Arbeitsgemeinschaft mit aktuellem Thema:

Topological Robotics

Mathematisches Forschungsinstitut Oberwolfach

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Introduction:

Topological robotics is a new mathematical discipline studying topological problems inspired by robotics and engineering as well as problems of practical robotics requiring topological tools. It is a part of a broader newly created research area called "computational topology". The latter studies topological problems appearing in computer science and algorithmic problems in topology.

Problems of topological robotics can roughly be split into three main categories: (A) studying special topological spaces, configuration spaces of important mechanical systems; (B) studying new topological invariants of general topological spaces, invariants which are motivated and inspired by applications in robotics and engineering; (C) studying algebraic topology of random topological spaces which arise in applications as configuration spaces of large systems of various nature. The meeting will focus on studying major problems and results of types (A), (B), (C).

A. **Topology of configuration spaces.**

Topology of classical configuration spaces (i.e. varieties of mutually distinct points of a given manifold) is an important subject of modern algebraic topology interacting with many sub-disciplines (the theory of knots and braids, embeddings and immersions of manifolds, topology of subspace arrangements). The meeting will survey major developments in this area, in particular we will analyze the Totaro spectral sequence which computes the cohomology algebras of configuration spaces.

One of the talks will describe a beautiful theorem of Światosław R. Gal which gives a general formula for Euler characteristics of configuration spaces $F(X, n)$ of n distinct particles moving in a polyhedron X, for all n. The Euler - Gal power series is a rational function encoding all numbers $\chi(F(X, n))$ and Gal's theorem gives an explicit expression for it in terms of local topological properties of the space.

We will study the topology of configuration spaces of mechanical linkages, a remarkable class of manifolds which appear in several fields of mathematics as well as in molecular biology and in statistical shape theory. Methods of Morse theory, enriched with new techniques based on properties of involutions, allow effective computation of their Betti numbers. We will also survey a recent solution of the conjecture raised by Kevin Walker in 1985. This conjecture asserts that the relative sizes of bars of a linkage are determined, up to certain equivalence, by the cohomology algebra of the linkage configuration space.

Another topic is the knot theory of the robot arm, a variation of the traditional knot theory question motivated by robotics. The main result is an unknotting theorem for planar robot arms proven recently by R. Connelly, E.Demaine and G. Rote.

B. **Topological complexity of robot motion planning algorithms.**

The concept of topological complexity of the robot motion planning problem $\mathsf{TC}(X)$ is an interesting topological invariant which measures navigational complexity of topological spaces and has obvious relevance to various robotics applications. It is a special case of the notion of Schwartz genus, a very general classical concept including also the Lusternik - Schnirelmann category $\mathsf{cat}(X)$. At the meeting we will survey the properties of the Lusternik - Schnirelmann category and its main applications in critical point theory.

Computing the topological complexity $\mathsf{TC}(X)$ meets serious difficulties in some cases. Very useful are general upper bounds in terms of the dimension and connectivity, and also the homotopy invariance of $\mathsf{TC}(X)$. The lower bounds for $\mathsf{TC}(X)$ use the structure of the cohomology algebra $H^*(X)$. These estimates can sometimes be significantly improved by applying the theory of weights of cohomology classes. One may also use stable cohomology operations to improve lower bounds on the topological complexity based on products of zero-divisors.

One of the talks will describe the symmetric and high-dimensional analogues of the notion $\mathsf{TC}(X)$, concepts relevant to situations when motion planning faces special requirements.

Surprisingly the number $\mathsf{TC}(\mathbf{RP}^n)$ computes the immersion dimension of the real projective space $\mathbf{RP}^n$ (with a few exceptions). Similarly, the symmetric topological complexity of $\mathbf{RP}^n$ computes its embedding dimension (again, with a few exceptions). Finally, we will discuss the algorithms for collision free motion of multiple particles in space and along graphs and compute the complexities of these problems.

C. Stochastic algebraic topology.

While dealing with large systems in application one cannot assume that all parameters of the system are known or can be measured without errors. A typical situation of this kind appears when one studies the configuration space of a linkage with a large number of sides $n \to \infty$. In such a case the topology of the configuration space depends on a large number of random parameters and it turns out that one may predict many statistical properties of the space with high confidence. Moreover, similar to the situation occurring in the statistical physics, the statistical predictions concerning the topology of a random space are extremely precise for large n.

We will discuss in detail the study of random linkages (random polygon spaces) where one may predict the asymptotics of their Betti numbers. We will also consider other probabilistic models producing random complexes of various dimensions, such as random graphs and random 2-dimensional complexes.

