

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Report No. 34/2009

DOI: 10.4171/OWR/2009/34

Mathematical Aspects of Hydrodynamics

Organised by
Gregory Seregin, Oxford - St. Petersburg
Vladimir Šverák, Minneapolis

July 19th – July 25th, 2009

ABSTRACT. The workshop was devoted to discussions of recent developments and possible future directions of research in the field of mathematical hydrodynamics. Many of the leading experts in the theory of PDE's arising in fluid dynamics participated in this event. The topics included:

- Regularity, uniqueness and well-posedness problems for the Navier-Stokes equations
- Stability of Navier-Stokes solutions
- Open problems concerning the steady-state Navier-Stokes solutions
- Statistical approach to 2d hydrodynamics
- Inviscid limits of Navier-Stokes solutions
- Anomalous weak solutions of Euler's equation
- Finding physically reasonable classes of weak solutions of Euler's equations
- Local well-posedness of Euler's equations in optimal spaces
- Stability of solutions of Euler's equations
- Water waves
- Model equations
- Geometric approach to hydromechanical equations
- Selected compressible flow problems

Mathematics Subject Classification (2000): 35xx, 76xx.

Introduction by the Organisers

The workshop *Mathematical Aspects of Hydrodynamics*, organized by Gregory Seregin (Oxford - St. Petersburg) and Vladimir Šverák (Minneapolis) was held July 19th – July 25th, 2009. The meeting was well attended, more than 45 mathematicians participated. The program of the workshop consisted of 23 talks presented by leading researchers in Mathematical Fluid Mechanics coming from all

around the world. The main topics covered by the workshop lectures, addressed 2D and 3D Euler and Navier-Stokes equations (stationary and non-stationary), Quasi-Geostrophic Equation, Hydrostatic Boussinesq equation and other model equations (Euler- α , Navier-Stokes-Voight etc.).

The lectures stimulated many interesting discussions and exchanges of ideas. Many participants appreciated the opportunity to learn more about a variety of approaches and points of views.

In addition to the scientific program, there were two events which might be worth mentioning. The first one was the traditional Wednesday afternoon hike to Oberwolfach-Kirche. The second event was an outstanding informal concert on Friday evening by Charles Doering (guitar) and László Székelyhidi (violin).

Many participants suggested that a conference devoted to mathematical aspects of fluid mechanics become a regular Oberwolfach workshop.

The unique atmosphere of the Institute was a significant factor in the success of the meeting. As always, the Oberwolfach staff contributed greatly by the perfect organizational work. The organizers would like to express once more their thanks to the Institute for the support and the flawless organization.

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Abstracts

New information provided by the shear flow for the 3d Euler equation

CLAUDE BARDOS

(joint work with Edriss S. Titi)

The "shear flow" is one of the most classical (or almost explicit) solutions of the 3d incompressible Euler equation. Analysis of this example leads to simple but may be important remarks on the Euler equation:

1. Existence of very unstable solutions (this was already observed by Di Perna and Lions).

2. Existence of initial data in $C^{0,\alpha}$ with solution in C^{0,α^2} this implies that in the Holder spaces the class $C^{0,\alpha}$ is not only sufficient but compulsory for well posedness.

3. All these pathological examples conserve the energy and this shows that relation between loss of regularity and energy decay for weak solution (the Onsager conjecture) is not completely true (regularity implies conservation of energy but conversely conservation of energy does not implies Holder regularity).

4. Existence of solution of the 3d Kelvin Helmholtz problem with (non analytic) singular interface. This statement is not true in 2d and comes from the fact that in 3d the linearized Kelvin Helmholtz equation is no more elliptic.

