OBERWOLFACH SEMINAR 2012 ON DISPERSIVE EQUATIONS

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1. Outline

The scope of the theory of nonlinear waves is very broad. It includes problems ranging from applied sciences to physics and even to algebraic geometry.

Despite encompassing a large class of equations, there are recurrent themes: dispersion, solitons and their stability, blow-up and scattering. In this workshop we want to pursue different coordinated threads: The nonlinear Schrödinger equation and generalized KdV, critical wave and Schrödinger equations, and geometric dispersive equations.

The goal of these lectures is to introduce the problems and equations, and to describe an array of ideas and techniques used in their study, leading up to the current results and open problems.

We aim at participants with a basic knowledge of real and functional analysis, as well as either partial differential equations or harmonic analysis at a level that can be reasonably expected from a beginning graduate student in one of these areas.

There will be three lectures at 90 minutes per day, two in the morning and one in the afternoon, plus an additional forum for discussions. Koch's lectures will focus on the nonlinear Schrödinger equation and the Korteweg-de-Vries equation, Tataru will deal with geometric dispersive equations, and Visan with the energy critical nonlinear Schrödinger and wave equation.

The first lectures will be devoted to basic notions and tools: Dispersive estimates, Strichartz estimates, local smoothing, bilinear estimates and adapted function spaces, existence and properties of bound states and schemes for the construction of solutions.

The nonlinear Schrödinger equations and the (generalized) Korteweg-de-Vries equation exhibit a fascinating and rich structure. They provide the simplest but nevertheless nontrivial context for many important techniques, and the simplest framework for open challenging questions.

Within the field of nonlinear dispersive equations, a special role is played by the so-called geometric dispersive equations, which arise from the standard Lagrangian or Hamiltonian but applied in a geometric context. The two simplest examples of such equations are the wave map and Schrödinger map equations.

Over the last decade, the induction on energy paradigm has grown into a powerful new tool for the large data analysis of partial differential evolution equations. It continues to develop in both depth and breadth and has already proven useful over a wide range of equations, from semilinear wave and Schrödinger equations to fluid equations and geometric flows. While enjoying many parallels to the calculus of variations (and often using its terminology), this new approach requires an independent set of techniques.