Talks:

1. **Classical configuration spaces.** The classical configuration space of n distinct particles in the topological space X is given by

$$F(X,n) \;=\; \{(x_1,\dots,x_n)\in X^{\times n}\,|\,x_i\neq x_j \text{ for } i\neq j\}.$$

This talk is going to focus primarily on the case $X = \mathbb{R}^{m+1}$, and the main aim is to describe the cohomology ring and minimal CW structure of the configuration space. Basic information on the homotopy group structure of this space is needed, and the main reference is the book [16], Chapters II, IV and V. An interesting goal for the lecturer is to discuss known properties about the topology and homotopy types of configuration spaces of spheres and projective spaces ([12], [33], and [46]).

2. **Complements of hyperplane arrangements.** A hyperplane arrangement $\mathcal{A}$ consists of a finite set of hyperplanes in a given vector space. By looking at the complement of these hyperplanes, we get a space $M(\mathcal{A})$, and by choosing appropriate hyperplanes, the configuration space $\mathbf{F}_k(\mathbb{C})$ can be recovered. Again, the structure of the cohomology ring can be determined. The main reference is [48], Chapter 5. Section 2 in [26] establishes a concrete link between this lecture and configuration spaces of the sphere S^2.

3. **Linkages and their configuration spaces.** An abstract linkage (L, ℓ) consists of a graph L and a vector ℓ which assigns each edge of L a positive real number, the length of the edge. The configuration space of a linkage can then be defined as the space of 'realizations' of (L, ℓ) in a Euclidean space $\mathbb{R}^n$, that is, maps of L into $\mathbb{R}^n$ such that if two vertices span an edge e, then their distance in $\mathbb{R}^n$ is exactly $\ell(e)$. One may also require certain vertices or edges to be fixed. If the graph L is homeomorphic to a circle, formulas for the homology of planar and spatial configuration spaces can be given explicitly in terms of the length vector ℓ. In the planar case, early work on this has been done by Niemann [47] and Walker [55]. In order to obtain the homology, the Morse theory of the distance function of a robot arm is studied. The main references are [17], Chapter 1, and [19].

4. **Cohomology of closed planar linkages and Walker's conjecture.** For closed linkages, the length vectors naturally fall into finitely many cells, such that the configuration spaces are diffeomorphic for all length vectors in a given cell. A natural question is whether different cells give rise to different configuration spaces. While homology is easily seen to not be able to distinguish all cells, Walker conjectured that cohomology can distinguish cells. By sharpening the results from the previous talk, partial results on the cohomology can be obtained which are sufficient to prove Walker's conjecture. The main reference is [18] which deals with most cells; the remaining cases are treated in [52]. For isomorphism problems of ring structures, see [6].

5. **Universality theorems for linkages.** Configuration spaces of mechanical linkages (understood more generally than in the previous talks) were studied for centuries as one of the basic topics of kinematics. The configuration spaces of mechanical linkages are compact real algebraic varieties naturally embedded in the Euclidean space. It is natural to ask whether, conversely, every compact real algebraic variety arises as the configuration space of some mechanical linkage. The universality theorems for linkages state that for any compact real-algebraic subset X of the Euclidean space there exists a planar linkage such that its configuration space is isomorphic (in a suitable sense) to a disjoint union of a number of copies of X. The main references for this talk are: [40], [39].

6. **The Totaro Spectral Sequence.** Quite often one wants to compute the cohomology algebras of configuration spaces

$$F(X,n) = \{(x_1,\dots,x_n) \in X^{\times n};\, x_i \neq x_j,\, i<j\}.$$

For this purpose one may use the Leray spectral sequence of the inclusion

$$F(X,n) \to X^{\times n}.$$

Totaro [54] suggested a convenient algebraic description of this spectral sequence. Moreover, he showed that the spectral sequence degenerates at the second term in the case of smooth complex projective algebraic varieties. The Totaro spectral sequence is a powerful tool broadly used in applications. Other related important publications: [3], [43].

7. **The Euler-Gal power series.** This talk will describe a beautiful result of S. Gal [28] which expresses explicitly the Euler characteristics of various configuration spaces associated with polyhedra. Given a finite polyhedron X one considers the Euler-Gal power series

$$\mathfrak{eu}_X(t) = \sum_{n=0}^{\infty} \chi(B(X,n))t^n = \sum_{n=0}^{\infty} \frac{\chi(F(X,n))}{n!} t^n$$

where $B(X,n)$ is the quotient of $F(X,n)$ with respect to the action of the symmetric group Σ_n. S. Gal [28] discovered that $\mathfrak{eu}_X(t)$ has a fairly simple expression while the individual numbers of the series are much more involved. Gal's theorem states that the formal power series $\mathfrak{eu}_X(t)$ is a rational function and gives an explicit expression for it in terms of the local topological properties of X. A detailed exposition of [28] is given in [17].

