgamated free products over Cartan subalgebra,* II. *Supplementary results and examples.* Advanced Studies in Pure Mathematics, Operator Algebras and Applications. **38** (2004), 239–265.

[21] S. Vaes & R. Vergnioux, *The boundary of universal discrete quantum groups, exactness and factoriality.* Duke Math. J. **140** (2007), 35-84.

[22] S. Vaes, *Rigidity results for Bernoulli actions and their von Neumann algebras (after S. Popa).* Séminaire Bourbaki, exposé 961. Astérisque **311** (2007), 237-294.

Relative property (T) for the subequivalence relations induced by the action of $\mathbf{SL_2}(\mathbb{Z})$ on $\mathbb{T}^2$

Adrian Ioana

In this talk we are concerned with the subequivalence relations $\mathcal{R}$ of the equivalence relation $\mathcal{S}$ induced by the action of $\mathrm{SL}_2(\mathbb{Z})$ on the 2-torus $\mathbb{T}^2$. Our main result asserts that any ergodic $\mathcal{R}$ is either hyperfinite or rigid in the sense of S. Popa [5]. This answers a question of D. Gaboriau and S. Popa [2].

To explain our result and its method of proof in more detail, recall that a countable ergodic measure preserving equivalence relation $\mathcal{R}$ on a probability space (X, μ) is said to be *rigid* if the inclusion of $L^\infty(X, \mu)$ into the von Neumann algebra $L(\mathcal{R})$ associated with $\mathcal{R}$ is rigid. Following [5], this means that any Hilbert $L(\mathcal{R})-$bimodule $\mathcal{H}$ which admits a sequence of (unit tracial) almost central vectors must have a non-zero $L^\infty(X, \mu)-$central vector.

The typical examples of rigid equivalence relations come from group theory. Starting with an automorphic action of a countable group Γ on a discrete abelian group A, the equivalence relation induced by the (Haar) measure preserving action of Γ on the dual $\hat{A}$ is rigid if and only if the pair of groups $(\Gamma \ltimes A, A)$ has relative property (T) of Kazhdan-Margulis [5]. Thus, we can view our result as a generalization of M. Burger's result showing that the pair $(\Gamma \ltimes \mathbb{Z}^2, \mathbb{Z}^2)$ has relative property (T), for any non-amenable subgroup Γ of $\mathrm{SL}_2(\mathbb{Z})$ [1].

To sketch the proof of the main result, assume that $\mathcal{R}$ is an ergodic subequivalence relation of $\mathcal{S}$ which is not rigid. Notice first that for a probability space (X, μ), any cyclic Hilbert $L^\infty(X, \mu)-$bimodule is of the form $L^2(X \times X, \nu)$, where ν is a measure on $X \times X$ whose coordinate projections onto X are equal to μ. Using this observation, the lack of rigidity of $\mathcal{R}$ implies that we can find a sequence of measures $\{\nu_n\}_{n \geq 1}$ on $\mathbb{T}^2 \times \mathbb{T}^2$ which, roughly speaking, concentrate around the diagonal $\Delta = \{(x, x) | x \in \mathbb{T}^2\}$ and become almost invariant under the diagonal product action of $[\mathcal{R}]$ (the full group of $\mathcal{R}$), as $n \to \infty$. A simple computation gives that the ν_n's are also almost invariant under the skew product action of $[\mathcal{R}]$ on $\mathbb{T}^2 \times \mathbb{T}^2$ defined by $\theta \circ (x, y) = (\theta(x), w(\theta, x)y)$, where $w(\theta, x)$ is the unique element of $\mathrm{SL}_2(\mathbb{Z})$ such that $\theta(x) = w(\theta, x)x$.

The next key element of the proof is that there exists a Borel map $\pi : (\mathbb{T}^2 \times \mathbb{T}^2) \setminus \Delta \to \mathbb{T}^2 \times \mathrm{P}^1(\mathbb{R})$ which is $\gamma-$equivariant in an open neighborhood of Δ, for every $\gamma \in \mathrm{SL}_2(\mathbb{Z})$. By pushing forward the ν_n's and taking a weak limit, we deduce that there exists a measure μ on $\mathbb{T}^2 \times \mathrm{P}^1(\mathbb{R})$ which is invariant under the skew product action of $[\mathcal{R}]$. Moreover, the projection of μ onto the $\mathbb{T}^2-$coordinate is equal to λ^2, hence we can disintegrate $\mu = \int_{\mathbb{T}^2} \mu_x d\lambda^2(x)$, where μ_x are probability measures on $\mathrm{P}^1(\mathbb{R})$. The uniqueness of the disintegration implies that $\mu_{\theta(x)} = w(\theta, x)\mu_x$, for all $\theta \in [\mathcal{R}]$ and almost every $x \in \mathbb{T}^2$. The final step of the proof consists of combining the existence of the measures μ_x with the topological amenability of the action $\mathrm{SL}_2(\mathbb{Z})$ on $\mathrm{P}^1(\mathbb{R})$ to conclude that $\mathcal{R}$ is hyperfinite.

Let us also mention that N. Ozawa has recently shown that any ergodic subequivalence relation $\mathcal{R}$ of $\mathcal{S}$ is either hyperfinite or strongly ergodic [4]. Related to this, note that it is not known whether rigidity implies strong ergodicity for equivalence relations.

References

[1] M. Burger, *Kazhdan constants for SL(3,$\mathbb{Z}$)*, J. Reine Angew. Math. **413** (1991), 36-67.
[2] D. Gaboriau, S. Popa, *An uncountable family of nonorbit equivalent actions of* $\mathbb{F}_n$, J. Amer. Math. Soc. **18** (2005), no. 3, 547-559 (electronic).
[3] A. Ioana, *Relative property (T) for the subequivalence relations induced by the action of* $SL_2(\mathbb{Z})$ *on* $\mathbb{T}^2$, in preparation.
[4] N. Ozawa *An example of a solid von Neumann algebra*, preprint arXiv:0804.0288.
[5] S. Popa: *On a class of type* II_1 *factors with Betti numbers invariants*, Ann. of Math. (2) **163** (2006), no. 3, 809-899.

Generalized CCR flows

Masaki Izumi
(joint work with R. Srinivasan)

An E_0-semigroup is a weakly continuous semigroup of unital $*$-endomorphisms of $\mathbb{B}(H)$, where H is a separable infinite dimensional Hilbert space. The notion of a product system was introduced by W. Arveson in his study of E_0-semigroups. He showed, on one hand, that the product system associated with an E_0-semigroup completely determines the cocycle conjugacy class of the E_0-semigroup, and on the other hand, that every product system arises from an E_0-semigroup (see [1]). Therefore classification of E_0-semigroups up to cocycle conjugacy is equivalent to that of product systems up to isomorphism.

Product systems, and hence E_0-semigroups, are classified into three categories, type I, type II, and type III, according to abundance of units. Type I E_0-semigroups are completely classified, and they are cocycle conjugate to so called CCR flows. By definition, type III E_0-semigroups have no units, and it is not so easy to construct them. Indeed, before Tsirelson's construction of uncountably many type III product systems [6], Powers' type III E_0-semigroup was the only example. The main purpose of this talk is to report on the recent development of type III E_0-semigroups.

Inspired by Tsirelson's construction of infinitely many mutually non-isomorphic type III product systems [6], Bhat and Srinivasan [2] introduced the notion of a sum system, which is a sort of "logarithm" of a product system giving rise to a product system via the Bosonic second quantization procedure. They obtained a dichotomy result about types, which says that every product system arising from a *divisible* sum system is either of type I or of type III.

One of the purposes of this talk is to show that every sum system is indeed divisible (see [4] for details). The proof goes through the notion of generalized CCR flows [5], which include the E_0-semigroups corresponding to Tsirelson's type III product systems [3]. In particular, every generalized CCR flow is either of type I or type III.

References

[1] W. Arveson, *Non-commutative dynamics and E-semigroups*, Springer Monograph in Math. (Springer 2003).
[2] B. V. R. Bhat and R. Srinivasan, *On product systems arising from sum systems* Infinite dimensional analysis and related topics, Vol. 8, Number 1, March 2005.
[3] M. Izumi, *A perturbation problem for the shift semigroup.* J. Funct. Anal. **251**, (2007), 498–545.
[4] M. Izumi, *Every sum system is divisible*, to appear in Trans. A.M.S. arXiv:0708.1591 [math.OA].
[5] M. Izumi and R. Srinivasan, *Generalized CCR flows.* Commun. Math. Phys. **281**, (2008), 529–571.
[6] B. Tsirelson, *Non-isomorphic product systems.* Advances in Quantum Dynamics (South Hadley, MA, 2002), 273–328, Contemp. Math., 335, Amer. Math. Soc., Providence, RI, 2003.

Bimodules and the orthogonal approach to the graded algebra of a planar algebra.

Vaughan Jones
(joint work with Kevin Walker, Dimitri Shlyakhtenko)

The Voiculescu trace on the algebra $\mathbb{C}\langle X_1, X_2, .., X_n\rangle$ of polynomials in n non-commuting self-adjoint variables is defined on a monomial by the number of possible Temperley-Lieb (planar) pairings of the letters in the word which pair only the same letters. Thus for instance $tr_V(X_1^2 X_2^4) = 2$ (and the trace of any word of odd length is zero). It is known that the inner product coming from this trace is positive definite and the result of the GNS construction is a II_1 factor isomorphic to the group von Neumann algebra of the free group on n generators. The Voiculescu trace may be obtained as the appropriately scaled limit of the trace of the corresponding word on n $N \times N$ independent random matrices as $N \to \infty$.

