Workshop 1202

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Title: ***Explicit versus tacit knowledge in mathematics***

Organisers:

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Abstract:

This workshop aims to bring together an international group of historians of mathematics to reflect upon the role played by tacit knowledge in doing mathematics at various times and places. The existence of tacit knowledge in contemporary mathematics is familiar to anyone who has ever been given an idea of how a particular proof or theory "works" by a verbal analogy or diagrammatic explanation that one would never consider publishing. Something of it is felt by every student of mathematics, when the process of learning mathematics often amounts to training the right reflexes. In more advanced contexts, the tacit understanding that a particular technique, instrument or approach is "the one to use" in a given circumstance gives another familiar instance. Tacit knowledge, a term introduced by the philosopher M. Polanyi, contrasts with the explicit knowledge that in almost all historical mathematical cultures is associated with mathematical text. The workshop invites a use of the categories of tacit and explicit knowledge to achieve a better knowledge of how mathematical creation proceeds, and also of how cultural habits play a tacit role in mathematical production. The proposed meeting offers the possibility of significant innovation and enrichment of historical method, as well as new and compelling insight into the process of creating mathematics in different times and places. The meeting will afford the opportunity for a presentation of selected case studies by leading experts and new scholars, with results that promise to be of significant interest not only to historians, but to the mathematical community more broadly.

Mathematical subject classification:

MSC: 01A16-61, 01A72, 01A85
IMU: 19

Objective of the conference:

The aim of this workshop is to bring together an international group of historians of mathematics to reflect upon the role played by *tacit, as opposed to explicit knowledge* in doing mathematics at various times and places. Methodological discussions on the use of this concept will alternate with specific case studies from the history of mathematics. Both will allow a better understanding of mathematical practices in given contexts. The theme impinges on the transmission of existing mathematics as well on the creation of new theories and results.

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The theme was specifically chosen for this meeting on the history of mathematics in view of its inspirational and unifying potential, and in the ways that it promises to shed light on methods for understanding mathematical texts and practices of the past. It suggests looking at cases that range from the most ancient history of mathematics to current developments, as the following examples show:

- The difference between algorithmic mathematics (like in ancient Mesopotamia or medieval China) and proof-oriented mathematics in the Euclidean tradition – and the intermediate stages, like Chinese two-column algorithmic texts which are proof-driven but not in the Euclidean style – are all too often analyzed without taking into account the parts of the practice that remain tacit and are not spelled out in the text, contributing thus to give a biased image of that difference.

- Tacit knowledge is present in various ways throughout the mathematical exchanges of the 17^{th} century. Correspondence by letters included knowledge on how to write a letter, without spelling out the rules of letter writing. In cases where these tacit codes were not applied, it is interesting to give an interpretation of this step aside. More generally, tacit rules of scientific exchange dictated what was to be made explicit or public in a mathematical proof, and which parts were not. On the mathematical level, curves were identified by a catalogue of properties, which was never explicitly listed in its entirety. For instance, as soon as a curve was found to have the property that its subtangent is the double of the abscissa, it was identified with a parabola.

- A good deal of the development of mathematics in the 19^{th} and 20^{th} centuries can be viewed as a process of making the practice of mathematics increasingly explicit, thereby reducing the amount of tacit knowledge and thus opening up a wide space of rational discussion and achievements. However, this tendency to greater technical explicitness, which is evident in the typical manuscripts posted by mathematicians on *ArXiv* every day, may induce historians of mathematics to neglect the persistence of tacit knowledge in the most recent mathematics. The identification of such tacit elements seems capable of affording significant insights into the development of mathematics today.

- Similarly, several large scale mathematical enterprises of the last 100 years like Bourbaki's *Éléments de mathématique* or – in a different manner – computer-based mathematical research, like the more recent projects towards automated theorem proving (ATP), appear at first as signposts of a massive pushing back of tacit knowledge. Looking more closely, however, at details like the occasional warning signs in the margins of Bourbaki's volumes, or at problems related with the user interface, one sees that these undertakings carry in fact their own heavy collection of tacit mathematical practice.

- Developments in the history of mathematics are often loosely described as moving from approximate, incompletely understood treatments, to fully explicit, formal statements and their rigorous proofs. (See for instance Breger's contribution to [1].) Paying attention to the kind of tacit knowledge which is mobilized before and after such a development often provides a much more satisfactory analysis of the historical process than the mere confrontation of precise *versus* imprecise methods. A case in point is the rewriting of Algebraic Geometry in the first half of the 20th century. In a 1926 letter to Hermann Weyl, Salomon Lefschetz significantly described the Italian school of Algebraic Geometry, not as lacking rigor, but as requiring "a terrible 'entraînement'". Later attempts, by Francesco Severi and others, to defend their classical Algebraic Geometry against growing criticism would invariably insist on the fact that all those technical assumptions or arguments which the modern algebraists could not find in the Italian papers where indeed tacitly assumed, and well-known to all geometers raised in the Italian school. The question whether the category of tacit knowledge may render such arguments historically convincing appears quite difficult, and can only be decided by very detailed case studies.

