Arbeitsgemeinschaft mit aktuellem Thema:
Quasiperiodic Schrödinger Operators
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Introduction:

The Arbeitsgemeinschaft will discuss quasiperiodic Schrödinger operators of the form

$$[H\psi](n) = \psi(n+1) + \psi(n-1) + V(n)\psi(n)$$

with potential $V : \mathbb{Z} \to \mathbb{R}$ given by

$$V(n) = f(\omega + n\alpha),$$

where $\omega, \alpha \in \mathbb{T}^k = \mathbb{R}^k/\mathbb{Z}^k$, $\lambda \in \mathbb{R}$ and $\alpha = (\alpha_1, \dots, \alpha_k)$ is such that $1, \alpha_1, \dots, \alpha_k$ are independent over the rational numbers, and $f : \mathbb{T}^k \to \mathbb{R}$ is assumed to be at least continuous. H acts on the Hilbert space $\ell^2(\mathbb{Z})$ as a bounded self-adjoint operator. While most of the talks will focus on this setting, occasionally we will also consider the multi-dimensional setting, where H acts on $\ell^2(\mathbb{Z}^d)$ and is again given by the sum of the discrete Laplacian and the multiplication operator generated by a quasiperiodic $V : \mathbb{Z}^d \to \mathbb{R}$. For the sake of simplicity, this introduction will consider the one-dimensional case.

The associated unitary group $\{e^{-itH}\}_{t\in\mathbb{R}}$ describes the evolution of a quantum particle subjected to the quasiperiodic environment given by V. For any $t \in \mathbb{R}$ and $n \in \mathbb{Z}$, $|\langle \delta_n, e^{-itH}\psi \rangle|^2$ is the probability of finding the particle, whose initial state at time zero is given by the ℓ^2-normalized ψ, at time t at site n. The long-time behavior of these probabilities is of interest and many relevant questions about them can be studied by means of spectral theory (i.e., by "diagonalizing" the operator H). The spectral theorem for self-adjoint operators associates a measure μ_ψ with the initial state ψ. Roughly speaking, the more continuous the measure μ_ψ is, the faster $e^{-itH}\psi$ spreads. For this reason, one wants to determine the spectral type of H. For example, H is said to have purely absolutely continuous (resp., purely singular continuous or pure point) spectrum if every μ_ψ is purely absolutely continuous (resp., purely singular continuous or pure point). Again roughly speaking, the absolutely continuous case corresponds to transport, whereas the pure point case typically corresponds to the absence of transport ("dynamical localization"), while the singular continuous case corresponds to intermediate transport behavior. In the case where not all spectral measures have the same type, one collects all those states whose measures have the same type in a single subspace, restricts the operator to the resulting three subspaces, and the spectra of these three restrictions are then called the absolutely continuous, singular continuous, and pure point spectrum of H, respectively.

In the recent past, the spectral analysis of quasiperiodic Schrödinger operators has seen great advances. It is the goal of this Arbeitsgemeinschaft to present many of these advances.

It is useful to regard quasiperiodic potentials as being dynamically defined in the sense that they are obtained by sampling along the orbit of an ergodic transformation with a real-valued sampling function. Concretely, if we consider the map $T : \mathbb{T}^k \to \mathbb{T}^k$, $\omega \mapsto \omega + \alpha$, it is invertible and has normalized Lebesgue measure as its unique invariant Borel probability measure. Then, the potential V may be obtained as $V(n) = f(T^n\omega)$. More generally, whenever we have such a dynamically defined situation with an ergodic (Ω, T, μ) and a (bounded) measurable $f : \Omega \to \mathbb{R}$, several fundamental results hold. Namely, the spectrum, as well as the absolutely continuous spectrum, the singular continuous spectrum, and the point spectrum, of H are μ-almost surely independent of ω and are denoted by $\Sigma, \Sigma_{\mathrm{ac}}, \Sigma_{\mathrm{sc}}, \Sigma_{\mathrm{pp}}$. Moreover, the density of states dk, given by

$$\int_\Omega \langle \delta_0, g(H)\delta_0 \rangle \, d\mu(\omega) = \int_{\mathbb{R}} g(E) \, dk(E)$$

and the Lyapunov exponent

$$L(E) = \lim_{n \to \infty} \frac{1}{n} \int_{\Omega} \log \|A_E^n(\omega)\| \, d\mu(\omega),$$

where

$$A_E^n(\omega) = \begin{pmatrix} E - f(T^{n-1}\omega) & -1 \\ 1 & 0 \end{pmatrix} \times \cdots \times \begin{pmatrix} E - f(\omega) & -1 \\ 1 & 0 \end{pmatrix},$$

are defined and determine Σ and Σ_{ac} as follows,

$$\Sigma = \operatorname{supp} dk, \qquad \Sigma_{\mathrm{ac}} = \overline{\{E \in \mathbb{R} : L(E) = 0\}}^{\mathrm{ess}}.$$

These two fundamental results can be considered classical and have been known since the early 1980's.