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Stokes and Navier-Stokes equations under Navier type Boundary conditions. Regularity and inviscid limit

HUGO BEIRÃO DA VEIGA

(joint work with Francesca Crispo)

We consider the evolutionary Navier-Stokes equations with a Navier slip-type boundary condition, and study the convergence of the solutions, as the viscosity goes to zero, to the solution of the Euler equations under the zero-flux boundary condition. We obtain quite sharp results in the 2-D and 3-D cases. However, in the 3-D case, we need to assume that the boundary is flat. Convergence is proved in $L^\infty(0, T; W^{k,p}(\Omega))$, for arbitrarily large values of k and p .

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Hydrostatic Boussinesq equations and Optimal Transport Theory

YANN BRENIER

(joint work with Mike Cullen)

We establish a connection between Optimal Transport Theory and classical Convection Theory for geophysical flows. Our starting point is the model designed few years ago by Angenent, Haker and Tannenbaum [AHT] to solve some Optimal Transport problems. This model can be seen as a generalization of the Darcy-Boussinesq equations, which is a degenerate version of the Navier-Stokes-Boussinesq (NSB) equations. In a unified framework, we relate different variants of the NSB equations (in particular what we call the generalized Hydrostatic-Boussinesq equations) to various models involving Optimal Transport (and the related Monge-Ampère equation). This includes the 2D semi-geostrophic equations and some fully non-linear versions of the so-called high-field limit of the Vlasov-Poisson system [NPS] and of the Keller-Segel for Chemotaxis. Mathematically speaking, we establish some existence theorems for local smooth, global smooth or global weak solutions of the different models. We also justify that the inertia terms can be rigorously neglected under appropriate scaling assumptions in the Generalized Navier-Stokes-Boussinesq equations.

We show how a "stringy" generalization of the AHT model can be related to the magnetic relaxation model studied by Arnold and Moffatt to obtain stationary solutions of the Euler equations with prescribed topology.

We prove that smooth solutions of the semigeostrophic equations in the incompressible $x - z$ setting can be derived from the Navier-Stokes equations with the Boussinesq approximation.

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Inviscid limit for damped and driven incompressible Navier-Stokes equations in \mathbb{R}^2

PETER CONSTANTIN

We consider the zero viscosity limit of long time averages of solutions of damped and driven Navier-Stokes equations in \mathbb{R}^2 . We prove that the rate of dissipation of enstrophy vanishes. Stationary statistical solutions of the damped and driven Navier-Stokes equations converge to renormalized stationary statistical solutions of the damped and driven Euler equations. These solutions obey the enstrophy balance.

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Transport and Dissipation in Turbulent Fluid Flows

CHARLES R. DOERING

The relation between the shape of the force driving a turbulent flow and the upper bound on the dimensionless dissipation factor β is presented. We are interested in non-trivial (more than two wave numbers) forcing functions in a three dimensional domain periodic in all directions. A comparative analysis between results given by the optimization problem and the results of Direct Numerical Simulations is performed. We report that the bound on the dissipation factor in the case of infinite Reynolds numbers have the same qualitative behavior as for the dissipation factor at finite Reynolds number. As predicted by the analysis, the dissipation factor depends strongly on the force shape. However, the optimization problem does not predict accurately the quantitative behavior. We complete our study by analyzing the mean flow profile in relation to the Stokes flow profile and the optimal multiplier profile shape for different force-shapes. We observe that in our 3D-periodic domain, the mean velocity profile and the Stokes flow profile reproduce all the characteristic features of the force-shape. The optimal multiplier proves to be linked to the intensity of the wave numbers of the forcing function.

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L^p -estimates for stationary compressible fluids

JENS FREHSE

(joint work with Mark Steinhauer, Wladimir Weigant)

We consider the Navier-Stokes equations for compressible isothermal flow in the steady two dimensional case and show the existence of a weak solution in the case of periodic and of mixed boundary conditions. Also we consider the three dimensional case and show the existence of a weak solution for homogeneous Dirichlet (no-slip) boundary conditions under the assumption that the adiabatic exponent satisfies some restriction. In particular we cover with our existence result the cases of a monoatomic gas and of air.