The ingredients of the above construction are a multiplication given by concatenation and a way to pair elements with Temperley-Lieb diagrams. These ingredients are supplied by any planar algebra (with suitable positivity) and in [1] it was shown that the Voiculescu trace on the graded algebra of a planar algebra gives II_1 factors in the same way. The noncommutative polynomials are simply the special case where the planar algebra is the "vertex model" planar algebra of [1]. Moreover there is an infinite family of different multiplications and embeddings of the ensuing II_1 factors, and embeddings of these factors one into another that realise the Jones tower for the subfactor given by the first two. The standard invariant for this subfactor is precisely the planar algebra one used to make the original construction.

An advantage of this construction is that the bimodules related to the subfactor can be realised in a very simple diagrammatic way by taking a graded Hilbert space with projections corresponding to the bimodules at the bottom, other strings at the top, and the left and right actions of the II_1 factor being given by left and right concatentation. Tensor product of bimodules is similarly given by concatentation.

In order to calculate the left and right Murray-von Neumann dimensions one works on the left and right separately and compares the factor M and its subfactor M. Somewhat surprisingly one discovers a canonical projection p in M of trace $[M : N]^{-1/2}$ and an isomorphism between N and pMp in suitably self-dual cases such as Temperley-Lieb and noncommutative polynomials. K. Walker found an explicit formula for p using an orthogonal version of the graded algebra. In [3] this orthogonal approach was used to provide a much simpler proof of the main result of [1]. The same method was discovered simultaneously and independently by Kodiyalam and Sunder in [4].

After changing to the orthogonal basis all the graded pieces of the algebra are orthogonal and the inner product on each one is the usual one in planar algebras. This makes positivity of the inner product coming from the Voiculescu trace trivial. In principle such an orthogonalisation could make the multiplication very complicated but in this case it remains very easy to work with the new product as it remains pictorial. The product between two elements in graded pieces is the sum of terms, the first of which is concatenation and the other terms each of which involves more and more contractions (strings joining the two graded pieces). Boundedness of the left multiplication operators is not difficult. The main trick in the proof of factoriality is to show that the element called "cup" (which is the only Temperley-Lieb diagram with two boundary points) generates a maximal abelian subalgebra. This is achieved by decomposing the whole graded algebra as mutually orthogonal bimodules (correpsondences) with respect to the von Neumann algebra generated by cup. The decomposition is very simple-there is the bimodule spanned by cup itself and then infinitely many copies of a single bimodule which is in fact the coarse correspondence. Maximal abelianness then follows from the absence of atoms in the spectrum of cup. To conclude that the whole algebra is a factor it then suffices to find an element that does not commute with anything in the algebra generated by cup. This is easy-one may choose the Temperley-Lieb element with four boundary points which is not the square of cup. Once again orthogonality makes the calculation rather simple.

The same tricks allow one to calculate the tower of relative commutants as the graded pieces of the initial planar algebra. The planar algebra structure is then identified by standard methods.

There is much work to be done in this area, perhaps the most interesting being the incorporation of a potential defining a perturbation of the Gaussian measure on random matrices. The planar algebra language seems to be well adapted for that.

References

[1] A. Guionnet, V. Jones and D. Shlyakhtenko, *Random matrices, free probability, planar algebras and subfactors.*, arXiv:0712.2904

[2] V. Jones, *Planar algebras I,* math.QA/9909027

[3] V. Jones, K. Walker and D. Shlyakhtenko, *An orthogonal approach to the subfactor of a planar algebra.*, arXiv:0807.4146

[4] V. Kodiyalam and V. Sunder,*From subfactor planar algebras to subfactors.*, arXiv:0807.3704

Moonshine, pariah groups and operator algebras

Yasuyuki Kawahigashi

The Moonshine conjecture asserts a surprising relation between the largest sporadic finite simple group, the Monster, and modular functions arising from discrete subgroups of $PSL(2, \mathbf{R})$. It was created by Conway and Norton, and has been now proved by Borcherds based on work of Frenkel, Lepowsky, and Meurman. The new algebraic structure for studying this is called a vertex operator algebra, which is an algebraic axiomatization of a certain family of operator-valued distributions on the circle. The "Moonshine vertex operator algebra" has the automorphism group isomorphic to the Monster group, and Monster elements correspond to discrete subgroups of $PSL(2, \mathbf{R})$ through certain power series, called the McKay-Thompson series. Longo and I constructed its operator algebraic counterpart with all expected nice properties including the automorphism group being isomorphic to the Monster group in [4]. A similar structure has been pursued for other sporadic finite simple groups by Duncan [2, 3], and he considered the Rudvalis group, which is one of the six "pariah" groups not involved in the Monster group. He constructed two super vertex operator algebras with automorphic actions of the Rudvalis group with certain Moonshine type properties on two-variable power series arising from group elements of the Rudvalis group. We now construct an operator algebraic counterpart for one of the two in [2], based on our general theory of super conformal nets of factors in [1]. The other super vertex operator algebra of Duncan in [3] has no unitarity, so it has no operator algebraic counterpart.

References

[1] S. Carpi, Y. Kawahigashi, R. Longo, *Structure and classification of superconformal nets*, Ann. Henri Poincaré **8** (2008), 1069–1121.

[2] J. F. Duncan, *Moonshine for Rudvalis's sporadic group I.*, math/0609449.

[3] J. F. Duncan, *Moonshine for Rudvalis's sporadic group II.*, math/0611355.

[4] Y. Kawahigashi, R. Longo, *Local conformal nets arising from framed vertex operator algebras*, Adv. Math. **206** (2006), 729–751.

Measurable rigidity for some amalgamated free products

Yoshikata Kida

The aim of this report is to construct rigid groups in the sense of measure equivalence.

Definition 1. *Two discrete countable groups Γ, Λ are said to be measure equivalent (ME) if there exist a standard Borel space (Σ, m) with a σ-finite positive measure and a measure-preserving action of $\Gamma \times \Lambda$ on Σ satisfying the following: There exist Borel subsets $X, Y \subset \Sigma$ such that $\Sigma = \bigsqcup_{\gamma\in\Gamma} \gamma Y = \bigsqcup_{\lambda\in\Lambda} \lambda X$ up to m-null sets.*

Measure equivalence is closely related with orbit equivalence. In fact, two groups Γ, Λ are ME if and only if there exist essentially free, measure-preserving actions $\Gamma \curvearrowright (X, \mu)$, $\Lambda \curvearrowright (Y, \nu)$ on standard probability spaces which are stably orbit equivalent. Furman [1] shows that if G is a center-free, non-compact, connected simple Lie group of $\mathbb{R}$-rank at least two, and if Γ is a lattice in G, then Γ is rigid in the following sense: If a group Λ is ME to Γ, then Λ is virtually isomorphic to a lattice in G. The author [2] shows that mapping class groups for compact orientable surfaces which are neither finite nor virtually $SL(2, \mathbb{Z})$ are ME rigid. That is, if a group Λ is ME to such a mapping class group, then the two groups are virtually isomorphic. This is the first example of ME rigid groups. The author recently studies rigidity phenomena for some amalgamated free products whose factor groups are rigid. Among other thing, we get the following construction of ME rigid groups.

Theorem 2. *Let G be a center-free, non-compact, connected simple Lie group of $\mathbb{R}$-rank at least two, and let Γ be a lattice in G. Let $A < \Gamma$ be a proper infinite subgroup satisfying the following conditions:*

(i) $LQN_\Gamma(A) = A$.
(ii) *The delta measure δ_e on the neutral element of C is the only probability measure on C which is invariant under conjugation by each element of A, where $C = \mathrm{Comm}_{\mathrm{Aut}(G)}(\Gamma)$.*

Then the amalgamated free product $\Gamma *_A \Gamma$ is ME rigid. That is, if a discrete group Λ is ME to $\Gamma *_A \Gamma$, then they are virtually isomorphic.

For a pair of a group Γ and a subgroup A, we denote by

$$LQN_\Gamma(A) = \{\gamma \in \Gamma : [A : \gamma A \gamma^{-1} \cap A] < \infty\}$$

the left quasi-normalizer of A in Γ, which is a subsemigroup of Γ containing A. The commensurator $\mathrm{Comm}_{\mathrm{Aut}(G)}(\Gamma)$ of Γ in $\mathrm{Aut}(G)$ by definition consists of all elements $g \in \mathrm{Aut}(G)$ such that $[\Gamma : g^{-1}\Gamma g \cap \Gamma] < \infty$ and $[\Gamma : g\Gamma g^{-1} \cap \Gamma] < \infty$.

Let us give an example satisfying the assumption in Theorem 2. Let $\{e_1, e_2, e_3\}$ be the standard basis for the vector space $\mathbb{R}^3$. Let F be the flag consisting of the subspaces $\{0\} \subset \langle e_1 \rangle \subset \mathbb{R}^3$. Let A be the stabilizer of F in $SL(3, \mathbb{Z})$, which consists of all matrices in $SL(3, \mathbb{Z})$ whose $(2,1)$-, $(3,1)$-entries are both 0. We can then prove that the pair of groups $A < SL(3, \mathbb{Z})$ satisfies the assumption in Theorem 2.

