OBERWOLFACH SEMINAR: CLUSTER ALGEBRAS AND REPRESENTATION THEORY

1. Dates

October 13-19, 2013

2. Organizers

- Christof Geiß (UNAM, Mexico),
- David Hernandez (Université Paris Diderot),
- Bernhard Keller (Université Paris Diderot),
- Bernard Leclerc (Université de Caen).

3. Programme

In this Oberwolfach seminar, the participants will be introduced to cluster algebras, quantum cluster algebras and their links to

- representations of quivers and finite-dimensional algebras (additive categorification) and
- representations of quantum affine algebras (monoidal categorification).

The overall aim is to show how these links allow one to approach some of Fomin–Zelevinsky's conjectures on cluster algebras, canonical bases and total positivity. There will be four lecture series, each of approximately four hours, on the following subjects

1. Introduction to cluster algebras
2. Cluster algebras and quiver representations
3. Cluster algebras and preprojective algebras
4. From quantum affine algebras to cluster algebras

In addition, there will be sessions devoted to exercises, discussion, and practical work on the computer.

4. Preparatory reading

The participants will be expected to be familiar with basic notions from Lie theory, notably the finite root systems and the structure of the finite-dimensional complex Lie algebras, cf. [7] [4]. Previous knowledge of cluster algebras is not necessary but may of course be helpful, cf. [9] and the other survey articles below.

References

[1] Sergey Fomin, *Total positivity and cluster algebras*, Proceedings of the International Congress of Mathematicians. Volume II (New Delhi), Hindustan Book Agency, 2010, pp. 125–145.

[2] Sergey Fomin and Andrei Zelevinsky, *Cluster algebras: notes for the CDM-03 conference*, Current developments in mathematics, 2003, Int. Press, Somerville, MA, 2003, pp. 1–34.

[3] Christof Geiß, Bernard Leclerc, and Jan Schröer, *Preprojective algebras and cluster algebras*, Trends in representation theory of algebras and related topics, EMS Ser. Congr. Rep., Eur. Math. Soc., Zürich, 2008, pp. 253–283.

[4] James E. Humphreys, *Introduction to Lie algebras and representation theory*, Graduate Texts in Mathematics, vol. 9, Springer–Verlag, 1972.

[5] Bernhard Keller, *Cluster algebras, quiver representations and triangulated categories*, Triangulated categories (Thorsten Holm, Peter Jørgensen, and Raphaël Rouquier, eds.), London Mathematical Society Lecture Note Series, vol. 375, Cambridge University Press, 2010, pp. 76–160.

[6] Bernard Leclerc, *Cluster algebras and representation theory*, Proceedings of the International Congress of Mathematicians. Volume IV (New Delhi), Hindustan Book Agency, 2010, pp. 2471–2488.

[7] Jean-Pierre Serre, *Algèbres de Lie semi-simples complexes*, W. A. Benjamin, inc., New York-Amsterdam, 1966.

[8] Andrei Zelevinsky, *Cluster algebras: origins, results and conjectures*, Advances in algebra towards millennium problems, SAS Int. Publ., Delhi, 2005, pp. 85–105.

[9] ———, *What is a cluster algebra?*, Notices of the A.M.S. **54** (2007), no. 11, 1494–1495.