# Oberwolfach Seminar 1242b:
# Algebraic Groups and Patching

*Organizers*:
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*Dates*: Oct 14 - Oct 20, 2012

*Description*:
Patching is a method that in the past was mostly used in Galois theory, to construct Galois extensions and to study absolute Galois groups. Recently, a new version of the method was introduced, which works over fields (rather than over rings) and turns out to be more suitable for applications in other areas of algebra. In particular, it can be applied to prove local-global principles for homogeneous spaces of linear algebraic groups over function fields of arithmetic curves; this in turn has consequences for quadratic forms and central simple algebras. It is expected that the method will have many more applications, and thus should be of interest to people starting a career in algebra and related areas.

The seminar will provide an introduction to the following relevant areas:

- Linear algebraic groups and homogeneous spaces

- Arithmetic curves

- Patching

- Quadratic forms and field invariants.

There will be lecture series on these four subjects, which will draw on each other more and more over the course of the week as the themes intertwine and applications are introduced.

*Prerequisites* are a strong background in algebra and some basic algebraic geometry (e.g. varieties over algebraically closed fields).

*Suggested Reading*: The items below may be useful in advance of the seminar, to provide background and context.

- On quadratic forms, *either* of these:

  - Tsit-Yuen Lam. Introduction to Quadratic Forms Over Fields. American Mathematical Society, 2005. (Especially the first 40 pages.)
  - Winfried Scharlau. Quadratic and Hermitian forms. Grundlehren der Mathematischen Wissenschaften, vol. 270. Springer-Verlag, 1985. (Especially the first 40 pages.)

- On linear algebraic groups:

  - Tamás Szamuely. Lectures on linear algebraic groups. 65 pages. Notes at `http://www.renyi.hu/~szamuely/lag.pdf`. (Especially sections 2, 3, 10, and 16-19.)
  - Fiona Murnaghan. Linear Algebraic Groups. In: Clay Mathematics Proceedings, vol. 4, 2005, pages 379–391. Available at `http://www.math.toronto.edu/fiona/courses/algp.pdf`. (Especially sections 4-7.)

- On patching and arithmetic curves:

  - David Harbater. Patching over fields (joint work with Julia Hartmann). In Oberwolfach Reports No. 26/2007 (vol. 4, issue 2), Arithmetic and Differential Galois Groups, pp. 1487–1489.
  - Julia Hartmann. A local-global principle for homogeneous spaces and applications (joint work with David Harbater and Daniel Krashen). In Oberwolfach Reports No. 05/2009 (vol. 6, issue 1), The Arithmetic of Fields, pp. 309–312.