Corollary 3. *The amalgamated free product $SL(3, \mathbb{Z}) *_A SL(3, \mathbb{Z})$ is ME rigid.*

We note that there are many flags of $\mathbb{R}^n$ with $n \geq 3$ such that the pair of $PSL(n, \mathbb{Z})$ and the stabilizers of the flags satisfy the assumption in Theorem 2 and therefore they are ME rigid. This corollary gives a new example of ME rigid groups. More details on these results will appear elsewhere.

References

[1] A. Furman, *Gromov's measure equivalence and rigidity of higher rank lattices*, Ann. of Math. (2) **150** (1999), 1059–1081.

[2] Y. Kida, *Measure equivalence rigidity of the mapping class group*, Ann. of Math., to appear.

K- and *L*-theory of group rings and topological rigidity

Wolfgang Lück
(joint work with Arthur Bartels)

Let G be a discrete group and let R be an associative ring with unit. We explain and state the following conjectures and discuss their relevance.

Kaplanski Conjecture. If G is torsionfree and R is an integral domain, then 0 and 1 are the only idempotents in RG.

Conjecture. Suppose that G is torsionfree. Then $K_n(\mathbb{Z}G)$ for $n \leq -1$, $\widetilde{K}_0(\mathbb{Z}G)$ and $\mathrm{Wh}(G)$ vanish.

Novikov Conjecture. Higher signatures are homotopy invariants.

Borel Conjecture. An aspherical closed manifold is topologically rigid.

Conjecture. The homomorphism induced by the Fuglede-Kadison determinant $\mathrm{Wh}(G) \to \mathbb{R}$ is trivial.

Farrell-Jones Conjecture. Let G be torsionfree and let R be regular. Then the assembly maps for algebraic K- and L-theory

$$\begin{array}{rcl} H_n\big(BG;\mathbb{K}_R\big) & \to & K_n(RG); \\ H_n\big(BG;\mathbb{L}_R^{\langle -\infty\rangle}\big) & \to & L_n^{\langle -\infty\rangle}(RG), \end{array}$$

are bijective for all $n \in \mathbb{Z}$.

There is a more complicate version of the Farrell-Jones Conjectures which makes sense for all groups and rings and allows twistings of the group ring. We explain that it implies all the other conjectures mentioned above provided that in the Kaplanski Conjecture R is a field of characteristic zero and in the Borel Conjecture the dimension is greater or equal to five. We announce the following result:

Theorem [Bartels-Lück]. Let $\mathcal{FJ}$ be the class of groups for which the Farrell-Jones Conjecture is true in its general form. Then:

(1) Hyperbolic groups, nilpotent groups and CAT(0) groups belong to $\mathcal{FJ}$;
(2) If G_0 and G_1 belong to $\mathcal{FJ}$, then also $G_0 * G_1$ and $G_0 \times G_1$;
(3) If G belongs to $\mathcal{FJ}$, then any subgroup of G belongs to $\mathcal{FJ}$;
(4) Let $\{G_i \mid i \in I\}$ be a directed system of groups (with not necessarily injective structure maps). If each G_i belongs to $\mathcal{FJ}$, then also the direct limit of $\{G_i \mid i \in I\}$.

Since certain prominent constructions of groups yield colimits of hyperbolic groups, the class $\mathcal{FJ}$ contains many interesting groups, e.g. limit groups, Tarski monsters, groups with expanders and so on. Some of these groups were regarded as possible counterexamples to the conjectures above but are now ruled out by the theorem above.

There are also prominent constructions of closed aspherical manifolds with exotic properties, e.g. whose universal covering is not homeomorphic to Euclidean space, whose fundamental group is not residually finite or which admit no triangulation. All these constructions yield fundamental groups which are CAT(0) and hence yield topologically rigid manifolds.

However, the Farrell-Jones Conjecture is open for instance for solvable groups, $\mathrm{SL}_n(\mathbb{Z})$ for $n \geq 3$, mapping class groups or automorphism groups of finitely generated free groups.

We explain that for a group which possesses a finite model for BG and all of whose L^2-Betti numbers vanish, one can define its L^2-torsion $\rho^{(2)}(G) \in \mathbb{R}$ provided that the homomorphism induced by the Fuglede-Kadison determinant $\mathrm{Wh}(G) \to \mathbb{R}$ is trivial. This invariant has nice properties. For instance, if G is the fundamental group of an odd-dimensional hyperbolic manifold, then $\rho^{(2)}(G)$ is up to a non-zero dimension constant the volume. We mention the following question.

Question. Let G_0 and G_1 be groups with finite models for BG_0 and BG_1. Suppose that the all L^2-Betti numbers of G_0 and G_1 vanish. Suppose that G_0 and G_1 are quasiisometric or that G_0 and G_1 are orbit equivalent. Does then $\rho^{(2)}(G_0) = 0 \Leftrightarrow \rho^{(2)}(G_0) = 0$ hold?

Finally we announce the following rigidity result about hyperbolic groups:

Theorem [Bartels-Lück-Weinberger] Let G be a torsionfree hyperbolic group and let n be an integer ≥ 6. Then the following statements are equivalent:

- The boundary ∂G is homeomorphic to S^{n-1};
- There is a closed aspherical topological manifold M such that $G \cong \pi_1(M)$, its universal covering $\widetilde{M}$ is homeomorphic to $\mathbf{R}^n$ and the compactification of $\widetilde{M}$ by ∂G is homeomorphic to D^n.

The manifold appearing above is unique up to homeomorphism.

References

[1] **Bartels, A., Lück, W. and Reich, H.**: *"Equivariant covers for hyperbolic groups"*, Geometry and Topology 12 (2008) 1799–1882.

[2] **Bartels, A., Lück, W. and Reich, H.**: *"The K-theoretic Farrell-Jones Conjecture for hyperbolic groups"*, Inventiones mathematicae 172 (2008), 29–70.

[3] **Bartels, A., Lück, W. and Reich, H.**: *"On the Farrell-Jones Conjecture and its applications"*, Journal of Topology 1 (2008), 57–86.

[4] **Bartels, Echterhoff, A. and Lück, W.**: *"Inheritance of Isomorphism Conjectures under colimits"*, Proceedings of the conference 'K-theory and non-commutative geometry'Ž in Valladolid, August/September 2006,Ž EMS Series of Congress reports, editors: Cortinaz, Cuntz, Karoubi, Nest, Weibel, European Mathematical Society (2008) 41–70.

[5] **Kreck, M. and Lück, W.**: *"Topological rigidity of non-aspherical manifolds"*, Preprintreihe SFB 478 — Geometrische Strukturen in der Mathematik, Heft 398, Münster, arXiv:math.GT/0509238, to appear in 2009 in Pure and Applied Mathematics Quarterly 5 (3), special issue in honor of Friedrich Hirzebruch part 2 (2005) 873–914.

[6] **Lück, W.**: *"L^2-Invariants: Theory and Applications to Geometry and K-Theory"*, Ergebnisse der Mathematik und ihrer Grenzgebiete 44, Springer (2002).

[7] **Lück, W. and Reich, H.**: *"The Baum-Connes and the Farrell-Jones Conjectures"*, Handbook of K-theory Volume 2, editors: É.M. Friedlander, D.R. Grayson, Springer (2005) 703 – 842.

II_1 factors with at most one Cartan subalgebra

Narutaka Ozawa
(joint work with Sorin Popa)

In recent few years, the classification program of finite von Neumann algebras has seen a remarkable progress. I focus on the the isomorphy problem for the group-measure-space construction: Let $\Gamma \curvearrowright X$ be an essentially-free, ergodic, probability-measure-preserving action on a standard probability space X. To what extent does the crossed product von Neumann algebra $L^\infty(X) \rtimes \Gamma$ remember the group Γ and the action $\Gamma \curvearrowright X$ that were used in the construction? In particular, I will look for rigidity phenomena: crossed product von Neumann algebras $L^\infty(X) \rtimes \Gamma$ that completely remember the group Γ and the action $\Gamma \curvearrowright X$. I will report a partial solution to this problem, obtained in joint works with S. Popa [1].

Recall that a second countable locally compact group G has the Haagerup property if there are a unitary G-representation $(\pi, \mathcal{H})$ and a 1-cocycle $b\colon G \to \mathcal{H})$ which is proper in the sense that $\|b(g)\| \to \infty$ as $g \to \infty$. In case π can be taken non-amenable, we say G has the property (HH). All connected simple Lie groups with the Haagerup property (i.e., $\mathrm{SO}(1,n)$ and $\mathrm{SU}(1,n)$) have the property (HH), and the property (HH) is preserved under taking products and passing to lattices.

Theorem. *Let Γ be a countable discrete group with the property* (HH) *and the complete metric approximation property. Then, the group von Neumann algebra $L\Gamma$ does not have a Cartan subalgebra, i.e., it does not admit a group-measure-space decomposition. Moreover, let $\Gamma \curvearrowright X$ be an action as before and assume it is profinite. Then, the crossed product von Neumann algebra $M = L^\infty(X) \rtimes \Gamma$ has a unique Cartan subalgebra up to unitary conjugacy, i.e., one can specify the position of $L^\infty(X)$ in M.*

I will talk how to apply Peterson's technique on real closable derivations to obtain the above theorem. Combined with a cocycle superrigidity type theorem, it implies:

Corollary. *Let $\Gamma_0 = \mathrm{PSL}(2, \mathbb{Z}[\sqrt{2}])$ and $\Gamma = \Gamma_0 \times \Gamma_0$. Let $p_1 < p_2 <$ be a sequence of prime numbers and $\Gamma \curvearrowright X = \varprojlim \mathrm{PSL}(2, (\mathbb{Z}/p_1 \cdots p_n)[\sqrt{2}])$ be the left-and-right translation action. Let $\Lambda \curvearrowright Y$ be any e.f.e.p.m.p. action of a* ***residually-finite*** *group Λ. If $L^\infty(X) \rtimes \Gamma \cong L^\infty(Y) \rtimes \Lambda$, then $\Gamma \curvearrowright X$ and $\Lambda \curvearrowright Y$ are virtually isomorphic (i.e., Γ and Λ have a common subgroup Δ of finite index and the actions $\Gamma \curvearrowright X$ and $\Lambda \curvearrowright Y$ are respectively induced by an action $\Delta \curvearrowright Z$).*

I strongly believe that the residual-finiteness assumption on Λ is redundant.

