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Stochastic Analysis in Finance and Insurance

Organised by
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4 May – 10 May 2014

Abstract. This workshop brought together leading experts and a large number of younger researchers in stochastic analysis and mathematical finance from all over the world. During a highly intense week, participants exchanged many ideas during talks and discussions, and laid foundations for new collaborations and further developments in the field.

Mathematics Subject Classification (2010): 91GXX, 60XX.

Introduction by the Organisers

The workshop *Stochastic Analysis in Finance and Insurance*, organised by René Carmona (Princeton), Martin Schweizer (Zürich) and Nizar Touzi (Paris), was held May 4 – May 10, 2014. The meeting had a total of 50 participants from all over the world with a deliberately chosen mix of more experienced researchers and many younger participants.

During the five days, there were a total of 24 talks with many lively interactions and discussions. In addition, there were three blocks of short communications, as explained below.

The topics presented in the talks covered a very wide spectrum. Some of the major developments included a focus on optimal transport problems in connection with robust pricing and hedging, microstructure and other modelling issues, aspects of numerical computations in high-dimensional systems, and as always a number of foundational questions. To stimulate discussions and maximise interactions, talks were deliberately not organised into groups by major topics. A short overview of the talks given day by day looks as follows.

Freddy Delbaen in the first talk of the workshop presented some new structure results on monetary utility functions (or equivalently risk measures) with a view towards requirements imposed or discussed by regulators and practitioners. *Miklós Rásonyi* presented new results on hedging, arbitrage and optimality for portfolio choice in financial markets with superlinear frictions. *Jan Obłoj* discussed a robust approach to pricing and hedging and showed how trading restrictions can lead there to the emergence of financial bubbles. *Josef Teichmann* gave a new convergence result for the Emery topology on semimartingales and explained how this could be used to give a streamlined and structured proof of the fundamental theorem of asset pricing. *Chris Rogers* proposed a very simple Bayesian approach to inference and action in financial econometrics which deals in a simple and unified way with a number of otherwise not well addressed issues. Finally, *Olivier Guéant* combined ideas from optimal control and option pricing to account for effects arising from execution costs and market impact.

Jean Jacod started the second day with an overview of backward stochastic differential equations (BSDEs) driven by a multivariate point process, and showed how such equations can be solved in a pathwise manner by a kind of backward recursion. *Yuri Kabanov* presented a new result on tradable local martingale deflators which could be viewed as a weak formulation for a fundamental theorem of asset pricing in frictionless markets. *Sebastian Herrmann* proposed a simple and tractable model for optimal investment in a setting where the underlying asset price process has a bubble, described by a combination of a Black–Scholes type model with a single-jump local martingale. He showed how this problem could be solved fairly explicitly and gave rise to a number of interesting phenomena. *Nicole El Karoui* used stochastic progressive utilities to describe and analyse models for long-term decision making (e.g. for question about mortality or pension funds) in a stochastic environment.

Tuesday afternoon was devoted to the traditional excursion to Sankt Roman; this was moved forward by one day from the usual Wednesday afternoon schedule in view of the weather forecast for the second half of the week (and this decision turned out to have been very wise).

On Wednesday, *Mete Soner* presented a continuous-time duality for robust hedging in models where price processes are assumed to be RCLL (or càdlàg). *Christoph Czichowsky* explained how optional strong supermartingales arise in the treatment via duality techniques of the problem of optimal portfolio choice under transaction costs. In addition, there were a number of short communications in a format which was introduced (and judged to be very successful) in an earlier meeting. Each presenter had 10 minutes to explain his result, which were then followed by 5 minutes of questions and discussion. This idea of explaining in a nutshell some current problems or results again met with great success; the list of speakers for giving a short presentation very quickly grew to a total of 15 names, and the corresponding talks were scheduled on Wednesday morning, Thursday morning and Thursday afternoon. Wednesday afternoon continued with *Terry Lyons* who showed how one could use sophisticated mathematical tools to extract in a very systematic way information from sequential data, without any a priori knowledge of the data or even the information contained in it. Finally, *François Delarue* gave a derivation of the master equation arising in mean field games and sketched a way to prove the existence of a classical solution to this forward-backward system of PDEs.

Thursday started with *David Hobson* who presented a number of results on fake diffusions, i.e. martingales whose univariate marginals (which in financial terms are determined by the prices of plain vanilla options) match a given family of probability measures with certain properties (again dictated by financial requirements). *Tom Hurd* presented the Gai–Kapadia model for systemic risk and gave some formal computations for obtaining the default probability distribution after a number of default cascade steps. A second block of short communications followed, leading again to intense discussions that continued into the afternoon and in the evenings. *Dan Lacker* then gave some new results on mean field games with a common noise term, which appear in (approximate) equilibria of symmetric stochastic differential games. *Umut Çetin* presented some recent developments in the microstructure models of Kyle and Glosten–Milgrom. The day was closed by a third block of short communications.

