Oberwolfach Seminar on Surgery Theory

Date

27 May - 2 June 2012

Organizers

Diarmuid Crowley, Bonn
Tibor Macko, Bonn
Andrew Ranicki, Edinburgh

Programme

See also the Manifold Atlas website:
http://www.map.mpim-bonn.mpg.de/Category:Oberwolfach_Surgery_Seminar_2012

A topological manifold is a space with Poincaré duality. The surgery theory of manifolds addresses two fundamental questions on the relationship between the topology and the homotopy theory of manifolds:

1. (Manifold existence) Is a space with Poincaré duality homotopy equivalent to a topological manifold?
2. (Manifold uniqueness) Is a homotopy equivalence of topological manifolds homotopic to a homeomorphism?

These questions have negative answers in general but the theory provides complete obstructions, at least in the high dimensions >4. Surgery theory arises out of a delicate combination of sophisticated techniques in both algebra and geometry. The aim of this seminar is to explain these techniques and their interaction by covering the following topics:

a) The geometric surgery exact sequence due to Browder-Novikov-Sullivan-Wall,

b) The algebraic surgery exact sequence due to Ranicki,

c) The relationship between the two sequences via the total surgery obstruction due to Ranicki.

On the geometric side the key concept is the notion of a degree one normal map from manifolds to Poincaré complexes. On the algebraic side the key concept is the L-theory of quadratic forms over a ring with involution, which can be regarded as the algebraic cobordism theory of quadratic chain complexes with Poincaré duality. The Wall surgery obstruction of a normal map can be viewed as the cobordism class of a quadratic Poincaré chain complex defined over the fundamental group ring. The algebraic surgery exact sequence requires the cobordism theory of chain complexes with more complicated structures. The structure can be symmetric, quadratic or hyperquadratic, defined in an ``additive category with chain duality''.

The above concepts will be collected together and used to prove the following: a Poincaré duality space of dimension >4 is homotopy equivalent to a manifold if and only if a certain algebraic quadratic Poincaré complex in null-bordant in an appropriate category of structured chain complexes.

The key invariants in the passage from geometry to algebra are certain “signatures”, which are elements in the L-groups arising in the geometric problems of surgery theory.

Participants of this seminar will gain an understanding of the above notions and how they can be used solve the fundamental problems mentioned above.

Prerequisites

The prerequisites for the seminar are a solid knowledge of the basics of differential and algebraic topology, meaning: manifolds, Poincaré duality, bundles, cobordism, transversality, generalized homology and cohomology, homotopy groups.

Participants should be familiar with the ideas covered in the first 7 chapters of the book

- A. Ranicki: Algebraic and Geometric Surgery.
  Oxford Mathematical Monograph, OUP (2002)

However material from sections 2.2., 4.2, 5.4, 7.3 will be covered during the seminar.

In addition participants should be familiar with the basics of spectra in stable homotopy theory. A good reference here is

- A. Hatcher: Algebraic Topology, section 4.F.
  CUP (2002), http://www.math.cornell.edu/~hatcher/AT/ATpage.html

References for the seminar

The following sources contain the material to be covered in the lectures:

- C.T.C. Wall: Surgery on compact manifolds
  2nd edition, Mathematical Surveys and Monographs 69, AMS (1999)
- W. Lück: A basic introduction to surgery theory
  in Topology of High-Dimensional Manifolds, ICTP, Trieste (2002)
  http://users.ictp.trieste.it/~pub_off/lectures/lns009/Lueck/ictp.pdf
- Ranicki: Algebraic L-theory and Topological Manifolds
  Cambridge Tract 102, CUP (1992)
- Ranicki: The total surgery obstruction,
  Proc. 1978 Arhus, Springer Lecture Notes 763, 275--316 (1979)
- P. Kuehl, T. Macko, A. Mole: The total surgery obstruction revisited
  http://arXiv:1104.5092v2 (2011)

Web sources

There are also many online sources. We suggest the following -- the first one contains a large collection of various materials related to surgery theory.

http://www.maths.ed.ac.uk/~aar/ (Edinburgh website of Andrew Ranicki)

http://www.maths.ed.ac.uk/~s1057008//surgerygroup/ (Edinburgh Surgery Theory Study Group)

http://www.map.him.uni-bonn.de/Category:Surgery (Manifold Atlas Project, Bonn)