References

[1] N. Ozawa and S. Popa, *On a class of* II_1 *factors with at most one Cartan subalgebra I, II*, arXiv:0706.3623, arXiv:0807.4270

Groups of intermediate rank: two examples

Mikaël Pichot
(joint work with Sylvain Barré)

The talk is devoted to a certain class of countable groups having intermediate rank properties. I will mostly concentrate on two concrete examples, namely:

- the bowtie group $G_{\bowtie}$
- the Wise group G_W

which are described more precisely below. In both cases the rank is situated strictly in between 1 and 2.

Most of the operator algebraic applications we derived so far concerned the reduced group C^*-algebra. In particular we show that both groups have property RD (of rapid decay) and satisy the Baum-Connes conjecture. The techniques are specific to each case but can be unified to some extend, via the concept of *groups of friezes*.

The framework is as follows. We let X be a polyhedral complex of dimension 2 with a CAT(0) structure, and G be a countable group acting freely and cocompactly on X by simplicial isometries. The 2-faces in X are of various shapes (in finite number) which we assume here to be flat, i.e. isometrically embeddable into the Euclidean $\mathbf{R}^2$.

1) The bowtie group $G_{\bowtie}$*.* The first example is a group taken from [1] and can be described as the fundamental group of a compact metric CW-complex $V_{\bowtie}$ with faces isometric to either : a "bowtie", a lozenge, or an equilateral triangle (see [1, Fig. 4]). The complex $V_{\bowtie}$ has 8 vertices: two of them have local rank 2 (their link is isomorphic to the incidence graph of the Fano plane) and the 6 others have local rank $\frac{3}{2}$ (see [1] for precisions and details of construction). Property RD for $G_{\bowtie}$ was proved in [1] by applying the following theorem to a natural subdivision of the universal cover $X_{\bowtie} = \tilde{V}_{\bowtie}$:

Let G be a group acting properly on a CAT(0) simplicial complex X of dimension 2 without boundary and whose faces are equilateral triangles of the Euclidean plane. Then G has property RD with respect to the length induced from the 1-skeleton of X. (see [1, Theorem 5])

2) The Wise group G_W*.* This group was introduced by Dani Wise in [5] and can be defined by the presentation

$$G_W = \langle a, b, c, s, t \mid c = ab = ba,\ c^2 = sas^{-1} = tbt^{-1} \rangle.$$

This is a non-Hopfian group acting on a polyhedral complex X_W of dimension 2 (see [5]) built out of the following 2 shapes: a square with edges of length 1 (one of them is divided into two), and an isocele triangle with 2 edges of length 1 and one of length $\frac{1}{2}$. In a paper in preparation [4] we will prove that:

The Wise group W has property RD.

This theorem was announced in [2] (with a quite detailed sketch of proof). It answers a question of Mark Sapir.

At the end of the introduction of [1] we noted some similarity between the bowtie group $G_{\bowtie}$ and the Wise group G_W (while studying their mesoscopic rank, see also [3] for more on this property). These analogies will be clarified by the notion of *frieze* in a CAT(0) polyhedral complex which we consider in [4]. The frizes of $X_{\bowtie}$ are flat strips alterning bowties and lozenges, while in X_W friezes are flat strips of squares.

Friezes allow to link property RD to the CAT(0) structure when they are *analytic* in an appropriate sense. This allows us to give a largely unified proof of property RD for the above two cases and to establish this property, as well as the Baum-Connes conjecture, for (infinitely) many new groups of friezes.

References

[1] Barré S., Pichot M., Intermediate rank and property RD, preprint Sept 2007.
[2] Barré S., Pichot M., Property RD for D. Wise non Hopfian group, preprint March 2008.
[3] Barré S., Pichot M., The 4-string braid group B_4 has property RD and exponential mesoscopic rank, preprint Sept. 2008.
[4] Barré S., Pichot M., Friezes in polyhedral complexes and applications, in preparation.
[5] Wise, Daniel T., A non-Hopfian automatic group. J. Algebra 180 (1996), no. 3, 845–847.

Biexactness and Orbit Equivalence Rigidity

Hiroki Sako

The class $\mathcal{S}$. There are several orbit equivalence rigidity (or OE) theorems which were proved by operator algebraic strategies. S. Popa gave cocycle and OE rigidity theorems for actions of Kazhdan groups or actions with spectral gaps ([7], [8]), using the deformation/rigidity arguments. A. Ioana gave cocycle and OE rigidity theorems for profinite actions of Kazhdan groups ([2]). In this note, we show an OE rigidity phenomenon which is also proved by operator algebraic strategy. We get a rigidity theorem for direct products of N. Ozawa's class $\mathcal{S}$ groups. The result is analogous to the result given by N. Monod and Y. Shalom ([3]).

The class $\mathcal{S}$ of countable groups is defined by means of Stone–Cech remainders.

Definition 1 (Ozawa, [5]). *Let Γ be a countable discrete group. The group Γ is said to be in the class $\mathcal{S}$, if the product group action $\Gamma \times \Gamma \curvearrowright \beta\Gamma \setminus \Gamma$ by the left and right translation is topologically amenable.*

Ozawa proved a remarkable theorem for group von Neumann algebras ([4]).

Theorem 2 (Ozawa, [4]). *Let Γ be a countable group in the class $\mathcal{S}$. The group von Neumann algebra $L(\Gamma)$ is solid, that is, for arbitrary diffuse subalgebra $\mathcal{A} \subseteq L(\Gamma)$, its relative commutant $\mathcal{A}' \cup L(\Gamma)$ is injective.*

Examples and basic properties of the class are written in Ozawa's paper [5].

Example 3. *Amenable groups are in the class $\mathcal{S}$, because any amenable group action on a compact space is amenable. Word-hyperbolic groups are in the class $\mathcal{S}$. In particular, the free groups are in the class. A wreath product of a class $\mathcal{S}$ group with an amenable base is in the class $\mathcal{S}$ (e.g. $\mathbb{Z} \wr \mathbb{F}_2$). We note that these wreath product groups are not dealt with in the paper* [3] *of Monod and Shalom. Groups in the class $\mathcal{C}$ (*[3]*) can not have infinite normal amenable subgroups.*

Orbit equivalence rigidity theorem for direct product groups. We will show the most typical case among claims which we can prove by our technique. Groups Γ and Λ are countable torsion free groups in the class $\mathcal{S}$. Groups G and H are arbitrary countable non-amenable groups which don't have nontrivial normal finite subgroup. The two product groups $G \times H$ and $\Gamma \times \Lambda$ freely act on a standard probability space X in measure preserving ways.

Theorem 4. *Assume that the G-action and the H-action are respectively ergodic. If the $G \times H$ action and $\Gamma \times \Lambda$ action give the same orbit equivalence relation, then there exists a bi-measurable isomorphism $\theta : X \to X$ which gives the following two conjugacies between two group actions:*

$$(G \curvearrowright X) \cong (\Gamma \curvearrowright X), \quad (H \curvearrowright X) \cong (\Lambda \curvearrowright X), \quad (\textit{or} \ \ \Gamma \leftrightarrow \Lambda).$$

This theorem is a complete analogue of Monod and Shalom's theorem. By Furman's technique, that is written in Monod and Shalom's paper, we also get the following corollary:

Corollary 5. *Let $\Gamma \times \Lambda \curvearrowright X$ be a free probability measure preserving action. Assume that Γ and Λ are non-amenable torsion free groups in the class $\mathcal{S}$ and that their actions are respectively ergodic. Let G be a countable group which has no nontrivial normal finite subgroup. Assume that G acts on X freely and that the action has no nontrivial recurrent set (mild mixing condition). If the $\Gamma \times \Lambda$-action and the G-action are orbit equivalent, then they are conjugate.*

This is a super-rigidity type theorem, but we still need some assumption on the G-action. The mild mixing condition is essential.

Three key points for the theorem. As the first step for Theorem 4, a C^*-algebraical continuity property for the class $\mathcal{S}$ has a vital role. We take two groups Γ and Λ in the class $\mathcal{S}$. From the action of the product group $\Gamma \times \Lambda$, we can construct a Cartan inclusion $(L^\infty(X) \subseteq \mathcal{M}) = (L^\infty(X) \subseteq L^\infty(X) \rtimes (\Gamma \times \Lambda))$. The von Neumann algebra $\mathcal{M}$ is standardly represented on $\mathcal{H} = L^2X \otimes \ell_2\Gamma \otimes \ell_2\Lambda$.