On the last day, *Xiaolu Tan* discussed martingale optimal transport with constraints on the one-dimensional marginals and explained some new ideas on how to connect such problems to Skorohod embeddings. *Arturo Kohatsu-Higa* gave a probabilistic representation of the parametrix method and explained how this could be used for numerical computations e.g. of prices for exotic options. *Shigeo Kusuoka* gave an overview of Monte Carlo methods for pricing Bermuda derivatives, highlighting in particular advantages and shortcomings of some alternative but competing approaches. *Monique Jeanblanc* presented a number of examples to illustrate some very subtle issues arising in the study of arbitrage theory in connection with an enlargement of filtration, as for example needed in the context of credit risk. *Dylan Possamaï* studied the properties of the solutions of backward stochastic differential equations (BSDEs) with a view towards obtaining in particular the existence of a density for both the state as well as the integrand processes. Finally, *Mathieu Rosenbaum* gave a number of limit theorems for nearly unstable Hawkes processes that appear in the context of microstructure modelling in financial markets.

Like in the workshop three years before, there were an enormous number of discussions, interactions and exchanges. Everyone felt privileged to be able to spend a highly productive and creative week at the unique place that has been created in Oberwolfach, and to profit from the excellent infrastructure, support and scientific environment. In particular, the younger participants and the first-time visitors to Oberwolfach unanimously said that the actual experience of the workshop and the overall scientific atmosphere still exceeded their already high anticipations.

As organisers and on behalf of all participants, we want to express, like in the last workshop three years earlier, our gratitude to the Mathematisches Forschungsinstitut Oberwolfach for giving us the opportunity of having this very successful workshop. We hope that we shall be able to come back at some time in the future.

Acknowledgement: The MFO and the workshop organizers would like to thank the National Science Foundation for supporting the participation of junior researchers in the workshop by the grant DMS-1049268, "US Junior Oberwolfach Fellows".

René Carmona
Martin Schweizer
Nizar Touzi

Workshop: Stochastic Analysis in Finance and Insurance

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Abstracts

Recent developments on the microstructure models of Kyle and Glosten–Milgrom

Umut Çetin

The seminal works of Kyle [1] and Glosten and Milgrom [2] aim at understanding the formation of prices in a financial market with asymmetrically informed traders. In an equilibrium framework they study how the private informations of the agents disseminate to the market and affect the market liquidity, and how bid-ask spreads emerge. The key to the analysis is the characterisation of the equilibrium strategies, if they exist, of the informed traders. From a mathematical point of view, optimal strategies of the informed traders correspond to the constructions of certain bridge processes in a Markovian framework. It turns out that the existence of equilibrium depends on a positive answer to the following question: Suppose that $Z_t = Z_0 + \int_0^t \sigma(s)a(Z_s)\,dB_s$ is a diffusion, where a is a sufficiently regular function and σ is deterministic. Let W be another Brownian motion independent of B. Can we find a function $\alpha : [0,1]\times\mathbb{R}\times\mathbb{R}$ such that the following three conditions hold?

(1) There exists a strong solution on $[0,1)$ to

$$Y_t = \int_0^t a(Y_s)\,dB_s + \int_0^t \alpha(s, Y_s, Z_s)\,ds.$$

(2) $Y_t = \int_0^t a(Y_s)\,dB_s^Y$, where B^Y is a Brownian motion with respect to the natural filtration of Y.

(3) $\lim_{t\to 1} Y_t = Z_1$.

In this talk we discuss the solution of the above problem and its further extensions along with the applications to financial equilibrium with asymmetrically informed agents.

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Strong supermartingales and portfolio optimisation under transaction costs

Christoph Czichowsky
(joint work with Walter Schachermayer)

Portfolio optimisation is one of the classical problems in mathematical finance and gives answers to the question how to invest optimally into a financial market. In this talk (that is based on [3]), we understand this problem as the one to maximise the expected utility from terminal wealth in a financial market with proportional transaction costs. Trading under transaction costs means here that whenever one buys stocks, one has to pay a higher ask price $(1+\lambda)S_t$, but only receives a lower bid price $(1-\lambda)S_t$ when selling them, where $\lambda \in (0,1)$ denotes the size of transaction costs.