Note that $L(E) \geq 0$ and hence one naturally distinguishes between the two cases $L(E) = 0$ and $L(E) > 0$. The result quoted above shows that the first case is connected to the absolutely continuous part, whereas the general tendency is for the second case to be connected to the pure point part. Indeed, among the major recent advances to be addressed in this Arbeitsgemeinschaft are ways to go from positive Lyapunov exponents to pure point spectral measures (and more, such as exponentially decaying eigenfunctions, dynamical localization, etc.) in the quasiperiodic case with sufficiently regular sampling function f and for most α.

Another major recent development is that in the case of $k = 1$ (i.e., $\alpha, \omega \in \Omega = \mathbb{T}$), the regime of zero Lyapunov exponents has been studied in a global sense and it has been shown for analytic f, that one typically has purely absolutely continuous spectrum there. A significant portion of the lectures will be devoted to these results.

As a consequence, one now understands the typical spectral type of a one-frequency quasiperiodic Schrödinger operator with analytic sampling function.

We will complement these results with a discussion of spectral phenomena in the multi-frequency case and/or for non-analytic sampling functions.

We will also devote time to the central example, the almost Mathieu operator, whose potential is given by

$$V(n) = 2\lambda \cos(2\pi(\omega + n\alpha)), \quad \lambda > 0, \ \alpha \in \mathbb{T} \setminus \mathbb{Q}, \ \omega \in \mathbb{T}.$$

This operator has been studied intensively for many decades, its spectral properties are almost completely understood, and works on it were a driving force behind the development of the theory for general analytic sampling functions. It is the central quasiperiodic model mainly due to being the one coming from physics and attracting continued interest there. It first appeared in the work of Peierls [69] and arises as being related, in two different ways, to a two-dimensional electron subject to a perpendicular magnetic field [48, 72]. It plays a central role in the theory of Thouless et al. of the integer quantum Hall effect [77]. Also, it seems to represent most of the nontrivial properties expected to be encountered in the more general case. On the other hand, it has a very special feature: the duality (essentially a Fourier) transform maps H_λ to $H_{1/\lambda}$,[1] and hence $\lambda = 1$ is the self-dual (also called critical) point. The regions $\lambda < 1$ and $\lambda > 1$ are referred to as subcritical and supercritical, respectively. The development of the theory of this model (and of general quasiperiodic operators along with it) was guided for a long time by two conjectures formulated by Aubry and André in [1] and heavily popularized in several articles by Simon in the early 1980's. One conjecture was on the metal-insulator transition at $\lambda = 1$, the other one was on the exact value of the Lebesgue measure of the spectrum. The third conjecture, by Azbel-Kac, the "ten Martini problem," on the Cantor nature of the spectrum added further interest. After 2000, the developments were centered around the "Simon problems" — three almost Mathieu related problems formulated in [74]. Two of these problems at the time of their formulation were unresolved only for zero measure sets of parameters, and the inclusion of those problems in this list of fifteen highlighted the fact that these issues were perceived more and more as a "number-theory" type problem, where recent advances have made it possible to seek very precise information for all values of the parameters. All three problems are now solved, but, for space considerations, only two were selected for this meeting. In the talks on the almost Mathieu operator, the arithmetic restrictions on the parameters, if any, should be exactly specified.

[1] Due to the crucial role played by the duality, we will make the dependence of H on λ explicit in the discussion of the almost Mathieu operator and write H_λ instead. The dependence on α and ω will still be left implicit in our notation, however.

Talks:

1. **Schrödinger operators and quantum dynamics**
   This talk should present the basics concerning the spectral theory of Schrödinger operators and the connections to the time-evolution given by the Schrödinger equation. It should be explained that for a bounded $V : \mathbb{Z}^d \to \mathbb{R}$, $H = \Delta + V$ defines a bounded self-adjoint operator on $\ell^2(\mathbb{Z}^d)$, and the Schrödinger equation $i\partial_t\psi = H\psi$, $\psi(0) = \psi_0$ is solved by $\psi(t) = e^{-itH}\psi_0$. Moreover, the long-time asymptotics of $\psi(t)$ may be studied in terms of the Lebesgue decomposition of the spectral measure associated with the pair (H, ψ_0). Notions such as absolutely continuous spectrum, singular continuous spectrum, and pure point spectrum should be defined, and their relevance concerning quantum dynamics should be explained through the RAGE theorem and related results. Extremal transport behavior, such as ballistic transport and dynamical localization, should be discussed and some examples should be given. Good sources include Teschl [76] (especially Chapters 3 and 5) for the general part up to the RAGE theorem and Last [64] for some results going beyond the RAGE theorem.