8. **Configuration Spaces of graphs.** The configuration space $F(\Gamma, n)$ of n distinct particles moving on a graph Γ can be used to model many important processes in engineering. This talk will discuss in detail the topology of $F(\Gamma, n)$ and will address the following issues: (a) Discrete models of the configuration space. (b) Nonpositive curvature of the discrete configuration space. (c) The asphericity of $F(\Gamma, n)$. (d) The upper bound on the homological dimension of the fundamental group of $F(\Gamma, n)$. In (a) one constructs a finite cell complex (in fact, *a cubical complex*) having the homotopy type of the configuration space $F(\Gamma, n)$ (which is not compact). In part (b) one observes that the discrete model is nonpositively curved in the sense of Gromov which implies its asphericity (c), i.e. vanishing of all homotopy groups $\pi_n F(\Gamma, n)$ with $n > 1$. The main references are [53], [29] and the monograph [5].

9. **Knot theory of the robot arm.** Classical knot theory studies subsets $K \subset \mathbf{R}^n$, called knots, viewed up to the equivalence relation of ambient isotopy. The precise meaning of the word knot depends on the context; most common knots are formed by embeddings of spheres (e.g. circles) or disks, subject to requirements of being smooth, piecewise linear, or locally flat. Unknotting theorems of knot theory state that under specific assumptions various knots are all equivalent to each other.

   This talk will describe a robotical variation of the knotting problem. A robot arm is a mechanism with hinges at its vertices and rigid bars at its edges. The hinges can be folded but the bars must maintain their length and cannot cross. Mechanisms of this kind appear widely in robotics. Related mathematical problems were studied in discrete and computational geometry, in knot theory, in molecular biology and polymer physics. The central problem is whether there exists a continuous motion of an arbitrary configuration of the robot arm bringing it into a straight line segment such that, in the process of the motion, no self-intersections are created and the lengths of the bars remain constant. The question "Can a robot arm be knotted?"is also known as "the carpenter's rule problem"and has a long and interesting history. The answer was found in 2003 by R. Connelly, E. Demaine and G. Rote [8] and their result will be the main topic of the talk. The book [17] contains an exposition of their result.

10. **A survey of the Lusternik - Schnirelmann category. Category weight.** The category (LS-cat) of a space was introduced in the 30's as an effort to understand the extent in which the topology of a manifold M determines the minimal number of critical points a given smooth function on M must have. The main

property of LS-cat comes from its homotopy invariance which opens up a fruitful gate to techniques in algebraic topology. The main omnibus reference for this lecture is [9] (Chapters 1–3 being best suited for the purpose of the lecture). The lecturer will also benefit from [56] where the first successful connection with homotopy theory was explored, as well as from [37] which gives a more condensed account than that of [9], fairly complete up to the time. Just as its applications, LS-cat has a large number of variants, the most important of which for the purpose of the lecture is that of category weight and its relativization with respect to prescribed fibrations. The idea started in [15], and [49] surveys the situation as to the end of last century, adding further applications of the concept. For connections with generalized multiplicative cohomology theories see [50]. A main goal of this lecture is to establish the multiplicative properties of category weight, as well as its characterization in terms of the iterated fiber-wise join power of a fibration. References for this are [14], [27], and [32].

11. **Motion planning algorithms, $\mathsf{TC}(X)$ and Schwartz genus.** This talk can be based on chapter 4 from [17]. One may start by describing the motion planning problem of robotics and its interpretation as the problem of finding a section of a fibration. For a system with noncontractible configuration space no continuous motion planning algorithm exists. This leads to the notion of topological complexity $\mathsf{TC}(X)$ which has several interpretations [22] in terms of the language of engineering. Mathematically, $\mathsf{TC}(X)$ is a special case of the notion of Schwartz genus. One may estimate $\mathsf{TC}(X)$ from below using the cohomology algebra; there is also an upper bound involving the notions of dimension and connectivity (these bounds should be discussed within the more general context of Schwartz genus). Possible other topics to be included: the product inequality, the method of navigation functions, simultaneous control of multiple objects, using cohomology operations and others.