Following the seminal works [1, 4] we investigate this concave maximisation problem by convex duality. The corresponding dual variables are then all pairs $(Q, \widetilde{S})$ of frictionless arbitrage-free price processes $\widetilde{S} = (\widetilde{S}_t)_{0 \leq t \leq T}$ evolving in the bid-ask spread $[(1-\lambda)S, (1+\lambda)S]$, and equivalent martingale measures Q for those. As our results allow to model the mid price $S = (S_t)_{0 \leq t \leq T}$ by a general càdlàg (right-continuous with left limits) stochastic process, our findings are surprisingly different from what can be expected from the frictionless case [4] or the case [1] of continuous price processes under transaction costs. The reason for this is that one needs a different limit to ensure the existence of a dual optimiser. For this, we have identified the notion of convergence in probability at all finite stopping times as a suitable topology to work with. In [2], we provide an extension of Komlós' subsequence theorem for those frictionless arbitrage-free prices, their left limits, and pathwise Riemann–Stieltjes integrals that works directly on the level of stochastic processes in that topology and price. Here the limits of the price processes and their left limits turn out to be an optional and a predictable strong supermartingale, respectively, which are two classical notions from the general theory of stochastic processes. Using these results as a substitute for compactness allows us to establish the existence of a dual optimiser. To obtain the interpretation of the latter as a generalised shadow price process for which the optimal trading strategy in the original market with transaction can be realised by frictionless trading, we need to extend the dual optimiser to a pair of two processes. One is needed for the approximating frictionless price process and another one for the result of trading before predictable events that is given by the limit of the left limits of the approximating frictionless price processes. Moreover, we give two examples that illustrate how and why new phenomena arise and that the generalised shadow price has to be of that form.

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The master equation for mean field games

François Delarue
(joint work with René Carmona, Jean-François Chassagneux, Dan Crisan)

Mean field games theory was introduced by Lasry and Lions in 2006 in order to describe asymptotic Nash equilibria among a population of players interacting with one another in a mean-field way and submitted to cost constraints; see [1]. Asymptotically, equilibria may be described by means of a system made of a forward Fokker–Planck equation and a backward Hamilton–Jacobi–Bellman equation. It turns out that whenever uniqueness of the equilibria holds, this forward-backward system admits a decoupling field solving a PDE on the Wasserstein space of probability measures. The purpose of this talk is to show how to derive the shape of this equation, see [2], and to indicate a way to prove the existence of a classical solution by means of stochastic flows, see [3].

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Monetary utility functions with convex level sets

Freddy Delbaen
(joint work with F. Bellini, V. Bignozzi, J. Ziegel)

Monetary utility functions are — except for the expected value — not of von Neumann–Morgenstern type. In case the utility function has convex level sets in the set of probability measures on the real line, we can give some characterisation that comes close to the vN–M form. For coherent utility functions, this was solved by J. Ziegel. The general concave case is more complicated. With some extra weak compactness property, Stefan Weber could settle this case already in 2004. We can now give a complete characterisation. Having convex level sets can be seen as a weakened form of the independence axiom in the vN–M theorem.

The notation is the standard notation of probability theory: $(\Omega, \mathcal{F}, \mathbb{P})$ is an atomless probability space, L^∞ the space of bounded random variables modulo equality a.s. L^1 is the space of integrable random variables modulo equality a.s.

Utility functions are defined on L^∞ and the convex set of their laws (or distributions) is denoted by the set $\mathcal{P}_c$. We use the following concept:

$u\colon L^\infty \to \mathbb{R}$ is called a *monetary utility function* if

(1) $u(0) = 0$ and $u(\xi) \geq 0$ if $\xi \geq 0$,
(2) for $a \in \mathbb{R}$ and $\xi \in L^\infty$: $u(\xi + a) = u(\xi) + a$,
(3) u is concave,
(4) $u(\xi_n) \downarrow u(\xi)$ for $\xi_n \downarrow \xi$.

The utility function u is completely characterised by the acceptance set

$$\mathcal{A} = \{\xi \mid u(\xi) \geq 0\},$$

indeed $u(\xi) = \sup\{y \mid \xi - y \in \mathcal{A}\}$. If u is also positively homogeneous, hence coherent, $\mathcal{A}$ is a cone and there is a closed convex set $\mathcal{S}$ of probability measures $\mathbb{Q} \ll \mathbb{P}$ such that

$$u(\xi) = \inf\left\{\mathbb{E}_\mathbb{Q}[\xi] \mid \mathbb{Q} \in \mathcal{S}\right\}.$$

In general, Fenchel duality yields the result of Föllmer–Schied: There is a convex lower semicontinuous function c, defined for all $\mathbb{Q} \ll \mathbb{P}$, taking values in $\overline{\mathbb{R}_+}$ and such that

$$u(\xi) = \inf\{\mathbb{E}_\mathbb{Q}[\xi] + c(\mathbb{Q}) \mid \mathbb{Q} \ll \mathbb{P}\}.$$

If the infimum is a minimum for all ξ, then the set $\{\mathbb{Q} \mid c(\mathbb{Q}) \leq m\}$ is weakly compact in L^1 for all $0 < m < \infty$, and conversely. If this property holds, we refer to it as the *weakly compact case*. This is also equivalent to $u(\xi_n) \uparrow u(\xi)$ for $\xi_n \uparrow \xi$ or to $u(\xi_n) \to u(\xi)$ for $\xi_n \to \xi$ and $\sup \|\xi_n\|_\infty < \infty$.