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Some large initial data to the Navier-Stokes equations giving rise to a global solution

ISABELLE GALLAGHER

(joint work with Jean-Yves Chemin, Marius Paicu)

Classes of initial data to the three dimensional, incompressible Navier-Stokes equations were presented, generating a global smooth solution although the norm of the initial data (periodic or defined in the whole space) may be chosen arbitrarily large. Such initial data giving rise to global solutions have particular oscillatory properties or varies slowly in one direction or the norm blows up as the small

parameter goes to zero. The proof uses the special structure of the nonlinear term of the equation.

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Three-dimensional Stability of Burgers Vortices

THIERRY GALLAY

(joint work with Yasunori Maekawa)

Although the two dimensional stability of the Burgers vortex has been well studied by now, its stability with respect to three dimensional perturbations has been less understood since the linearized equation becomes much more complicated in this case. In Gallay-Wayne [1-2] it is mathematically proved that the Burgers vortex is locally stable for three dimensional perturbations at least for sufficiently small circulation numbers. New results concerning stability of Burgers Vortices in the symmetric and non-symmetric case are discussed. The talk is focused on this three dimensional stability problem in the case of arbitrary circulation numbers.

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Strong L^p solutions to fluid-solid interaction problems

MATTHIAS HIEBER

(joint work with Matthias Geissert, Karoline Goetze)

In this talk we consider the movement of a rigid body in a Newtonian or Non-Newtonian fluid under the influence of gravitation. We show that in the first case the system consisting of the Navier-Stokes equations coupled with the balance laws for the momentum and the angular momentum admits a unique, local, strong solution in the L^p -setting. Note that the fluid-solid interface is a moving one and has to be found as part of the solution process. Our result will then be extended to the case of certain Non-Newtonian fluids. This is joint work with Matthias Geissert and Karoline Goetze.

A variation on a theme of Caffarelli and Vasseur

ALEXANDER KISELEV

(joint work with Fedor Nazarov)

The 2D surface quasi-geostrophic (SQG) equation has recently been a focus of significant research effort. This equation appears in geophysics, emerging under certain assumptions from Boussinesq approximation equations describing fluid in a three dimensional strongly rotating half-space. Part of the recent interest to the SQG equation may be due to the fact that it is the simplest-looking equation of fluid dynamics for which the question of global existence of smooth solutions is still open. Recently, there has been a progress in understanding SQG equation with critical dissipation. There are two very different proofs of global regularity in this case: a nonlocal modulus of continuity approach by Kiselev, Nazarov and Volberg and a method of Caffarelli and Vasseur, based on DiGiorgi-type iterative techniques. The KNV approach is shorter, while the CV approach is more general, showing Hölder regularization of solution to drift-diffusion equation with BMO norm estimate on the drift. The SQG equation result is then a particular consequence. In this talk, I will outline a third approach which allows to understand and recover some of Caffarelli-Vasseur results by more elementary means. This is a joint work with F. Nazarov.

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- [3] A. Kiselev, F. Nazarov, *A variation on a theme of Caffarelli and Vasseur*, preprint arxiv:math/0908.0923, 9 pages.

Leray's inequality in 3D multi-connected domains

HIDEO KOZONO

(joint work with Taku Yanagisawa)

We consider the stationary Navier-Stokes equations in 3D bounded domains the boundary of which consists of several connected surfaces. It is known that if the given boundary data satisfies the "restricted" flux condition which means that the flux on each component of the boundary vanishes, then we have an existence theorem. However, it has been an open question whether or not the same existence result does hold under the "general" flux condition which implies that the total sum of each flux on the boundary component is zero. This problem was proposed by Leray in 1933 who had established the former existence theorem by means of the Leray inequality. Roughly speaking, we may regard the Leray inequality as the quadratic estimate of the nonlinear convection term in terms of the Dirichlet integral. In this talk, we will see that, in many cases, the Leray inequality necessarily yields the restricted flux condition.