- In contemporary mathematics, blogs and Wikis – the most famous probably being Terence Tao's – currently provide an extended form of oral culture in which less formal, formerly tacit approaches are written down and opened to a broad mathematical public according to shifting and variable rules.

The term "tacit knowing" or "tacit knowledge" which we propose to explore here in its bearing on the history of mathematics, comes from a philosophical context, but has been mobilized before for the history of science. Michael Polanyi introduced "tacit knowing", or "tacit knowledge" in order to describe abilities which cannot be fully described or explained (see [4]). In the history of science, the concept has been mobilized in the study of the craft aspects of experimental science from the 17th century to the present day. The philosophical theory of tacit knowledge has been much discussed over the years – for instance also in the context of mathematical education and curricula, which is not the purpose of the workshop proposed here. More recently, the sociologist Harry Collins reassessed this notion in [2], in particular distinguishing several types of tacit knowledge.

The theory of tacit knowledge marks a counterpoint to the "ideal of wholly explicit knowledge" which took shape through the scientific revolution of the seventeenth century.
Among the different interpretations which have been given of the concept of "tacit knowledge", from a conscious under-articulation or deliberate secrecy to the strong thesis that there are specific kinds of knowledge that cannot in principle be fully articulated – the standard example being here riding a bike - the application to the history of mathematics will focus on the weak

sense: tacit knowledge is what mathematicians selectively conceal, avoid articulating or under-articulate, consciously or not. This does include the possible concealment of information by mathematicians competing with others, as well as the case of descriptions which are left incomplete because their authors assume, or know by experience, that their readers share a certain knowledge with them. Tacit knowledge is then built on experience or action, and cannot be fully described by rules or words. It concerns any type of knowledge or skill used as subsidiary to the performance and control of a mathematical task.

The notion of tacit knowledge could be applied to the history of mathematics, as suggested by Breger ten years ago who used the greater level of abstraction created by the ongoing development of mathematics to detect tacit elements in earlier texts. This is a challenging thesis but obviously history of mathematics should not be reduced to just re-reading old texts through the spectacles of more modern mathematical achievements.

At this point, more recent methods in the history of mathematics come to the rescue: following a tradition that can be traced back to Ludwig Wittgenstein and other authors of the 1930s and 1940s, the second half of the 20th century has seen authors such as Imre Lakatos, Paul Feyerabend, and Hans-Jörg Rheinberger placing the detailed analysis of scientific practice at the heart of history of science. This goes hand in hand with the realization that tacit scientific knowing is acquired by the individual scientist through a social context or network whose members share a common know-how. Although unstated know-how tends to be difficult to identify in a single mathematical text, shared tacit knowledge or know-how is more accessible, often by way of comparison with other local mathematical cultures or broader networks. It also tells a lot about mathematical (and strategic) practices in a specified time period.

In the case of mathematics, Epple has adapted Rheinberger's approach to the history of mathematics in his book [3] on the history of knot theory. The notions of *epistemic objects* and *epistemic techniques* are his key concepts to describe the ways of the active researchers to handle the complex web of established theories ready for use, formal and informal operational skills to deal with new phenomena, and often vague general ideas about the kind of mathematical object under focus.

Furthermore, the mathematical tools made use of in specific contexts or sites are in most cases abstract techniques or objects, but may also be material devices, from the measuring rod and compass to the analog integrator and computer. In the Renaissance and early modern periods, the design and use of such instruments was a core feature of mathematical practice, and the tacit knowledge involved in acquiring the techniques of use or design was considerable. Yet such knowledge has left historical evidence: Albrecht Dürer, most famously, tried to describe explicitly what perpective artists were actually doing, including the gestures transmitted through long workshop traditions. One aim of the conference will be to assess the degree of continuity between these older traditions and those in evidence in more recent mathematical practice.

Our main objective for the conference proposed here is therefore to use the peculiar bias of the distinction between tacit and explicit knowledge in order to re-invigorate discussions about how the analysis of social networks on the one hand and of the research practice of mathematicians on the other come together to afford a close-up understanding of the historical process which we call mathematics. Last but not least it will allow a better understanding of how mathematical practices depend on larger cultural habits, or are embedded in larger cultural contexts, including language, writing cultures, literary and rhetorical devices, and craft knowledge.

References

[1] Herbert BREGER, Emily GROSHOLZ (eds.), *The Growth of Mathematical Knowledge.* Dordrecht 2000.

[2] Harry M. COLLINS, *Tacit and Explicit Knowledge.* Chicago 2010.

[3] Moritz EPPLE, *Die Entstehung der Knotentheorie. Kontexte und Konstruktionen einer modernen mathematischen Theorie***.** Braunschweig 1999.

[4] Michael POLANYI, *Personal knowledge. Towards a post-critical philosophy*. London 1969.