To state the continuity property, we need two other C^*-algebras. For finite subsets $\mathcal{G} \subset \Gamma, \mathcal{L} \subset \Lambda$, we define the two orthogonal projections $e_\mathcal{G}$ and $e_\mathcal{L}$ by

$$e_\mathcal{G} \colon \mathcal{H} \twoheadrightarrow L^2X \otimes \ell_2\mathcal{G} \otimes \ell_2\Lambda, \quad e_\mathcal{L} \colon \mathcal{H} \twoheadrightarrow L^2X \otimes \ell_2\Gamma \otimes \ell_2\mathcal{L}.$$

We define the two C^*-algebras $I, D \subseteq \mathcal{B}(\mathcal{H})$ by

$$\begin{aligned} I &= \overline{\bigcup_{\mathcal{G},\mathcal{L}} (e_{\mathcal{G}} \vee e_{\mathcal{L}})\mathcal{B}(\mathcal{H})(e_{\mathcal{G}} \vee e_{\mathcal{L}})}^{\text{norm}}, \\ D &= C^*(L^\infty X, JL^\infty XJ, u_\Gamma, Ju_\Gamma J, u_\Lambda, Ju_\Lambda J, 1 \otimes \ell_\infty\Gamma \otimes 1, 1 \otimes 1 \otimes \ell_\infty\Lambda). \end{aligned}$$

Here $u_.$ denotes the implementing unitary for the action of $\Gamma \times \Lambda$ and J denotes the canonical conjugation for $\mathcal{M}$. We note that D is in the idealizer of I.

Proposition 6. *The natural $*$-homomorphism*

$$\Phi\colon L^\infty X \rtimes_{\text{red}} (\Gamma \times \Lambda) \otimes_{\mathbb{C}} JL^\infty X \rtimes_{\text{red}} (\Gamma \times \Lambda)J \longrightarrow (D + I)/I$$

is continuous with respect to the minimal tensor norm.

This proposition tells us that if the two groups are in the class $\mathcal{S}$, the equivalence relation satisfies this weakened amenability condition. In the proof, we make use of the natural embedding $\ell_\infty\Gamma/c_0\Gamma$, $\ell_\infty\Lambda/c_0\Lambda$ into $(D + I)/I$. The unitaries $\{u_{\gamma_1} J u_{\gamma_2} J + I \mid (\gamma_1, \gamma_2) \in \Gamma \times \Gamma\} \subseteq (D + I)/I$ and $\ell_\infty\Gamma/c_0\Gamma \subseteq (D + I)/I$ give a covariant system for the $\Gamma \times \Gamma$-action on $\ell_\infty\Gamma/c_0\Gamma = C(\beta\Gamma \setminus \Gamma)$. This is also true for $\Lambda\times\Lambda$-action on $\ell_\infty\Lambda/c_0\Lambda = C(\beta\Lambda\setminus\Lambda)$. Using a C^*-algebraical characterization of amenable actions by C. Anantharaman-Delaroche [1], we get Proposition 6.

The second step of the theorem is the following proposition.

Proposition 7. *Let $G\times H \curvearrowright X$ be a free action which gives the same equivalence relation as that of the $\Gamma\times\Lambda$ action. If $L^\infty X \rtimes G \not\preceq_{\mathcal{M}} L^\infty X \rtimes \Gamma$ and $L^\infty X \rtimes G \not\preceq_{\mathcal{M}} L^\infty X \rtimes \Lambda$, then H is amenable.*

Here the notation "$\preceq_{\mathcal{M}}$" means the embedding of a corner inside $\mathcal{M}$. This notion was introduced by S. Popa. It is characterized by some finite dimensionality of bimodules. The Cartan inclusion $L^\infty X \subseteq L^\infty X \rtimes (G\times H)$ is naturally identified with $L^\infty X \subseteq L^\infty X \rtimes (\Gamma \times \Lambda)$. The assumption of this proposition tells that the G-action $\{\mathrm{Ad}(u_g)\}$ "mixes" I. In other words, for any finite subsets $\mathcal{G} \subset \Gamma, \mathcal{L} \subset \Lambda$, there exists $g \in G$ satisfying

$$u_g e_{\mathcal{G}} u_g^* \text{ "almost" } \perp\ e_{\mathcal{G}}, \quad u_g e_{\mathcal{L}} u_g^* \text{ "almost" } \perp\ e_{\mathcal{L}}.$$

Using the continuity of Φ in Proposition 6, we get the fact that the following natural $*$-homomorphism is continuous with respect to the minimal tensor norm:

$$\Psi\colon C^*_{\text{red}}(H) \otimes_{\mathbb{C}} C^*_{\text{red}}(H)^{\text{op}} \longrightarrow \mathcal{B}(\ell_2 H).$$

This is equivalent to the amenability of H.

At the the last step, we construct group isomorphisms and bi-measurable map on X. The key point is the following observation.

Proposition 8. *If the G-action and the H-action on X are respectively ergodic and G and H are non amenable, then the following two von Neumann algebras are atomic abelian, i.e., isomorphic to ℓ_∞:*

$$\begin{aligned} \widetilde{\mathcal{A}}_\Gamma^G &= (L^\infty X \rtimes G)' \cap \langle \mathcal{M}, L^\infty X \rtimes \Gamma\rangle, \\ \widetilde{\mathcal{A}}_\Lambda^H &= (L^\infty X \rtimes H)' \cap \langle \mathcal{M}, L^\infty X \rtimes \Lambda\rangle, \quad (\textit{or } \Gamma \leftrightarrow \Lambda). \end{aligned}$$

Here, $\langle \mathcal{M}, \mathcal{N} \rangle$ means the basic construction for an inclusion $\mathcal{N} \subseteq \mathcal{M}$.

By the previous proposition and the non amenability of H, there exists a projection $e \in \widetilde{\mathcal{A}}_\Gamma^G$ (or $\in$ "$\widetilde{\mathcal{A}}_\Lambda^G$") whose value of the canonical semi-finite trace is finite. It turns out that there exists a minimal projection in $\widetilde{\mathcal{A}}_\Gamma^G$. The groups H and Λ act on $\widetilde{\mathcal{A}}_\Gamma^G$ from the left and right. These actions commute with each other. It is also proved that the Λ-action on the set of minimal projections in $\widetilde{\mathcal{A}}_\Gamma^G$ is singly transitive. Hence we naturally get an injective group homomorphism from H to Λ. Using the same technique, we also get an injective group homomorphism from G to Γ. Finally, it turns out that the product of a minimal projection in $\widetilde{\mathcal{A}}_\Gamma^G$ and a minimal projection in $\widetilde{\mathcal{A}}_\Lambda^H$ gives a graph of bi-measurable map θ on X. Furthermore, θ gives a conjugacy in the conclusion of Theorem 4.

References

[1] C. Anantharaman-Delaroche, *Systèmes dynamiques non commutatifs et moyennabilité,* Math. Ann. **279** (1987), no. 2, 297–315.

[2] A. Ioana, *Cocycle Superrigidity for Profinite Actions of Property (T) Groups,* Preprint arXiv: 0805.2998.

[3] N. Monod, Y. Shalom, *Orbit equivalence rigidity and bounded cohomology,* Ann. of Math. (2) **164** (2006), no. 3, 825–878

[4] N. Ozawa, *Solid von Neumann algebras,* Acta Math. **192** (2004), no. 1, 111–117.

[5] N. Ozawa, *A Kurosh-type theorem for type* II_1 *factors,* Int. Math. Res. Not. (2006), Art. ID 97560, 21 pp.

[6] N. Ozawa, S. Popa, *Some prime factorization results for type* II_1 *factors,* Invent. Math. **156** (2004), no. 2, 223–234.

[7] S. Popa, *Cocycle and orbit equivalence superrigidity for malleable actions of w-rigid groups,* Invent. Math. **170** (2007), no. 2, 243–295.

[8] S. Popa, *On the superrigidity of malleable actions with spectral gap,* J. Amer. Math. Soc. **21** (2008), no. 4, 981–1000.

A cocycle rigidity theorem for hyperbolic lattices

Roman Sauer

(joint work with Uri Bader, Alex Furman)

We recall the notion of measure equivalence that gained intense attention in recent years by the work of Gaboriau, Furman, Monod, Popa, Shalom and others.

Definition 1. *We say that two countable groups Γ, Λ are* measure equivalent *if there is a standard measure space (Ω, μ) (called a* measure coupling*) such that both actions commute, are μ-preserving, and possess fundamental domains of finite measure.*

Next we explain the notion of integrability – in the context of lattices.

Definition 2. *Let Λ be a finitely generated lattice in a second countable, locally compact group H. For a choice of a word-metric on Λ let $l(\lambda) \in \mathbb{N}$, for $\lambda \in \Lambda$, denote the length of λ. For a Λ-fundamental domain F in H let $\alpha_F : H \times F \to \Lambda$*

denote the cocycle given by the condition $\alpha(h,x)xh \in F$*. We say that* Λ *is* integrable *if there is* Λ*-fundamental domain* $F \subset H$ *such that*

$$\int_F l\big(\alpha(g,x)\big)\, d\mu_{\mathrm{Haar}}(x) < \infty$$

holds for all $g \in G$*.*

Note that all cocompact lattices are integrable. Furthermore, all lattices in G as above or in any simple Lie groups of higher rank [2] are integrable.