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Some Remarks on Euler Equation and Triebel-Lizorkin Spaces

PIERRE-GILLES LEMARIÉ-RIEUSSET

The solvability of the Cauchy problem for the non-stationary Euler equations in Triebel-Lizorkin Spaces is discussed.

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Behavior of solutions to Navier-Stokes equations in the scaling invariant spaces

NATAŠA PAVLOVIĆ

(joint work with Jean Bourgain, Pierre Germain and Gigliola Staffilani)

In this talk we will discuss the Navier-Stokes equations in scaling invariant spaces. In particular, we will briefly recall regularity of so called "mild" solutions to the Navier-Stokes equations evolving from small initial data in a critical space in \mathbb{R}^n (joint work with Pierre Germain and Gigliola Staffilani). Then we will describe a result on ill-posedness of the Navier-Stokes equations in the largest critical space in 3D (joint work with Jean Bourgain).

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2-D Navier-Stokes Equation and Intermediate Asymptotics

MARIO PULVIRENTI

(joint work with Emanuele Caglioti, Frederic Rousset)

We introduce a modified version of the two-dimensional Navier-Stokes equation, preserving energy and momentum of inertia, which is motivated by the occurrence of different dissipation time scales and related to the gradient flow structure of the 2-D Navier-Stokes equation. The hope is to understand intermediate asymptotics.

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Liquid crystals and Weil-Petersson geodesics

TUDOR S. RATIU

(joint work with François Gay-Balmaz, Jerrold E. Marsden)

The approach develops the theory of affine Euler-Poincaré and affine Lie-Poisson reductions and applies these processes to various examples of complex fluids, including Yang-Mills and Hall magnetohydrodynamics for fluids and superfluids, spin glasses, microfluids, and liquid crystals. As a consequence of the Lagrangian approach, the variational formulation of the equations is determined. On the Hamiltonian side, the associated Poisson brackets are obtained by reduction of a canonical cotangent bundle. A Kelvin-Noether circulation theorem is presented and is applied to these examples.

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Well-posedness for the free-boundary 3-D compressible Euler equations in physical vacuum

STEVE SHKOLLER

(joint work with Daniel Coutand, Hans Lindblad)

We prove a priori estimates for the three-dimensional compressible Euler equations with moving *physical* vacuum boundary, with an equation of state given by $p(\rho) = C_\gamma \rho^\gamma$ for $\gamma > 1$. The vacuum condition necessitates the vanishing of the pressure, and hence density, on the dynamic boundary, which creates a degenerate and characteristic hyperbolic *free-boundary* system to which standard methods of symmetrizable hyperbolic equations cannot be applied.

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What is a weak solution of the Euler Equation?

ALEXANDER SHNIRELMAN

I am going to discuss the construction of a more realistic 3-dim weak solution, whose kinetic energy monotonically decreases in time. This solution is also everywhere discontinuous and unbounded, while has some realistic features. The construction starts from a simple mechanical system having the property that the kinetic energy decreases, while there is no explicit friction; it requires Generalized Flows, introduced by Y. Brenier. At last, I am going to discuss the ways to the construction and theory of true, physically reasonable weak solutions of the Euler equations.

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Recent results on strong and weak solutions of the Navier-Stokes Equations

HERMANN SOHR

(joint work with Reinhard Farwig)

The first part of this talk yields the optimal initial value condition for the existence of a local (in time) unique strong solution of the Navier-Stokes equations in a smooth bounded domain. This condition is not only sufficient - there are several well-known sufficient conditions in this context - but also necessary, yielding therefore the largest possible class of such local strong solutions. A restricted result holds for completely general domains. In the second part, the well-known class of (global in time) Leray-Hopf weak solutions with zero boundary conditions and zero divergence will be extended to a larger class with corresponding nonzero conditions where boundary values and divergence are given satisfying certain compatibility conditions.

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On a problem of magneto-hydrodynamics in multi-connected domains

VSEVOLOD A. SOLONNIKOV

A proof of the solvability for the initial boundary value problem arising in magneto-hydrodynamics in multi-connected domains is presented.