   Spectral properties can often by linked to the behavior of generalized eigenfunctions. Some of the major results should be stated, such as Schnol's theorem [31, Section 2.4] and Gilbert-Pearson's theory of subordinate solutions [34, 41, 42, 54, 55]. Of the latter, only the key result on support of absolutely continuous spectral measures [42] should be stated, in the discrete formulation of [54, 55]. The α-continuity/singularity need not be mentioned.

   The following situation will be of special interest to us: the dimension is $d = 1$ and the potential V is dynamically defined. That is, for an invertible discrete-time dynamical system $T : \Omega \to \Omega$ and a map $f : \Omega \to \mathbb{R}$, potentials of the form $V(n) = f(T^n\omega)$ will be considered. In this case we will say that the potential is dynamically defined. Choosing a T-ergodic measure μ, it turns out that the basic spectral quantities associated with H (such as the spectrum and the three subspectra discussed earlier) are μ-almost surely independent of the choice of ω in the definition of V. This fundamental result should be explained. In addition, it should be proved that for quasiperiodic operators, the spectrum is constant in ω for **all** ω. The density of states dk, defined by

$$\int_\Omega \langle \delta_0, g(H)\delta_0 \rangle \, d\mu(\omega) = \int_{\mathbb{R}} g(E) \, dk(E),$$

should be introduced and the description of the μ-almost sure spectrum Σ as the topological support of dk should be explained. Textbook treatments may be found in [29, Chapter V], [31, Chapter 9], [68, Section 2].

2. **Basic properties of cocycles and connections to spectral theory**

   If the potential is dynamically defined, the generalized eigenvalue problem for H will be related to certain extensions of T, namely a one-parameter family of $\mathrm{SL}(2,\mathbb{R})$ cocycles. Explicitly, consider for $E \in \mathbb{R}$, the map

$$A_E : \Omega \to \mathrm{SL}(2,\mathbb{R}), \quad \omega \mapsto \begin{pmatrix} E - f(\omega) & -1 \\ 1 & 0 \end{pmatrix},$$

   and the following extension of T,

$$(T, A_E) : \Omega \times \mathbb{R}^2 \to \Omega \times \mathbb{R}^2, \quad (\omega, v) \mapsto (T\omega, A_E(\omega)v).$$

   Iterates are given by $(T, A_E)^n = (T^n, A_E^n)$ with a suitable map $A_E^n : \Omega \to \mathrm{SL}(2,\mathbb{R})$. Then, $u : \mathbb{Z} \to \mathbb{R}$ solves the difference equation

$$u(n+1) + u(n-1) + f(T^n\omega)u(n) = Eu(n)$$

   if and only if

$$\begin{pmatrix} u(n) \\ u(n-1) \end{pmatrix} = A_E^n(\omega) \begin{pmatrix} u(0) \\ u(-1) \end{pmatrix}.$$

   The map (T, A_E) or, by abuse of notation just A_E, is an $\mathrm{SL}(2,\mathbb{R})$ cocycle over T. The fundamental concepts of uniform and non-uniform hyperbolicity, reducibility, and the fibered rotation number should be discussed for $\mathrm{SL}(2,\mathbb{R})$ cocycles and the connection between the fibered rotation number for $\{A_E\}_{E\in\mathbb{R}}$ and the density of states of H should be explained. For references on these topics, see [50, 62, 71].

   Moreover, the Lyapunov exponent $L(E)$, describing the average rate of exponential growth of $A_E^n(\omega)$, should be defined and the proof of the Thouless formula,

$$L(E) = \int_{\mathbb{R}} \log |E' - E| \, dk(E'),$$

relating the Lyapunov exponent and the density of states, should be sketched. For this part of the talk, see [29, Chapters V and VI], [31, Chapter 9], and [68, Section 11].

3. **Kotani theory: Lyapunov exponents and the absolutely continuous spectrum**
   This talk should explain the description of the almost sure absolutely continuous spectrum in terms of the Lyapunov exponent. Indeed, for μ-almost every ω, the absolutely continuous spectrum of H is given by the essential closure of the set of energies E for which $L(E) = 0$. This result is due to Ishii [51], Pastur [67], and Kotani [60]; see [32, 73] and [29, Chapter VII] for a proof of this result in our setting. The talk should present this result along with its proof. It should also describe some of the results that are naturally obtained in addition once one has Kotani theory set up, and that will be used in later talks, namely the paper [35] by Deift and Simon (and in particular Theorems 1.4 and 7.1 from that paper) and the short note [61] by Kotani.