If Λ and Γ are finitely generated one can impose an analogous integrability condition on fundamental domains of their measure coupling as in the case of lattices. If a measure coupling Ω has integrable fundamental domains we say that Ω is an ℓ^1*-measure coupling.*

The following is our main result:

Theorem 3. *Let* $G = \mathrm{Isom}(\mathbb{H}^n)$*,* $n \geq 3$*, let* $\Gamma < G$ *a lattice, and let* Λ *be some finitely generated group* ℓ^1*-measure equivalent to* Γ*. Then there exists a short exact sequence* $1 \to \Lambda_0 \to \Lambda \to \bar{\Lambda} \to 1$*, where* Λ_0 *is finite and* $\bar{\Lambda}$ *is a lattice in* G*.*

Moreover, if (Ω,μ) *is an ergodic* ℓ^1*-ME coupling of* Γ *and* Λ*, then there exists a measurable map* $\Phi : \Omega \to G$ *satisfying*

$$\Phi(\gamma\omega) = \gamma\,\Phi(\omega), \qquad \Phi(\lambda\omega) = \Phi(\omega)\rho(\lambda)^{-1}$$

and $\Phi_*\mu$ *is either the Haar measure on* G^0*, or the Haar measure on* G*, or the counting measure on a lattice* Γ' *containing* Γ *and a conjugate of* $\bar{\Lambda}$ *as finite index subgroups.*

In the last case one may assume that Γ and $\bar{\Lambda}$ are actually contained in a possibly larger lattice Γ' upon adjusting ρ and Φ by a fixed $g_0 \in G$.

For lattices in semisimple Lie groups of higher rank one has measure equivalence rigidity without imposing an integrability condition [1]. The problem is that we do not have an analog of Margulis-Zimmer superrigidity. For the proof we prove a certain orbit equivalence rigidity theorem that cocyclifies Mostow rigidity rather than superrigidity.

The methods involve bounded cohomology and geometric group theory.

References

[1] A. Furman, *Gromov's measure equivalence and rigidity of higher rank lattices*, Ann. Math. **150** (1999), 1059–1081.

[2] Y. Shalom, *Rigidity, unitary representations of semisimple groups, and fundamental groups of manifolds with rank one transformation group*, Ann. Math. **152** (2000), 113–182.

L^2-Betti numbers and the Atiyah conjecture

Thomas Schick
(joint work with Peter Linnell)

Let Γ be a discrete group such that $\{|F| \mid F \subset \Gamma \text{ a finite subgroup}\}$ is bounded, and let d be the least common multiple of these numbers. Let $R \subset \mathbb{C}$ be a field which is closed under complex conjugation.

The Atiyah conjecture for $R\Gamma$ asserts that for any $A \subset M_n(R\Gamma)$ we have $d \cdot \dim_\Gamma(A \cdot L\Gamma^n) \in \mathbb{Z}$. Here $\dim_\Gamma(A \cdot L\Gamma^n) = tr_\Gamma(p)$ for the unique orthogonal projection $p \in M_n(L\Gamma)$ such that $p \cdot L\Gamma^n = A \cdot L\Gamma^n$. Here $L\Gamma$ is the group von Neumann algebra of Γ with its canonical trace tr_Γ. For details on L^2-invariants and the Atiyah conjecture compare [3]

Consider the diagram

$$\begin{array}{ccc} R\Gamma & \longrightarrow & L\Gamma \\ \downarrow & & \downarrow \\ D\Gamma & \longrightarrow & U\Gamma. \end{array}$$

Here $U\Gamma$ is the ring of affiliated operators, and $D\Gamma$ is the division closure of $R\Gamma$ in $U\Gamma$, i.e. the smallest subring containing $R\Gamma$ closed under taking inverses which exist in $U\Gamma$.

The Atiyah conjecture has strong implications for the structure of $D\Gamma$. In particular, we discuss the following theorem of [2]:

Theorem. If Γ contains no finite normal subgroup, then the Atiyah conjecture for $R\Gamma$ holds if and only if $D\Gamma$ is a $d \times d$-matrix ring over a skew field.

We discuss also the situation where Γ has a maximal finite normal subgroup $\Delta \neq \{1\}$. Note that then the center of $R\Gamma$ (and of $L\Gamma$) is non-trivial.

In this situation, we introduce a strengthening of the Atiyah conjecture. Its assertion is that not only $\dim_\Gamma$, but also the value of the center valued dimension is restricted in a precise way (determined by the R-representations of the system of finite subgroups of Γ).

In work in progress we prove that this stronger Atiyah conjecture is equivalent to a very precise description of $D\Gamma$ as direct sum of matrix rings over skew fields (the summands are given by minimal central idempotents supported on $R\Gamma$, the size by the R-representation theory of finite subgroups of Γ).

We discuss that this stronger Atiyah conjecture (and therefore its implications for $D\Gamma$ is true essentially for all groups for which the Atiyah conjecture holds. In parts, this is joint with Anselm Knebusch and based on work of [1].

References

[1] Anselm Knebusch, *Approximation of center-valued Betti-numbers*, preprint, arXiv:0804.0530, to appear in Houston Journal of Mathematics.

[2] Peter Linnell and Thomas Schick, *The Atiyah conjecture and Artinian rings*, preprint, arXiv:0711.3328

[3] Wolfgang Lück, *L^2-invariants: theory and applications to geometry and K-theory.* Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge. A Series of Modern Surveys in Mathematics [Results in Mathematics and Related Areas. 3rd Series. A Series of Modern Surveys in Mathematics], 44. Springer-Verlag, Berlin, 2002.

Random matrices, Free probability and Subfactor Planar Algebras

Dimitri Shlyakhtenko
(joint work with Alice Guionnet, Vaughan Jones, Kevin Walker)

Let P be a planar algebra in the sense of Jones [1]. We introduce a new operation on P, called the graded multiplication. In the case that P is the planar algbera of polynomials, this operation corresponds to (ordinary) multiplication of polynomials. We then show that a canonical trace found by Voiculescu in the context of his free probability theory [3, 4], given by the sum over all non-crossing pairing, makes sense in the arbitrary planar algebra context and gives rise to a positive trace on P (viewed as an algebra with the graded multiplication). This trace Tr is given as the inner product with the sum of all Temperley-Lieb diagrams in P. We also show that the trace Tr arises as a limit of random matrix models. Thus the graded multiplication and the trace Tr occur in the contexts of free probability, random matrices and subfactors.

Using the graded algebra structure and a trace we give a natural construction of a factor-subfactor pair associated in a canonical way to every planar algebra. This reproves a theorem of Popa [2] showing that every abstract system of higher relative commutants does arise from a subfactor.

References

[1] V. Jones, *Planar algebras*, Preprint, Berkeley, 1999
[2] S. Popa, *An axiomatization of the lattice of higher relative commutatns of a subfactor*, Inv. Math. 120 (1995) 427–445
[3] D.-V. Voiculescu, *Symmetrices of some reduced free product C^*-algebras*, Operator Algebras ad their Connections with Topology and Ergodic Theory, Lecture Notes in Mathematics vol. 1132, Springer, 1985, 556– 588.
[4] D.-V. Voiculescu, K. Dykema, A. Nica, *Free Random Variables*, CRM Monograph Series, AMS 1992

Haagerup property for wreath products

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(joint work with Yves de Cornulier, Alain Valette)

A countable group is *Haagerup* if it has a metrically proper isometric action on a Hilbert space. Groups with the Haagerup Property are also known as *a-T-menable* groups as they generalize amenable groups. However, they include a wide variety of non-amenable groups, like free groups, and more generally groups having a proper action on a space like a CAT(0)-cubical complex, e.g. any Coxeter group, a real or complex hyperbolic symmetric space, e.g. discrete subgroups of $\mathrm{SO}(n, 1)$, or a product of several such spaces, e.g. the Baumslag-Solitar group $BS(p,q)$, which acts properly on the product of a tree and a real hyperbolic plane.

The class of countable groups with Haagerup Property is obviously closed under taking subgroups. However, unlike the class of amenable groups, it is not closed under taking quotients, nor extensions, even semidirect products, as Kazhdan proved that $(\mathbf{Z}^2 \rtimes \mathrm{SL}_2(\mathbf{Z}), \mathbf{Z}^2)$ has the relative Property (T). The most important result about extensions is that the class of countable Haagerup groups is closed under extensions with **amenable** quotients [2, Example 6.1.6]; it relies on the characterization of the Haagerup Property in terms of unitary representations [8, Proposition 2.3].

Here we report on stability by taking wreath products. Recall that the standard wreath product (often referred as wreath product) of two groups H and G, denoted by $H \wr G$, is the semidirect product $H^{(G)} \rtimes G$, where G acts by shifting the copies of H in the direct sum $H^{(G)} = \bigoplus_{g\in G} H$. As obviously H is Haagerup if and only if $H^{(G)}$ is, stability under wreath products appears as a stability under a special kind of extensions.

Main result

In case H is Haagerup and G is **amenable**, it was known that $H \wr G$ is Haagerup (by applying the above result on extensions). More trivially, if $H \wr G$ is Haagerup, G and H have to be Haagerup since they both embed in $H \wr G$. The converse is our main theorem.

Theorem 1. [5] *Let G, H be countable groups. If G and H have the Haagerup Property, then so does the standard wreath product $H \wr G$.*

The proof of Theorem 1 relies on the characterization due to [11, 3] of the Haagerup Property by actions on spaces with measured walls. Namely, starting with measured walls structures on G and H, we construct such a structure on the wreath product $H \wr G$. Here is a particular case which is sufficient for the applications discussed below.

Theorem 2. [4] *For any $n \geq 1$ and any finite group K the wreath product $K \wr \mathbf{F}_n$ has the Haagerup property.*

Its proof is simpler; in particular, we can deal with spaces with walls instead of spaces with measured walls.