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Weak solutions of the Euler Equations: Non-uniqueness and dissipation

LÁSZLÓ SZÉKELYHIDI JR.

(joint work with Camillo De Lellis)

In this paper we propose a new point of view on weak solutions of the Euler equations, describing the motion of an ideal incompressible fluid in \mathbb{R}^n with $n \geq 2$. We give a reformulation of the Euler equations as a differential inclusion, and in this way we obtain transparent proofs of several celebrated results of V. Scheffer and A. Shnirelman concerning the non-uniqueness of weak solutions and the existence of energy-decreasing solutions. Our results are stronger because they work in any dimension and yield bounded velocity and pressure.

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Inviscid Regularization of Hydrodynamics Equations: Global Regularity, Numerical Analysis and Statistical Behavior

EDRISS S. TITI

Inviscid Regularization of Hydrodynamics Equations is described. Some results on solvability and regularity of the corresponding solutions and their numerical analysis and statistical behavior are discussed.

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Recent results in fluid mechanics

ALEXIS F. VASSEUR

We will present, in this talk, new applications of De Giorgi's methods and blow-up techniques to fluid mechanics problems. Those techniques have been successfully applied to show full regularity of the solutions to the surface quasi-geostrophic equation in the critical case.

We will present, also, a new nonlinear family of spaces allowing to control higher derivatives of solutions to the 3D Navier-Stokes equation. Finally, we will present a regularity result for a reaction-diffusion system which has almost the same supercriticality than the 3D Navier-Stokes equation.

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Is free surface potential Hydrodynamics integrable?

VLADIMIR E. ZAKHAROV

If the spectral density of ocean wave is narrow in comparison with steepness, the modulation instability occurs. We have studied the nonlinear shape of this instability by performing massive numerical simulations of the Euler equations for ideal fluid with free surface. Our conclusion is the following - the modulational instability generates rogue waves.

We started with the exact Euler equation for potential flow of infinitely deep incompressible fluid in two dimensions and performed canonical map of the domain filled by fluid onto the lower half-plane. Under this map, the hydrodynamic equations are transformed to the elegant Dyachenko equations, which are suitable for numerical solution by implementation of the standard spectral code. We used the code with adjusting time step and adjusting number of spectral modes. This number varied within the limits $10^4/10^6$. All experiments were done in the standard wave tank of the length 2π .

In the first group of experiments we studied the development of modulational instability of slightly perturbed Stokes wave. Both steepness and wave numbers of the initial wave could be essentially varied. The most impressive results are obtained for the Stokes with wave number $n = 100$ and $\mu \cong 0.1$. In this case the modulational instability is a slow process. On the initial stage we observed exponential growth of perturbation followed by formation of a kind of one-dimensional turbulence. After more than ten inverse growth rates of the instability (two or more periods of initial wave) the process ends up by formation of the freak wave of an amplitude exceeding initial average level of surface elevation in four-five times. Onset of the freak wave is a catastrophic event taking only a few wave periods.

In the second group of experiments we studied the formation of freak waves from envelope quasisolitons. The quasisolitons of small steepness, $\mu < 0.1$, propagate peacefully and could be fairly approximated by the Nonlinear Schrodinger equation (NLSE). Quasisolitons of a moderate amplitude, $0.1 < \mu < 0.14$, still propagate without changing their form, however, they are essentially asymmetric. The quasisolitons of high steepness, $\mu > 0.1$, are unstable. Their evolution leads to a formation of freak wave. The freak waves appear also in collisions of relatively small amplitude solitons.

In the third group of experiments we generated stochastic quasi-monochromatic waves from the white noise by including into equations of the narrow-band weak instability (so far in the range of wave numbers 30/40). At a certain level of nonlinearity, the growth of instability is arrested by a sporadic wave breaking. On this background we saw rare events of rogue wave formation.

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