If $u(\xi)$ only depends on the law of ξ, then we say that u is law determined or *law invariant*. This is equivalent to $c(\mathbb{Q}) = c(\mathbb{Q}')$ as soon as $\frac{d\mathbb{Q}}{d\mathbb{P}}$ and $\frac{d\mathbb{Q}'}{d\mathbb{P}}$ have the same distribution or law. In that case one can show easily (using the law of large numbers) that for any sub-σ-algebra $\mathcal{C} \subset \mathcal{F}$ and any $\xi \in L^\infty$: $u(\mathbb{E}_\mathbb{P}[\xi \mid \mathcal{C}]) \geq u(\xi)$.

There seems to be an interest in law determined utilities since many people associate uncertainty with a probability law on the outcome. In case u is law determined, u factorizes over $\mathcal{P}_c$ and we write $u(\mu)$ where $\mu \in \mathcal{P}_c$. No confusion because of this abuse of notation will arise.

We say that u has **convex level sets** if $u(\mu) = u(\nu)$ implies for all $0 \leq \lambda \leq 1$ that

$$u(\lambda\mu + (1-\lambda)\nu) = u(\mu) = u(\nu).$$

This is a weak form of the independence axiom in the von Neumann–Morgenstern theory. This axiom says that if $\mu \sim \nu$ — meaning the economic agent is indifferent between the lotteries μ and ν — then for every $\kappa \in \mathcal{P}_c$ and every $0 \leq \lambda \leq 1$: $\lambda\mu + (1-\lambda)\kappa \sim \lambda\nu + (1-\lambda)\kappa$.

The property of having convex level sets is a consequence of the property called *elicitability*. This concept from statistics plays a role when dealing with estimation and convergence properties. The property of elicitability does not have to be defined on $\mathcal{P}_c$, but can be defined on any set of probability laws that is closed under taking convex combinations. Also the mapping ϕ below can be replaced by a set-valued function.

Let $\mathcal{P}_c$ be the convex set of probability measures with bounded support (laws of elements of L^∞). If

$$\phi\colon \mathcal{P} \to \mathbb{R}$$

is a function, then we say that ϕ is *elicitable* if there is a function

$$s\colon \mathbb{R} \times \mathbb{R} \to \mathbb{R}$$

such that for all $\mu \in \mathcal{P}$:

$$\int s(\phi(\mu), x)\, \mu(dx) = \inf_{y \in \mathbb{R}} \int s(y, x)\, \mu(dx).$$

Examples: for $s(x,y) = (x-y)^2$ we find the expected value; for $s(x,y) = |x-y|$ we get the median (or better the set of medians).

The relation with monetary utility functions (not necessarily coherent but only concave) is explained as follows. [Pisa lectures 2000, Osaka lecture notes 2008]

Let $\varphi\colon \mathbb{R} \to \mathbb{R}$ be concave, increasing and $\varphi(0) = 0$. Put

$$\mathcal{A} = \{\xi \mid \mathbb{E}[\varphi(\xi)] \geq 0\}$$

and let

$$u(\xi) = \sup\{y \mid \xi - y \in \mathcal{A}\}.$$

In other words, $u(\xi)$ is the solution of the equation

$$\mathbb{E}[\varphi(\xi - y)] = 0.$$

$u(\xi)$ can also be obtained as the minimum of

$$y \to \int_{\mathbb{R}} \Phi(x-y)\mu(dx),$$

where μ is the law of ξ and $\Phi(x) = \int_0^x \varphi(t)\, dt$. Clearly these utilities are elicitable. u is coherent if the function φ is homogeneous. This means it is of the form: there are numbers

$$0 < \gamma \leq \beta$$

and

$$\varphi(x) = \beta x \text{ for } x \leq 0; \quad \varphi(x) = \gamma x \text{ for } x \geq 0.$$

Stefan Weber's result is phrased using some different notations, but one can show that it is equivalent to the following. Weber only uses some continuity for Bernoulli variables, but one can show the equivalence.

Suppose that u has convex level sets in $\mathcal{P}_c$ and satisfies the weak compactness property; then u is of the form as in the previous section, i.e., there is a nondecreasing concave function $\varphi\colon \mathbb{R} \to \mathbb{R}$ such that $\mathcal{A} = \{\xi \mid \mathbb{E}[\varphi(\xi)] \geq 0\}$.