   In addition, the work [65] of Last and Simon on absolutely continuous spectrum, their description of an essential support of the absolutely continuous part, and their corollary that the absolutely continuous spectrum of one-dimensional quasiperiodic operators is constant for **all** ω should be briefly described as well.

4. **Positivity of the Lyapunov exponent at large coupling**
   From this point on we will focus exclusively on quasiperiodic potentials. These potentials are dynamically defined; the underlying dynamics is given by a minimal translation $T\omega = \omega + \alpha$ on a finite-dimensional torus $\mathbb{T}^k$. In addition, we will place regularity assumptions (usually analyticity) on the function $f : \mathbb{T}^k \to \mathbb{R}$ used in the definition of the potentials, $V(n) = f(\omega + n\alpha)$. This talk should focus on sufficient conditions for $L(E)$ to be uniformly bounded away from zero for all energies E. The talk should start by presenting Herman's method [50] for f a trigonometric polynomial. As an immediate application, one obtains the desired positivity and thus the absence of absolutely continuous spectrum for the super-critical almost Mathieu operator. The basic idea is developed in Section 2 of [50], the relevant example for Schrödinger cocycles is given in Subsection 4.7 (see also [31, Section 10.2]).

Herman's ideas were extended to general analytic f by Sorets-Spencer [75]. The talk should cover the lattice case presented in Sections 2 and 3 of [75], in particular the proof of Theorem 2. A useful summary of both methods is also given in Chapter 3 of [21]. Extensions to shifts on $\mathbb{T}^k$, $k > 1$, obtained by Bourgain in [22] should be mentioned.

5. **Regularity of the Lyapunov exponent I**

6. **Regularity of the Lyapunov exponent II**
   The goal of talks 5–6 is to present the result from [43] on Hölder continuity of the Lyapunov exponent in energy in the case of Diophantine α and the result from [27] on the joint continuity of the Lyapunov exponent in cocycle and frequency in the general case, as well as develop some analytic tools needed for the proof of nonperturbative localization in a later talk. The presentation should start with a bound on the Fourier coefficients of subharmonic functions with bounded extension (a suggested source is Corollary 4.7 in [21] but another proof would also be fine). The talk should continue to prove the large deviation theorem for the shift model as in Theorem 5.1 [21] as well as formulate and explain changes needed for the version of [27] (Lemma 4 in [27]). The avalanche principle [43] should be proved and the related Young's method [79] should be mentioned without giving details. The remaining time should be devoted to explaining how the Hölder regularity [43] and joint continuity [27] follow. For Hölder regularity, it is suggested to use the concise presentation in Chapter 7 of [21]. For continuity, there is only the original source and the speaker should explain the general scheme. The part of the proof related to the Liouville scales could be omitted, however the presentation should make it clear why continuity in the frequency holds. For shifts on $\mathbb{T}^k$ with $k > 1$, continuity of the LE was obtained in [22] (see Main Theorem, page 314), this should be mentioned without proof.

   The two speakers will be encouraged to coordinate the break-up of the material and the preparation of their lectures.

7. **Classical KAM theory**
   The goal of this talk is to introduce some basic results of KAM theory and their application to quasiperiodic cocycles close to constant. The

basic references are the appendix of [49] for a version of the classical KAM theorem on tori, and Section 5 of [50] for the application. Finally, an application to quasiperiodic Schrödinger operators with Diophantine frequencies and small potential should be given: the existence of an absolutely continuous spectral component (using [65], presented in talk 3, or [41, 42], presented in talk 1, for the derivation).

8. **KAM theory for Schrödinger operators I**
   This talk will present the results of [38] on quasiperiodic Schrödinger operators with Diophantine frequencies and quasiperiodic cocycles with Diophantine frequencies and small analytic potential. The focus should be on a detailed proof of reducibility under a full measure non-resonance condition on the fibered rotation number. The non-standard aspects of the iterative scheme should be emphasized, especially its use of large changes of coordinates and its ability to keep track of the full spectrum, even in the presence of the topologically generic failure of reducibility. The speaker should note that [38] is concerned with the slightly different setting of cocycles over quasiperiodic flows, the adaptation to the discrete setting having been made in several references since ([47], [39]).

9. **KAM theory for Schrödinger operators II**
   This talk will present results of [11] on one-frequency quasiperiodic Schrödinger operators with analytic potential close to constant and arbitrary irrational frequency. A detailed proof should be given of the existence of an analytic conjugacy to a cocycle of rotations, under a positive measure non-resonance condition on the fibered rotation number.

   The speaker will also discuss (following [5]) how it is currently known that all cocycles close to constant are almost reducible (as discussed in talk 23), and that this allows one to replace the positive measure condition on the fibered rotation number by a full measure condition.

