

Oberwolfach Seminar: Random Networks

Date

May 19-24, 2013.

Organizers

Shankar Bhamidi (University of North Carolina, Chapel Hill, USA)

Remco van der Hofstad (Eindhoven University of Technology, The Netherlands)

Frank den Hollander (Leiden University, The Netherlands)

Asaf Nachmias (University of British Columbia, Vancouver, Canada)

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Programme

An introduction to recent developments in the area of random networks, with four mini-courses:

- (1) Dynamic random graphs (Shankar Bhamidi)
- (2) Percolation in high dimensions (Asaf Nachmias)
- (3) Routing on random networks (Remco van der Hofstad)
- (4) Random walks in dynamic random environments (Frank den Hollander)

Each mini-course consists of 2 lectures (1.5 hours each), 2 proof sessions (1 hour each) and 1 problem session (45 minutes). Sometimes complete proofs are given, sometimes only highlights of the ideas behind them.

Lecture (1) starts by reviewing the beautiful proof by David Aldous of the $2/3$ scaling exponent of the largest cluster size in the critical Erdős-Rényi random graph, and then moves to the study of more recent results on dynamic models with limited choice and the nature of emergence of the giant component. Time permitting we will also discuss recent work and conjectured results on “explosive percolation” as well as dynamic models of tree growth with choice.

Lecture (2) describes critical percolation on various high-dimensional graphs, such as lattices in dimension larger than six, Cayley graphs of finitely generated non-amenable groups, and also on finite graphs, such as the complete graph, the Hamming hypercube and expander families. The fractal geometry of high-dimensional percolation will be studied, with focus on the uniqueness of infinite clusters, critical exponents, scaling windows and random walks on percolation clusters.

Lecture (3) discusses the effect of random weights on the metric properties of random graphs. The weights can have the interpretation of the cost of using the edge, or the time needed to traverse it. The number of edges and the weight of smallest-weight paths will be studied and how these relate to the underlying topology of the random graph in question.

Lecture (4) describes random walks in dynamic random environments. The main question is to what extent the dynamics affects the anomalous scaling that is known to occur in certain static environments. The dynamics may “wash away” the effect of “local traps” in the environment that slow the random walk. Especially dynamics driven by interacting particle systems will be considered.

Preparatory reading

For mini-course (1): the books by Harris (1963) or Athrey and Ney (1972), and the 1997 paper of Aldous on critical random graphs.

For mini-course (2): “Percolation” by G. Grimmett.

For mini-course (3): the book by Durrett (2006) or the lecture notes by van der Hofstad at <http://www.win.tue.nl/~rhofstad/NotesRGCN.pdf>.

For mini-course (4): the Saint-Flour lectures by Zeitouni (2001).

Deadline for applications

April 1, 2013