The construction of φ is nontrivial. The basic ingredient is to argue that the set $D = \{\mu \mid u(\mu) \geq 0\}$ is "closed" and convex in $\mathcal{P}_c$. Also the set $C = \{\mu \mid u(\mu) < 0\}$ is convex. Since $C \cap D = \emptyset$ one can hope to use the separation theorem. However several technicalities arise.

For coherent utilities Johanna Ziegel could sharpen the result as follows. If u is coherent and has convex level sets, then

either $u = \text{ess inf}$

or u has a weakly compact scenario set.
The latter means that there is a Young function Φ such that $\xi_n \to \xi$ in L^Φ implies $u(\xi_n) \to u(\xi)$. Using the result of Stefan Weber, this implies that u is an *expectile*.

For general concave monetary utility functions, the alternative between the weakly compact case and the ess inf is wrong. We can prove that *either* u is ess inf *or* there is $-\infty \le k < 0$ and a concave increasing function $\phi\colon (k, +\infty) \to \mathbb{R}$ with $\phi(0) = 0$ such that $\mathcal{A} = \{\xi \mid \mathbb{E}[\phi(\xi)] \ge 0\}$. The function ϕ should take the value $-\infty$ for $x < k$. What happens at the point k can be characterised as well, but this goes beyond this summary. We have numerous examples for which $k > -\infty$.

If $k = -\infty$, we are back in the weak compact case. If ϕ takes finite values on $\mathbb{R}$, then for $0 \ge k > -\infty$ we can make a perturbation such that

$$\mathcal{A} = \{\xi \mid \mathbb{E}[\phi(\xi)] \ge 0 \text{ and } \xi \ge k\}.$$

However, not every u with convex level sets is of this form. Our approach uses the same techniques as in Weber's theorem, but we must use more precise separation theorems.

The basic lemma is to show that if there is $k < 0$ and $a > 0$ such that

$$\{\alpha \mid 0 \le \alpha \le 1 : u(\alpha\delta_k + (1-\alpha)\delta_a) \ge 0\} \ne \{0\},$$

then the same property holds for every $a > 0$. Also in that case for

$$\alpha(k, a) = \max\{\alpha \mid 0 \le \alpha \le 1 : u(\alpha\delta_k + (1-\alpha)\delta_a) \ge 0\}$$

we have

$$u(\alpha(k,a)\delta_k + (1-\alpha(k,a))\delta_a) = 0 \text{ and not just } \ge 0.$$

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Dynamic utilities and long term decision making

Nicole El Karoui
(joint work with M. Mrad, C. Hillairet)

Motivated by the long term financial problems, we propose to use an adaptive utility criterion to adjust preferences to new economic information. Then we use the Ramsey rule to link the discount rate with the marginal utility of consumption at the economic equilibrium.

First, we recall the properties of market consistency, forward utilities and their dynamic characterisation, both as solutions of SDEs or more explicitly as well-defined from the optimal wealth process and the optimal state price density in explicit form by

$$U_x(t,x) = Y_t^*\big(u_x'(X_t^*(x))^{-1}\big),$$

where both processes $X_t^*(x)$ and $Y_t^*(y)$ are shown to be increasing with respect to their initial condition. Then we solve an optimisation forward problem with consumption and use the results to make links with a backward classical optimisation problem and indifference pricing valuation in both the forward and backward point of view. Then we use the Ramsey rule to link the marginal utility of consumption (as in the economic point of view) and the classical point of view in finance where interest rates are expressed in terms of ("indifference") prices of zero coupon bonds. Mathematical arguments provide a pathwise version of the Ramsey rule since

$$\frac{u_c'(c_t^*)}{u_c'(c_0)} = \frac{Y_t^*(y)}{y},$$

where $y = u_c'(c_0)$, and by indifference pricing, the links between the equilibrium point of view and the financial point of view. Both forward and backward problems are analysed, showing the strong influence of the maturity τ_H of the optimisation problem, suggesting the use of different yield curves for different τ_H. This difficulty disappears in the forward case, making the problem more consistent. In the backward case, for a Gaussian economy, we observe a non-standard behaviour for the investors with maturity τ_H when τ_H goes to infinity.

When option pricing meets optimal execution

Olivier Guéant
(joint work with Jiang Pu, Guillaume Royer)

The goal of the talk was to present two papers that account for execution costs and market impact in option pricing and hedging, in a way that is inspired from the literature on optimal execution. The first paper [1] is dedicated to vanilla options. We introduce nonlinear execution costs and market impact, and we look for an optimal hedging strategy in an expected utility framework. Both cash and physical settlement are considered. In this framework, the indifference price is obtained as the solution of a PDE that is the usual Black–Scholes PDE with additional nonlinear terms. The second paper [2] deals with accelerated share repurchase (ASR) contracts. These contracts exhibit an Asian payoff with Bermudan exercise dates and are usually associated with a very large nominal. A numerical method is presented that uses pentanomial trees with cubic growth of the number of nodes.