10. **Renormalization I**

11. **Renormalization II**
    The goal of talks 10–11 will be to present an application of renormalization to quasiperiodic cocycles with one-frequency. The beginning

should be more conceptual (based on [2]), explaining the general idea of "global to local" reduction, which aims at applying a perturbative technique (such as KAM) developed near a renormalization attractor to its entire basin. The typical importance of *a priori* bounds to ensure convergence to a renormalization attractor should be emphasized, and the need to assume at least the vanishing of Lyapunov exponents should be noted.

This should be followed by a detailed presentation of the results of [14], including the precise definition of the renormalization operator at the level of $\mathbb{Z}^2$-actions, the full proof of *a priori* bounds, and the particular implementation of global to local reduction in this setting. Finally, the applicability of renormalization to the study of general irrational frequencies (following [11, Introduction]) should be discussed.

The two speakers will be encouraged to coordinate the break-up of the material and the preparation of their lectures.

12. **The metal-insulator transition for the almost Mathieu operator**

The purpose of the first part of this talk is to prove localization for the super-critical almost Mathieu operator with Diophantine frequency α and almost every phase ω [52]. In later talks this result will be used to establish absolutely continuous spectrum in the subcritical case, thereby establishing a sharp metal-insulator transition for this model.

The talk should use the previously established Herman bound on the Lyapunov exponent as a starting point. It should be emphasized that super-criticality enters in the proof through the positivity of the Lyapunov exponent only. Since the estimate on the Lyapunov exponent and thus the required coupling does not depend on the frequency, such a result is called non-perturbative. The proof should be given in detail; however, the proof of Lemma 5 should be skipped (it could be noted that this is a version of a known Remez-type inequality and a similar statement follows from Cartan's lemma.) Also Lemmas 11–12 should be replaced by a streamlined argument based on Lemma 9.6 of [12].

At the end of the proof, the speaker should define resonances as in [13] and non-resonant phases, and point out that the result is proved precisely under the non-resonance condition on the phase.

An extension to the case of weakly Liouville frequencies, as in [12], should be formulated without proof, and the conjecture on the arithmetic boundary of localization (presented, e.g., in [12]) should be pointed out.

The talk should then proceed to the notion of almost localization (Section 3.1 of [13]). Theorem 5.1 of [13] should be formulated directly for the long range operator, without mentioning duality, and it should be explained why its proof, for case of the almost Mathieu operator, is essentially contained in the proof just presented. After that, a sketch of modifications needed for the long-range case should be presented following the argument of [26] or Chapter 11 of [21]. It should be pointed out that the theorem holds for weakly Liouville frequencies.

13. **Duality and the almost Mathieu operator**
An important symmetry underlying the almost Mathieu operator is known as Aubry duality. It relates the sub- and supercritical regions, heuristically establishing a correspondence between "localized states" and Bloch waves. The goal of the first part of this talk is to formulate two rigorous versions of duality and use one to deduce the exact formula for the Lyapunov exponent and the other, dynamical formulation, to deduce the duality between localization and rotation-reducibility. The talk should start with invariance (modulo energy scaling) of the density of states dk and the spectrum under the duality. There are a number of proofs in the literature and the speaker should quickly sketch one (suggestions range from the very first one [16] to the most recent one [56, Section 8], which is presented for a more general context; in any case, the fact that the duality in this sense holds for general continuous potentials should be emphasized). An interpretation of duality in terms of the corresponding two-dimensional magnetic model [66, Sections 2–3)] should be briefly mentioned. Based on the Thouless formula, the invariance of the density of states under duality yields a formula connecting Lyapunov exponents at λ and $1/\lambda$, which provides another proof of the positivity of the Lyapunov exponent for the supercritical almost Mathieu operator. The short argument can be found in [16] (see Corollary 6.4 therein) and should be briefly presented.

Next, a dynamical formulation of Aubry duality, as given in Theorem 2.5 of [13] should be covered. Within these frameworks, fixing

$\lambda < 1$, pure point spectrum with exponentially localized eigenfunctions for almost every phase for the supercritical almost Mathieu operator $H_{1/\lambda}$ [52] implies rotation-reducibility in the dual regime at each eigenvalue. While, using subordinacy theory (cf. talk 1), this is only a quick argument away from absolutely continuous spectrum for almost every phase for H_λ, this should not be elaborated upon, as a much more subtle all phase result will be presented in a later talk.

Rotation-reducibility implies $L(E) = 0$ for a dense subset of the spectrum. Using continuity [27], one concludes $L(E) = 0$ throughout the spectrum in the subcritical case, and, applying again duality, $L(E) = \log \lambda$ throughout the spectrum in the supercitical case.