Applications

Theorem 2 has interesting consequences in view of a recent result of Ozawa and Popa [10]. A countable group G is said to be *weakly amenable* if its *Cowling-Haagerup constant*, denoted by $\Lambda(G)$, is finite. We refer to [6] for the definition of $\Lambda(G)$. Combining [10, Corollary 2.11], basic results in [6] and our Theorem 2, one can deduce:

Corollary 3. *Let H be a non-trivial finite group. The iterated wreath product $(H \wr \mathbf{F_2}) \wr \mathbf{Z}$ is a-T-menable but not weakly amenable.*

This disproves a conjecture of Cowling (see page 7 in [2]), stating that the class of a-T-menable groups coincides with the class of groups G satisfying $\Lambda(G) = 1$. Whether every such group is a-T-menable, is still an open question.

Let us indicate another application in the same vein: it was recently proved by Guentner and Higson [7] that any group G acting metrically properly, isometrically on a **finite-dimensional** $CAT(0)$ cube complex satisfies $\Lambda(G) = 1$. In contrast, our techniques to prove Theorem 2, together with results in [1, 9], lead to the following.

Corollary 4. *Let H be a non-trivial finite group. The iterated wreath product $(H \wr \mathbf{F_2}) \wr \mathbf{Z}$ admits a metrically proper, isometric action on a $CAT(0)$ cube complex.*

We already mentioned that this group is not weakly amenable. Of course, the complex arising in Corollary 4 is not finite dimensional.

References

[1] Indira Chatterji and Graham A. Niblo. From wall spaces to CAT(0) cube complexes. Internat. J. Algebra Comput. 15 (2005), no. 5&6, 875–885, .

[2] P.-A. Cherix, M. Cowling, P. Jolissaint, P. Julg, and A. Valette. *Groups with the Haagerup property*, *Progress in Mathematics*, vol. 197. Birkhäuser Verlag, Basel, 2001.

[3] P.-A. Cherix, F. Martin, and A. Valette. Spaces with measured walls, the Haagerup property and property ((T)). Ergodic Theory Dynam. Systems 24 (2004), no. 6, 1895–1908.

[4] Y. de Cornulier, Y. Stalder, A. Valette. Proper actions of lamplighter groups associated with free groups. C. R. Acad. Sci. Paris, Ser. I 346 (2008).

[5] Y. de Cornulier, Y. Stalder, A. Valette. Proper actions of wreath products and generalizations. In preparation.

[6] Michael Cowling and Uffe Haagerup. Completely bounded multipliers of the Fourier algebra of a simple Lie group of real rank one. Invent. Math. 96 (1989), no. 3, 507–549.

[7] Erik Guentner and Nigel Higson. Weak amenability of CAT(0) cubical groups. `arXiv:math.OA/0702568 v1`

[8] Paul Jolissaint. Borel cocycles, approximation properties and relative property T. Ergodic Theory Dynam. Systems 20 (2000), no. 2, 483–499.

[9] Bogdan Nica. Cubulating spaces with walls. Algebr. Geom. Topol., 4 (2004) 297–309 (electronic).

[10] Narutaka Ozawa and Sorin Popa. On a class of II_1 factors with at most one Cartan subalgebra. `arXiv:math.OA/0706.3623 v3`

[11] G. Robertson, T. Steger. Negative definite kernels and a dynamical characterization of property T. Ergodic Theory Dynam. Systems 18 (1998), no. 1, 247–253.

Positive definite functions and equivalence relations

Todor Tsankov

(joint work with Adrian Ioana, Alexander S. Kechris)

This extended abstract summarizes work from [4].

Consider a *standard probability space* (X, μ), i.e., a space isomorphic to the unit interval with Lebesgue measure. We denote by $\mathrm{Aut}(X, \mu)$ the automorphism group of (X, μ), i.e., the group of all Borel automorphisms of X which preserve μ (where two such automorphisms are identified if they are equal μ-a.e.).

To each countable, measure preserving equivalence relation E one can assign the positive-definite function $\varphi_E(S)$ on $\mathrm{Aut}(X,\mu)$ given by $\varphi_E(S) = \mu(\{x : S(x)Ex\})$. Intuitively, $\varphi_E(S)$ measures the amount by which S is "captured" by E. This positive-definite function completely determines E.

We use this function to measure the proximity of a pair $E \subseteq F$ of countable, measure preserving equivalence relations. We prove the following result, where we use the following notation: If a countable group Γ acts on X, we also write γ for the automorphism $x \mapsto \gamma \cdot x$; if $A \subseteq X$ and E is an equivalence relation on X, then $E|A = E \cap A^2$ is the restriction of E to A; if $E \subseteq F$ are equivalence relations, then $[F : E] = m$ means that every F-class contains exactly m classes; if F is a countable, measure preserving equivalence relation on (X,μ), then $[F]$ is the *full group* of F, i.e., $[F] = \{T \in \mathrm{Aut}(X,\mu) : T(x)Fx, \mu\text{−a.e.}(x)\}$.

Theorem 1. *Let Γ be a countable group and consider a measure preserving action of Γ on (X,μ) with induced equivalence relation $F = E_\Gamma^X$. If $E \subseteq F$ is a subequivalence relation and $\inf_{\gamma\in\Gamma} \varphi_E(\gamma) = \varphi_E^0 > 0$, then there is an E-invariant Borel set $A \subseteq X$ of positive measure such that $[F|A : E|A] = m \leq \frac{1}{\varphi_E^0}$, so that if $\varphi_E^0 > \frac{1}{2}$, $F|A = E|A$.*

Remark 2. *Popa has pointed out that versions of this result were known in the field of operator algebras; see Popa* [6].

Theorem 1 has various applications, among which some estimates for the cost of property (T) groups. We note that currently no property (T) groups with cost greater than 1 are known to exist.

Theorem 3. *Let Γ be an infinite group with property (T) and (Q,ϵ) be a Kazhdan pair for Γ. If $|Q| = n$, then*

$$\mathrm{cost}\,\Gamma \leq n\left(1 - \frac{\epsilon^2}{2}\right) + \frac{n-1}{2n-1}.$$

If, moreover, Q contains an element of infinite order, we have the two additional estimates:

$$\mathrm{cost}\,\Gamma \leq n - \epsilon^2/2 \quad \textit{and} \quad \mathrm{cost}\,\Gamma \leq n - (n-1)\epsilon^2/8.$$

We also consider a recent co-inducing construction of Epstein [1]. Given a measure preserving, ergodic action b_0 of a countable group Γ on (X,μ) with associated equivalence relation $F = E_\Gamma^X$ and a free, measure preserving action a_0 of a countable group Δ on (X,μ) with associated equivalence relation $E = E_\Delta^X \subseteq F = E_\Gamma^X$, Epstein's construction gives for any measure preserving action a of Δ on a space (Y,ν), a measure preserving action b of Γ on a space (Z,ρ), called the *co-induced action of a modulo* (a_0,b_0), in symbols $b = \mathrm{CInd}(a_0,b_0)_\Delta^\Gamma(a)$. This construction has important applications in the study of orbit equivalence, see Epstein [1].

For further potential applications of this method, it seems that one should have a better understanding of the connection of ergodic properties between a, b as above. We show, for example, that if b_0 is free, mixing and a_0 is ergodic, then: a is mixing $\Rightarrow$ b is mixing. There are however interesting situations under which

b is always mixing for arbitrary a. It turns out that this phenomenon, for given (a_0, b_0), is connected to the positive-definite function discussed earlier. We show the following:

Theorem 4. *If b_0 is mixing, the following are equivalent:*

(1) *For all actions a of Δ, $b = \mathrm{CInd}(a_0, b_0)^{\Gamma}_{\Delta}(a)$ is mixing,*
(2) *$\varphi_E(\gamma) \to 0$ as $\gamma \to \infty$.*

The condition (2) in Theorem 4 somehow asserts that E is "small" relative to F. In the opposite case we have the following fact. If $\varphi^0_E = \inf_{\gamma \in \Gamma} \varphi_E(\gamma) > 0$, then b is ergodic $\Rightarrow a$ is ergodic.

It is well-known that for any ergodic b_0 as above one can find a free, mixing action a_0 of $\Delta = \mathbf{Z}$ with $E \subseteq F$ (see, e.g., Zimmer [7], 9.3.2). We show that when b_0 is mixing, one can find such an a_0 so that (2) of Theorem 4 holds. This gives a method of producing, starting with arbitrary measure preserving $\mathbf{Z}$ actions, apparently new types of measure preserving, mixing actions of any infinite group Γ.

Theorem 5. *Let Γ be an infinite countable group, and let b_0 be a free, measure preserving, mixing action of Γ on (X, μ). Then there is a free, measure preserving, ergodic action a_0 of $\mathbf{Z}$ on (X, μ) such that $E = E^X_{\mathbf{Z}} \subseteq F = E^X_{\Gamma}$ and $\varphi_E(\gamma) \to 0$ as $\gamma \to \infty$.*

After a series of earlier results that dealt with various important classes of non-amenable groups, Epstein [1] finally showed that in general any non-amenable group admits uncountably many non-orbit equivalent free, measure preserving, ergodic actions. This was proved earlier by Ioana [3] in the case where $F_2 \leq \Gamma$, and his main lemma in that proof could be also used to derive, in this case, the stronger fact that the equivalence relation E_0 (on $2^{\mathbf{N}}$, where $xE_0y \Leftrightarrow \exists n \forall m \geq n(x(m) = y(m))$ can be Borel reduced to OE on the space of free, measure preserving, ergodic actions of Γ. Moreover, OE on that space cannot be classified by countable structures (see [5], Section 17, **(B)**). However, it was not known whether this non-classification result extends to *all* non-amenable groups and whether every non-amenable group admits uncountably many non-orbit equivalent, free, measure preserving, *mixing* actions. Putting together Theorem 4, the main result of Gaboriau–Lyons [2] (and the additional observation that the subequivalence relation generated by $\mathbf{F}_2$ that they produce can be taken to satisfy condition (2) of Theorem 4), and the work of Epstein [1] leads now to the following positive answer. This is a joint result with I. Epstein.