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Optimal investment in a Black–Scholes model with a bubble

Sebastian Herrmann
(joint work with Martin Herdegen)

How does an investor behave in the presence of a financial bubble? To tackle this question, we study the problem of maximising expected utility from terminal wealth for a power utility investor in a simple extension of the Black–Scholes model.

Our financial market consists of a positive riskless asset ("bond") $B = (B_t)_{t\in[0,T]}$ normalised to 1 and a risky asset ("stock") $S = (S_t)_{t\in[0,T]}$ whose dynamics are given by

$$(1) \qquad \frac{\mathrm{d}S_t}{S_{t-}} = \mu\,\mathrm{d}t + \sigma\,\mathrm{d}W_t + \mathrm{d}\mathcal{M}_t^G\phi, \quad S_0 = 1,$$

where $\mu, \sigma > 0$ and $\mathcal{M}^G\phi$ is a martingale of finite variation which has a single negative jump at a random time γ independent of the Brownian motion W. More precisely, $G \in C^2[0,T)$ is the distribution function of γ, $\phi \in C^2[0,T)$, and each trajectory $\mathcal{M}^G_\cdot\phi(\omega)$ follows the deterministic function ϕ on $[0,\gamma(\omega))$, jumps downwards at the random time $\gamma(\omega)$ and is constant on $[\gamma(\omega),T]$; cf. [1].

We consider a small investor with initial capital $x > 0$ and constant relative risk aversion $p > 0$, who can trade in this market. Denote by X^π the wealth process corresponding to a predictable process π that describes the fraction of the investor's wealth invested in the stock. The investor's goal is to maximise the expected utility $E[U(X_T^\pi)]$ over all strategies π such that $X^\pi > 0$; here, $U(x) = \frac{x^{1-p}}{1-p}$ for $0 < p \neq 1$ and $U(x) = \log x$ for $p = 1$.

We prove existence and uniqueness of the optimal strategy $\hat{\pi}$ and its dual minimiser $\hat{Q}$ (an equivalent local martingale measure (ELMM)) and characterise it in terms of the solution to an integral equation (or to a first-order ODE). Existence of a solution to this equation is non-trivial since ϕ may explode at the time horizon T. Moreover, we decompose the optimal strategy $\hat{\pi} = \pi^{\mathrm{m}} + \pi^{\mathrm{h}}$ into its myopic demand π^{m} and its hedging demand π^{h}. Since our model behaves just as the Black–Scholes model after the bubble has burst, we can show that $0 \leq \pi^{\mathrm{m}} \leq \frac{\mu}{p\sigma^2}$ and that

$$\pi^{\mathrm{h}} \leq 0 \text{ for } p \in (0,1), \quad \pi^{\mathrm{h}} = 0 \text{ for } p = 1, \quad \text{and} \quad \pi^{\mathrm{h}} \geq 0 \text{ for } p > 1.$$

In particular, the optimal strategy $\hat{\pi}$ never involves short-selling if $p > 1$. In addition, we give a necessary and sufficient condition on G and ϕ such that S becomes a *strict* local martingale under the dual minimiser $\hat{Q}$.

Finally, we present numerical illustrations of the optimal strategy and the welfare loss compared to the Black–Scholes model.

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Fake diffusions

David Hobson

Suppose we are given a family of centred probability measures which are increasing in convex order; for example, they may arise as the marginals of a martingale diffusion.

The issue is to construct a martingale whose univariate marginals match the given measures. Assuming only that the target laws have a density which is differentiable in time, and a dispersion assumption which sees mass leave a central region, we show how to construct a martingale with the right properties. The key to the construction is a certain picture. The construction has the special property that amongst all martingales with the given laws, the process has smallest (in expectation) total variation.