The above-mentioned dynamical formulation of duality also allows one to prove the absence of point spectrum for the critical almost Mathieu operator, H_1, for all frequencies and all but countably many phases. The proof can be found in [7] and should be covered entirely, taking advantage of facts already established. It should be noted that together with results on the measure of the spectrum, this achieves purely singular continuous spectrum of the critical almost Mathieu operator for all but a countable set of phases. Earlier non-dynamical work of Delyon [36] should be mentioned here.

The talk should conclude with results establishing necessity of arithmetic conditions for the localization theorem: absence of point spectrum for Liouville α (see [45, 16] and also [31, Theorem 10.4]) or resonant ω ([58]). Both short arguments should be presented as completely as possible.

14. **Measure of the spectrum of the almost Mathieu operator**
In [1], Aubry-André conjectured that the measure of the spectrum for the almost Mathieu operator ($f(\omega) = 2\lambda \cos(2\pi(\omega + n\alpha))$) is given by $4|1 - |\lambda||$ for all irrational α and all real λ.

Avron-van Mouche-Simon [15] established the first partial results on the conjecture for non-critical values of λ, $|\lambda| \neq 1$. The key idea was to establish measure estimates for the sets $\sigma_+(p/q) := \bigcup_\omega \sigma(p/q, \omega)$ and $\sigma_-(p/q) := \bigcap_\omega \sigma(p/q, \omega)$, for rationals p/q approximating α. Here, $\sigma(p/q, \omega)$ denotes the phase-dependent spectrum of the periodic operators obtained when replacing α by p/q. The talk should present the key ideas of [15], starting with the important Chambers formula for the discriminant (Proposition 3.1 in [15]). Crucial for passing to the limit of irrational α was 1/2-Hölder continuity of the map $\beta \mapsto \sigma_+(\beta)$, $\beta \in \mathbb{T}$, in the Hausdorff metric, proven in Section 7 of [15]. The latter result should at least be mentioned.

Based on this continuity result, Last [63] verified the Aubry-André conjecture for all λ, including the critical value, and almost every α. The underlying arguments allowing one to pass to the limit of irrational α should be given (see Section 4 in [63]). In particular, the relation between the above mentioned 1/2-Hölder continuity of the map $\beta \mapsto \sigma_+(\beta)$ and the full measure condition on α in [63] should be explained. Later results, allowing one to remove the full measure condition on α for non-critical and critical values of λ were obtained in [53] and [14], resolving Simon's Problem 5 in [74]. Both these results should be mentioned.

A recent stronger statement on almost everywhere convergence of characteristic functions of both spectra and absolutely continuous spectra upon rational approximation [57] (the main theorem there) should be briefly discussed. It should be mentioned that such convergence holds for general analytic potentials.

15. **Non-perturbative localization for analytic potentials I**

16. **Non-perturbative localization for analytic potentials II**
The goal of talks 15–16 is to present a non-perturbative result on localization for the general analytic case [24]. The large deviation theorem from talks 5–6 can be used as a starting point and given without proof. From this, one should derive the basic Green's function estimate (e.g., Proposition 7.19 in [21].) The necessary facts from the theory of semi-algebraic sets should be stated, following either [24] or Chapter 9 of [21]. The rest of the proof should be done more or less fully, following either [24] or Chapter 10 of [21]. It will be fine for the speaker to concentrate on the one-frequency case, only briefly mentioning changes needed for the multi-frequency case. If time permits, the proof of dynamical localization ([28] or part 4 in Chapter 10 in [21]) should be presented.

17. **Absence of non-perturbative results in higher dimensions**

The goal of this talk is to show that non-perturbative results do not hold in general, neither for multi-dimensional operators, nor for absolutely continuous spectrum at small disorder in the multi-frequency case. This is material from [20] (with some technique developed in [19]), also partially presented in Chapter 13 of [21]. The talk should start out by stating without proof the multi-dimensional long-range localization theorem [30] and explaining a simple version of Aubry duality that allows one to deduce continuous spectrum for the dual model from localization, without worrying about absolute continuity. This should follow the proof of Theorem 11.6 in [21], adapted to the multi-dimensional setting. Then a sketch of the proof of Theorem 13.3 [21] should be given, thus establishing that for a given small λ, localization for some frequencies coexists with continuous spectrum for others. Then it should be mentioned that duality can be applied to the localization result again to obtain such coexistence for the multi-dimensional model at high coupling. In this regard, the multi-dimensional localization theorem of [23] or [25] should be stated.