12. **Symmetric and high-dimensional analogues of $\mathsf{TC}(X)$.** In practice the motion planning problem might be subject to a number of restrictions imposed by the particular problem under consideration. For instance, no motion at all might be required when the initial and final configurations agree (motion optimality). Another usual restriction is that the motion from configuration A to configuration B should be the time-reverse movement of that from B to A (symmetry). These and other restricted instances were studied in [13] which is the first main reference for this lecture. The speaker should also consider [36], from where the optimality condition can be identified as the problem for having homotopy in-

variance. Another natural restriction in motion planning arises when repeated motion planning cycles are required, in each of which partial prescribed configurations should be attained (e.g. in a industrial manufacturing process). This type of restrictions was considered in [13] and [51]. The latter reference introduces the concept of high-dimensional TC, which is the second main goal for this lecture. The analysis of the symmetric version of TC requires a good working control of the algebraic topology of unordered configuration spaces $B(X, n)$. When X is a manifold, the mod-2 cohomology of $B(X, n)$ was first worked out in [34], and a summarized account is given in [13]. The ordered case essentially comes from [45], and serves as a convenient comparison point for the Totaro spectral sequence of lecture # 7.

13. $\mathsf{TC}(X)$ **and the immersion and embedding problems.** Determining the smallest dimension of Euclidean spaces where a given manifold admits an immersion/embedding is a (generally open, hard, and) classic goal in algebraic and differential topology and geometry. The problem reached its golden era in the 60's and has experienced a recent resurrection motivated in part by a surprising connection, in the case of projective spaces, with the concept of topological complexity developed in the previous two lectures. The present lecture will be based on the original work [25] where such a connection was established for the immersion problem of real projective spaces via the concept of axial maps, and on the latter work [30] where the corresponding connection for the embedding problem was settled. For lens spaces the connection is not as sharp, but explicit calculations of their TC were made in [14] using the category weight of cohomology classes, and in [31] where cobordism methods were employed. Both works can be seen as efforts for adapting Hopf's classic condition for the existence of symmetric bilinear forms. As originally shown in [10], the cobordism approach appears as a particularly promising method for future research.

14. **Collision free motion planning.** This talk may be based on the papers [23] and [24], see also [22] and [17]. Firstly, one wants to compute the topological complexity of the configuration space $F(\mathbf{R}^m, n)$ of n distinct particles in $\mathbf{R}^m$. The second goal is to compute the topological complexity of the configuration space $F(\Gamma, n)$ where Γ is a graph. The answers are very different: $\mathsf{TC}(F(\mathbf{R}^m, n))$ is linear in n and $\mathsf{TC}(F(\Gamma, n))$ is bounded by a constant independent of n.

15. **A survey of random graphs.** The theory of random graphs is the best developed part of stochastic algebraic topology. Many beautiful and deep results

about random graphs are described in the literature, notably in several monographs on the subject [1], [4], [38]. The talk must first describe some models of random graphs and then state some typical results. The topics which could be mentioned include: the containment problem, connectivity of random graphs and information about the evolution theory of random graphs.

16. **Random 2-dimensional complexes.** Random 2-dimensional complexes are generalizations of random graphs. They were studied recently by Linial–Meshulam in [42], and Meshulam–Wallach in [44]. One generates a random 2-dimensional simplicial complex Y by considering the full 2-dimensional skeleton of the simplex Δ_n on vertices $\{1, \dots, n\}$ and retaining 2-dimensional faces independently with probability p. Note that in this construction Y contains the one-dimensional skeleton of Δ_n. The work of Linial–Meshulam and Meshulam–Wallach provides threshold functions for the vanishing of the first homology groups of random complexes with coefficients in a finite abelian group. Additional information can be found in the preprints [2], [7] and [41]. Many intriguing questions about random 2-complexes remain open, see [35].

17. **Random polygon spaces.** Interesting examples of random manifolds are provided by configuration spaces of mechanical linkages and polygon spaces with random (or unknown) side lengths. These random manifolds can be used for modeling large systems in biology and statistics. One wants to use methods of probability theory in dealing with the variety of diffeomorphism types of these configuration spaces for n large. The topological invariants of these manifolds become random functions and we want to analyze their expectations and other statistical properties. The talk may focus on the mathematical expectations of the Betti numbers of random manifolds and may be based on papers [20] and [21]. Additional information can be found in the preprint [11].

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Participation:

The idea of the Arbeitsgemeinschaft is to learn by giving one of the lectures in the program.

If you intend to participate, please send your full name and full postal address to

`michael.farber@durham.ac.uk`

by 20.08.2010 at the latest.

You should also indicate which talk you are willing to give:
First choice: talk no. ...
Second choice: talk no. ...
Third choice: talk no. ...

You will be informed shortly after the deadline if your participation is possible and whether you have been chosen to give one of the lectures.

The Arbeitsgemeinschaft will take place at Mathematisches Forschungsinstitut Oberwolfach, Lorenzenhof, 77709 Oberwolfach-Walke, Germany. The institute offers accomodation free of charge to the participants. Travel expenses cannot be covered. Further information will be given to the participants after the deadline.