The Gai–Kapadia model of systemic risk

Tom Hurd

Questions about stability of financial networks, or systemic risk, have obvious economic importance. This talk aims to present the details of a large network asymptotic result that lies at the core of systemic risk modelling. We focus on pure default cascades that develop within deliberately simplified models of interbank networks. These models boil down to accounting for the impact, on the capital buffers of creditor banks, of shocks sent from defaulting debtor banks. Two similar models differ in one respect: the Eisenberg–Noe 2001 model assumes no bankruptcy costs at default, while the Gai–Kapadia 2010 model assumes 100% bankruptcy losses. Methods of random graph theory going back to Erdös and Renyi, combined with stochastic balance sheets, lead to the possibility of rigorous analysis of cascades in the Gai–Kapadia 2010 paradigm. We give a formal computation, for $N = \infty$, of the default probability distribution $\{\pi_{jk}^{(n)}\}$ after n cascade steps for a bank with j debtor banks and k creditor banks. The general understanding of such a result is that for a "well-behaved" sequence of random financial networks $\{G^{(N)}\}_{N=1,2,\ldots,}$, where $N = \mathbb{E}[|G^N|]$, the fraction $\pi_{jk}^{(N,n)}$of (j,k) banks which are defaulted by n cascade steps will be "approximately $\pi_{jk}^{(n)}$ with high probability", that is,

$$\pi_{jk}^{(N,n)} = \pi_{jk}^{(n)} + o(N)$$

as $N \to \infty$, in probability.

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Backward differential equations driven by a point process: an elementary approach

Jean Jacod

(joint work with Fulvia Confortola, Marco Fuhrman)

We consider a BSDE driven by a point process, or a multivariate point process, and show how to derive a solution of the BSDE by pasting together solutions to ordinary differential equations.

More specifically, we have a probability space, a fixed time horizon T, and a non-explosive point process N with successive jump times $S_1, S_2, \dots$ (with the convention $S_n = \infty$ on the set $\{S_n > T\}$). We denote by $(\mathcal{F}_t)$ the filtration generated by N and by A the predictable compensator of N. The BSDE we consider takes the form

$$(1) \qquad Y_t + \int_t^T Z_s\, dN_s = \xi + \int_t^T f(s, Y_s, Z_s)\, dA_s, \qquad t \in [0, T],$$

where the "generator" f is a predictable function on $\Omega \times [0,T] \times \mathbb{R} \times \mathbb{R}$ (as usual, we omit to mention the sample point ω in (1) and below), and the terminal condition ξ is an $\mathcal{F}_T$-measurable random variable.

More generally, we could replace N by a multivariate point process, represented by a random measure $\mu(dt, dx) = \sum_{n \geq 1:\, S_n \leq T} \delta_{(S_n, \chi_n)}(dt, dx)$, where the S_n are as above and the χ_n are random variables taking values in some Polish space E. Then if ν is the predictable compensator of μ, the BSDE takes the form

$$(2) \quad Y_t + \int_{(t,T]} \int_E Z(s,x)\, \mu(ds, dx) = \xi + \int_{(t,T]} \int_E f(s, x, Y_{s-}, Z(s,x))\, \nu(ds, dx).$$

All results given below for (1) hold for (2) as well, but for simplicity, here we restrict ourselves to (1).

A *solution* of (1) is a pair (Y, Z), with Y a càdlàg adapted process and Z a predictable process. However, at least as soon as A is continuous (that is, all jump times S_n are totally inaccessible), any solution is such that Y is continuous outside the times S_n, and on the set $\{S_n \leq T\}$ we have $\Delta Y_{S_n} = Z_{S_n}$, so actually Z is completely determined by Y.

Two kinds of assumptions are made:

Assumption (A) on the point process: The compensator A is continuous, and $\mathbb{P}[S_{n+1} > T \mid \mathcal{F}_{S_n}] > 0$ a.s. for all n.

Assumption (B) on the generator: We have $\int_0^T |f(s,0,0)|\, dA_s < \infty$ a.s., and $|f(\omega, t, y', z') - f(\omega, t, y, z)| \leq L'|y' - y| + L|z' - z|$ for two constants L, L'.

Due to the nature of the filtration generated by the point process, we have a special structure of the ingredients appearing in (1). Let H_n be the class of $D=\{t_0,\dots,t_n\}$ with $0=t_0\le t_1\le\cdots\le t_n$ and $t_j\in[0,T]\cup\{\infty\}$ and $t_j<t_{j+1}$ if $t_j\le T$. With $S_0=0$, we also consider the random set $D_n=\{S_0,\dots,S_n\}$.

First, for each $D\in H_n$, there is a continuous decreasing positive function $t\mapsto G_D(t)$ on $[0,T]$, with $G_{D_n}(t)=\mathbb{P}[S_{n+1}>t\mid\mathcal{F}_{S_n}]$ on $\{S_n\le T\}$, and A is

$$A_t=\sum_{n=0}^{\infty}a^n_{D_n}(t\wedge S_{n+1}),\qquad a^n_D(t)=-\log G^n_D(t).$$

Next, for each $n\ge 0$, there is a measurable map $D\mapsto u^n_D$ on H_n, with

$$S_n(\omega)\le T<S_{n+1}(\omega)\quad\Longrightarrow\quad \xi(\omega)=u^n_{D_n(\omega)}.$$