18. **Global theory of one-frequency Schrödinger operators I**

19. **Global theory of one-frequency Schrödinger operators II**

20. **Global theory of one-frequency Schrödinger operators III**
While the previous talks and the upcoming talk 21 describe (in particular) how a good understanding of Schrödinger operators with analytic potential can be achieved for either large or small potentials (in one case by making use of the positivity of the Lyapunov exponent, and in the other case by exploring a KAM scheme), they do not shed light on the behavior in the intermediate range. The talks 18–20 should explain the recent advances towards the description of the behavior of the general operator in the one-frequency case, obtained in [3] and [4].

The beginning must be more conceptual. It should initially discuss how the Schrödinger cocycles can be formally separated into three classes (supercritical/critical/subcritical) according to the behavior of the Lyapunov exponent, and how a global picture can in principle be obtained from a geometric description of the locus of criticality in the infinite dimensional parameter space as a thin interface between stable regimes, together with a dynamical understanding of subcritical cocycles. The role of the Almost Reducibility Conjecture (ARC) in accomplishing the latter part of the program should be described. The results on the geometry of the critical locus obtained in [3] and [4] should then be formulated, followed by a deduction of the Spectral Dichotomy (assuming the ARC), and a discussion of its consequences. It should then be discussed how the desired geometric understanding of the critical locus seems at odds with the relative lack of regularity of the Lyapunov exponent, and that this can be reconciled through the notion of stratified regularity.

Detailed proofs of the results of [3] and [4] should then be given.

The three speakers will be encouraged to coordinate the break-up of the material and the preparation of their lectures.

21. **Quantitative duality**
The goal of this talk is to describe results of [13] on quasiperiodic Schrödinger operators with small analytic potential, in the case of one frequency. The initial focus should be on the "quantitative duality" proof of almost reducibility as a consequence of a statement of "almost localization" of eigenfunctions for the dual model, under a Diophantine condition. The almost localization statement should be stated precisely, but does not need to be proved (a proof will have been sketched in talk 12). It should be shown how to deduce directly the 1/2-Hölder regularity of the integrated density of states from the dynamical estimates. It should be noted that for the almost Mathieu operator, the smallness condition turns out to be sharp, and a brief discussion of the application of quantitative duality to the "Dry Ten Martini Problem" should follow.

22. **Absolutely continuous spectrum in the subcritical regime**
The goal of this talk is to discuss the problem of establishing absolutely continuous spectrum without exceptional frequencies or phases, throughout the entire subcritical regime. The focus is on the almost Mathieu operator, for which this is now known. Initially, the treatment of the exponentially Liouville case by the method of periodic approximation should be covered, following [10] and [6]. This should be followed by a discussion of the subexponentially Diophantine case, with the direct proof of absolute continuous spectrum via quantitative

duality given in [6]. For this latter part, the speaker will be encouraged to coordinate the material with the speaker of talk 21.

23. **Progress towards the Almost Reducibility Conjecture**
The goal of this talk is to present a proof of the Almost Reducibility Conjecture (ARC) for exponentially Liouville frequencies, following [5]. Two important consequences (obtained by combination with results coming from quantitative duality) should be highlighted: almost reducibility of one-frequency cocycles close to constant, and global stability of almost reducibility.

24. **Beyond analyticity I: C^0-generic singular continuous spectrum**
This is the first talk in a series of three that present results for sampling functions f that are not necessarily analytic. Recall that $f : \mathbb{T}^k \to \mathbb{R}$ needs to be at least continuous so that $V(n) = f(\omega+n\alpha)$ is almost periodic in the Bochner/Bohr-sense. Thus, $C^0(\mathbb{T}^k, \mathbb{R})$ is a class of sampling functions whose study is well-motivated from this perspective. Within this class, the generic behavior is quite well understood. This talk will investigate the spectral type. The final result, whose proof should be presented in as much detail as possible, is the following. For every minimal translation $T : \mathbb{T}^k \to \mathbb{T}^k$, $\omega \mapsto \omega + \alpha$, there is a dense G_δ set $\mathcal{SC} \subseteq C^0(\mathbb{T}^k, \mathbb{R})$ such that for every $f \in \mathcal{SC}$ and Lebesgue almost every $\omega \in \mathbb{T}^k$, the operator $H = \Delta + f(\omega+n\alpha)$ has purely singular continuous spectrum. This result is obtained by generically excluding absolutely continuous spectrum and point spectrum separately, and then by taking the intersection of these two dense G_δ sets of sampling functions f. Generic absence of absolutely continuous spectrum is proved in [9] and generic absence of point spectrum is proved in [18].

    Both proofs rely on the fact that the absence of absolutely continuous spectrum and the absence of point spectrum are easier to prove for discontinuous f. Regarding the absence of absolutely continuous spectrum, this is an immediate consequence of Kotani theory (cf. the third talk) for sampling functions f taking only finite many values, as was observed by Kotani in a short 1989 follow-up paper to his earlier work [61]. An extension to discontinuous functions can be found in [33]. The proof given in [9] indeed uses approximation of continuous functions by discontinuous functions in the L^1 topology. Regarding the absence of point spectrum, this has been studied in many papers for functions taking only finitely many values (see, e.g., [37]), and the proof given in [18] borrows some ideas from the earlier work, in particular the use of the Gordon lemma (cf the end of the talk 13).