Theorem 6 (with I. Epstein)**.** *Let Γ be a non-amenable countable group. Then E_0 can be Borel reduced to OE on the space of free, measure preserving, mixing actions of Γ and OE on this space cannot be classified by countable structures.*

References

[1] I. Epstein, Orbit inequivalent actions of non-amenable groups, *Math. arXiv*; math.GR, 0707.4215.

[2] D. Gaboriau and R. Lyons, A measure theoretic solution to von Neumann's Problem, *Math. arXiv*; math.GR, 0711.1643.
[3] A. Ioana, Orbit inequivalent actions for groups containing a copy of $\mathbf{F}_2$, *Math.arXiv*; math.GR, 0701027.
[4] A. Ioana, A.S. Kechris, and T. Tsankov, Subequivalence relations and positive definite functions, preprint.
[5] A.S. Kechris, *Global Aspects of Ergodic Group Actions and Equivalence Relations*, http://www.math.caltech.edu/people/kechris.html.
[6] S. Popa, On a class of type II_1 factors with Betti numbers invariants, *Ann. of Math.*, **163** (2006), 809–899.
[7] R.J. Zimmer, *Ergodic Theory and Semisimple Groups*, Birkhäuser, 1984.

Superrigid actions of lattices in $\mathrm{SL}(n,\mathbb{R})$ on homogeneous spaces

Stefaan Vaes
(joint work with Sorin Popa)

This talk is devoted to the following natural group actions: the linear action of a lattice Γ in $\mathrm{SL}(n,\mathbb{R})$ on the vector space $\mathbb{R}^n$ and the projective action of a lattice Γ in $\mathrm{PSL}(n,\mathbb{R})$ on the projective space $\mathrm{P}^{n-1}(\mathbb{R})$ or, more generally, on the flag variety X of signature $(d_1,\ldots,d_l,n)$. Our main result shows that for n sufficiently large, these actions are orbit equivalent superrigid. A more precise statement is stated below, but this roughly means that the orbit equivalence relation given by these actions, remembers the group and the action. It should be noted that the actions $\Gamma \curvearrowright \mathbb{R}^n$ are infinite measure preserving, while the above actions on $\mathrm{P}^{n-1}(\mathbb{R})$ or the flag variety X do not preserve a finite or infinite measure (belonging to the Lebesgue measure class).

Let $n \geq 4$ and $\Gamma < \mathrm{SL}(n,\mathbb{R})$ a lattice. In [5], we prove that the restriction $\mathcal{R}$ of the orbit equivalence relation $\mathcal{R}(\Gamma \curvearrowright \mathbb{R}^n)$ to a subset of finite Lebesgue measure, is a II_1 equivalence relation having property (T) in the sense of Zimmer [6], yet having fundamental group $\mathbb{R}_+$. It follows that $\mathcal{R}$, nor any of its amplifications $\mathcal{R}^t$, $t>0$, can be implemented by a free action of a countable group or by any action of a property (T) group. Indeed, in both cases, the fundamental group of $\mathcal{R}$ would be forced to be countable. The main ingredient of the proof is the notion, due to Zimmer [6], of property (T) for non-singular actions of locally compact groups and the result of Anantharaman-Delaroche [1] showing that $\Gamma \curvearrowright G/\Lambda$ has property (T) if and only if $\Lambda \curvearrowright G/\Gamma$ does, whenever G is a second countable locally compact group and Γ,Λ are closed subgroups of G.

In [5], we prove cocycle superrigidity for the linear action $\Gamma \curvearrowright \mathbb{R}^n$, whenever $n \geq 5$ and $\Gamma < \mathrm{SL}(n,\mathbb{R})$ is a lattice. More precisely, consider the class $\mathcal{U}_{\mathrm{fin}}$, introduced by Popa [4], of Polish groups that can be realized as the closed subgroup of the unitary group of a II_1 factor. This class $\mathcal{U}_{\mathrm{fin}}$ contains all countable groups and all second countable compact groups. Every measurable 1-cocycle $\omega : \Gamma \times \mathbb{R}^n \to \mathcal{G}$, for $\mathcal{G} \in \mathcal{U}_{\mathrm{fin}}$, is shown to be cohomologous to a group morphism $\Gamma \to \mathcal{G}$. The idea of the proof is the following.

In [4], Popa defines the notion of malleability for a probability measure preserving (p.m.p.) action $\Gamma \curvearrowright (X, \mu)$. Oversimplifying things, this means that there exists a flow $\mathbb{R} \curvearrowright X \times X$ commuting with the diagonal Γ-action and being equal to the flip in $t = 1$. The main examples of malleable p.m.p. actions are the Bernoulli actions $\Gamma \curvearrowright (X_0, \mu_0)^\Gamma$ and the Gaussian actions. But malleability can be equally defined for infinite measure preserving actions. The linear actions $\Gamma \curvearrowright \mathbb{R}^n$ satisfy this property by considering the rotation on $\mathbb{R}^n \times \mathbb{R}^n$.

Popa proves in [4] that every weakly mixing, malleable, p.m.p. action of a property (T) group, is $\mathcal{U}_{\rm fin}$-cocycle superrigid. In [5], $\mathcal{U}_{\rm fin}$-cocycle superrigidity is shown for certain malleable, infinite measure preserving actions. In that case, property (T) of the acting group has to be replaced by property (T) for the action $\Gamma \curvearrowright X$, while weak mixing has to be replaced by ergodicity of the 4-fold diagonal action $\Gamma \curvearrowright X \times X \times X \times X$. In order to understand this last point, observe that if $\Gamma \curvearrowright (X, \mu)$ is p.m.p., then the diagonal action $\Gamma \curvearrowright X \times X$ is ergodic if and only every k-fold diagonal action $\Gamma \curvearrowright X \times \cdots \times X$ is ergodic. Such a statement fails for infinite measure preserving actions – just look at $\mathrm{SL}(n, \mathbb{Z}) \curvearrowright \mathbb{R}^n$.

Other cocycle superrigidity theorems are available in the literature, the main ones being Zimmer's cocycle superrigidity for Zariski dense cocycles for higher rank lattices with values in linear algebraic groups (see [7]) and Ioana's cocycle superrigidity for profinite actions of property (T) groups, with values in arbitrary countable groups (see [3]).

Once cocycle superrigidity is proven, standard techniques allow to derive orbit equivalence superrigidity theorems. I just state the following sample theorems from [5]. Let $\Gamma < \mathrm{SL}(n, \mathbb{R})$ be a lattice and $n \geq 5$. If the linear action $\Gamma \curvearrowright \mathbb{R}^n$ is stably orbit equivalent with the essentially free, non-singular action $\Lambda \curvearrowright Y$, then the latter action is conjugate to an induction of $\Gamma \curvearrowright \mathbb{R}^n$ or, in case $-1 \in \Gamma$, an induction of $\Gamma/\{\pm 1\} \curvearrowright \mathbb{R}^n/\{\pm 1\}$.

Let X be the real flag variety of signature $(d_1, \ldots, d_l, n)$ and assume that $n \geq 4d_l + 1$. If $\Gamma < \mathrm{PSL}(n, \mathbb{R})$ is a lattice and the action $\Gamma \curvearrowright X$ is stably orbit equivalent with the essentially free, non-singular action $\Lambda \curvearrowright Y$, then the latter action is conjugate to an induction of $\Gamma \curvearrowright X$ or an induction of one of the finite cover actions of $\Gamma \curvearrowright X$ labeled by the subgroups of the finite abelian group $(\mathbb{Z}/2\mathbb{Z})^{\oplus l}$.

Finally, we classify the linear actions $\Gamma \curvearrowright \mathbb{R}^n$ and the projective actions on flag varieties up to stable orbit equivalence, by combining our orbit equivalence superrigidity results with Furman's classification [2] of these actions up to conjugacy.

References

[1] C. Anantharaman-Delaroche, Cohomology of property T groupoids and applications. *Ergod. Th. & Dynam. Sys.* **25** (2005), 977-1013.

[2] A. Furman, Measurable rigidity of actions on infinite measure homogeneous spaces, II. *J. Amer. Math. Soc.* **21** (2008), 479-512.

[3] A. Ioana, Cocycle superrigidity for profinite actions of property (T) groups. *Preprint.* `arXiv:0805.2998`

[4] S. Popa, Cocycle and orbit equivalence superrigidity for malleable actions of w-rigid groups. *Invent. Math.* **170** (2007), 243-295.
[5] S. Popa and S. Vaes, Cocycle and orbit superrigidity for lattices in $\mathrm{SL}(n,\mathbb{R})$ acting on homogeneous spaces. *Preprint.* arXiv:0810.3630
[6] R.J. Zimmer, On the cohomology of ergodic actions of semisimple lie groups and discrete subgroups. *Amer. J. Math.* **103** (1981), 937-951.
[7] R.J. Zimmer, Ergodic theory and semisimple groups. Birkhäuser, Boston, 1984.

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