25. **Beyond analyticity II: C^0-generic Cantor spectrum**
This talk is in the same spirit as the previous one in that it shows that a certain spectral property holds generically in the continuous sampling functions. Here one is concerned with Cantor spectrum, that is, the fact that the spectrum is nowhere dense. The description of the complement of the spectrum in terms of uniform hyperbolicity of the associated cocycles, which was explained in the second talk, is crucial for this result. From this perspective, showing that the complement of the spectrum is dense comes down to proving that uniform hyperbolicity is dense in a certain class of cocycles. Results of this kind were obtained in [8]. The talk should present this paper, discuss the approach to proving Cantor spectrum via denseness of uniform hyperbolicity, and then prove that for every minimal translation $T : \mathbb{T}^k \to \mathbb{T}^k$, $\omega \mapsto \omega + \alpha$, there is a dense G_δ set $\mathcal{C} \subseteq C^0(\mathbb{T}^k, \mathbb{R})$ such that for every $f \in \mathcal{C}$, the (ω-independent) spectrum of the operator $H = \Delta + f(\omega + n\alpha)$ is nowhere dense.

26. **Beyond analyticity III: Examples of discontinuity of the Lyapunov exponent**
The purpose of this talk is to present recent results on discontinuity of the Lyapunov exponent, illustrating the optimality of the continuity results for analytic potentials. It has been known for a while that the Lyapunov exponent is in general discontinuous for continuous f. In this context, Furman [40] proved that for irrational α, the Lyapunov exponent is discontinuous at every non-uniformly hyperbolic cocycle. More recently, in [17], Bochi proved that cocycles with zero Lyapunov exponent are dense in $C^0(\mathbb{T}, \mathrm{SL}(2, \mathbb{R}))$. Until very recently, it had however been unclear what happens in $C^l(\mathbb{T}, \mathrm{SL}(2, \mathbb{R}))$, $1 \leq l \leq \infty$. The missing link is provided by Wang and You [78], who have recently constructed counter-examples proving discontinuity of the Lyapunov exponent in all of these intermediate spaces, including C^∞. The construction is based on Young's method to prove positivity of the Lyapunov exponent [79].

First, the key ideas of [79] should be explained briefly and connections to the avalanche principle [43] mentioned earlier should be discussed.

Next, the ideas of Wang and You's construction should be explained in more detail. The counter-examples are of the form

$$D_l := \begin{pmatrix} \lambda & 0 \\ 0 & \lambda^{-1} \end{pmatrix} R_\phi,$$

where R_ϕ is the rotation by ϕ, and $\phi \in C^l(\mathbb{T}, \mathbb{R})$. In [78, Sections 2–3], the method of Young [79] is adapted to construct a sequence A_n, which approximates D_l in $C^l(\mathbb{T}, \mathrm{SL}(2, \mathbb{R}))$ and possesses some finite hyperbolicity property (see Definition 2.1 in [78]). This is shown to give rise to a lower bound of the Lyapunov exponent, $L \geq (1-\delta)\lambda$, for some $0 < \delta < 1$. On the other hand, in [78, Sections 4–5], the sequence A_n is modified to $\tilde{A}_n$, still satisfying $\tilde{A}_n \to D_l$ in $\mathcal{C}^l$, however so that $L \leq (1-\epsilon)\lambda$ with $\delta < \epsilon < 1$. The talk should mainly focus on [78, Sections 1–5]. The necessary modifications for the $\mathcal{C}^\infty$ case, considered in [78, Section 6], may only be mentioned.

Wang and You's examples are constructed for α associated with a bounded sequence of elements in its continued fraction expansion. It has been shown in in talks 5–6 that, given a Schrödinger cocycle with analytic f, the Lyapunov exponent is continuous in the frequency at every irrational α [27]. Continuity, however, in general fails at rational α, even in the analytic category. A simple counterexample can for example be found in [3, Remark 5] and should conclude the presentation.

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Participation:

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by February 20, 2012 at the latest. We would like you to name three of the talks listed above which you are willing to give.

You will be informed shortly after the deadline if your participation is possible and about the title of your talk. The Arbeitsgemeinschaft will take place at Mathematisches Forschungsinstitut Oberwolfach, Lorenzenhof, 77709 Oberwolfach-Walke, Germany. The institute offers accommodation free of charge to the participants. Travel expenses cannot be covered. Further information will be given to the participants after the deadline.