Next, if Y is càdlàg adapted and continuous outside the times S_n, for each n and $D\in H_n$, there is a continuous function $t\mapsto y^n_D(t)$ on $[0,T]$, measurable in D, with

$$S_n(\omega)\le t<S_{n+1}(\omega),\ t\le T\quad\Longrightarrow\quad Y_t(\omega)=y^n_{D_n(\omega)}(t). \tag{3}$$

Similarly, for each n and $D\in H_n$, there are Borel maps $(t,y,z)\mapsto f^n_D(t,y,z)$ with

$$S_n(\omega)<t\le S_{n+1}(\omega)\wedge T\quad\Longrightarrow\quad f(\omega,t,y,z)=f^n_{D_n(\omega)}(t,y,z).$$

The following (simple) lemma is a key point for our analysis.

Lemma *A process Y solves* (1) *if and only if, outside a $\mathbb{P}$-null set, the functions y^n_D associated by* (3) *satisfy for all n and $t\in[0,T]$:*

$$y^n_{D_n}(t)=u^n_{D_n}+\int_t^T f^n_{D_n}(s,y^n_{D_n}(s),y^{n+1}_{D_n\cup\{s\}}(s)-y^n_{D_n}(s))\,da^n_{D_n}(s). \tag{4}$$

If further $\mathbb{P}[S_{M+1}=\infty]=1$ for some finite integer (equivalently, N has at most M points), we also have

$$t\in[0,T]\quad\Longrightarrow\quad y^M_{D_M}(t)=u^M_{D_M}=\xi. \tag{5}$$

The case of finitely many points: When N has at most M points, (5) gives us y^M_D, and then all equations (4) are ordinary (backward) differential equations, which can be solved recursively in n, starting with $n=M$: At each stage, $u^n_{D_n}$ is known, and the coefficient $f^n_{D_n}$ is Lipschitz, so we have a unique solution as soon as $\int_t^T|f^n_{D_n}(s,0,y^{n+1}_{D_n\cup\{s\}}(s))|\,da^n_{D_n}(s)$ is finite, and it can be shown that the latter condition amounts to having ξ integrable. In other words:

Theorem *When N has at most M points almost surely, and if ξ is integrable,* (1) *has a unique solution.*

The general case: When $\mathbb{P}[S_n\le T]>0$ for all n, the problem is more difficult because we cannot start the previous backward ODEs with (5) at some M. The idea is to "stop" the process N (and A as well) at time $S_n\wedge T$, for any given n, and replace the terminal condition ξ by $\xi^{(n)}=\xi\,1_{\{T<S_n\}}$. According to what precedes, we solve in a unique way each stopped equation, and then we let $n\to\infty$. To obtain

the convergence of the stopped solution toward a solution to (1), we unfortunately need some *a priori* estimates, and stronger conditions on ξ.

More specifically, for any $\alpha, \beta > 0$ we denote by $\mathcal{L}_{\alpha,\beta}$ the set of all pairs (Y, Z) of measurable processes such that

$$\mathbb{E}\left[\int_0^T (|Y_s| + |Z_s|) e^{\beta A_s} \alpha^{N_s} \, dA_s\right] < \infty.$$

Then one can prove the following:

Theorem *Assume that for some $\beta > 1 + L + L'$ and $\alpha > L$, we have*

$$\mathbb{E}\left[e^{\beta A_T} \alpha^{N_T} |\xi| + \int_0^T \alpha^{N_s} e^{\beta A_s} |f(s, 0, 0)| \, dA_s\right] < \infty.$$

Then (1) *admits one and only one (up to null sets) solution* (Y, Z) *which belongs to* $\mathcal{L}_{\alpha,\beta}$.

Finally, one can try to see whether Assumption (A) is really needed. Although we do not have a full answer to this question, we can mention two interesting properties:

If A is continuous but $\mathbb{P}[S_1 > T] = 0$ (so the second condition in (A) fails for $n = 0$): Assuming further that there is a single point S_1, and for the simple generator $f(\omega, t, y, z) = z$, then as soon as ξ is integrable, for any real y, equation (1) has a unique solution satisfying further $Y_0 = y$ (thus, we – strangely enough – have exactly one solution if we fix *both* the terminal condition ξ, and the initial condition y).

If A is discontinuous: Again in the case of a single point S_1, one can show in some cases that even when ξ is bounded, there might be no solution at all. This is the case, for example, when S_1 takes only the values T (with probability p) and $+\infty$ (with probability $1 - p$), and the generator is $f(y, z) = \frac{y}{p} + g(z)$, and when $\xi = a$ if $S_1 = T$ and $\xi = b$ if $S_1 = \infty$. In this case, we have infinitely many solutions if $b + pg(a - b) = 0$, and none at all otherwise.

Arbitrages and progressive enlargement of filtrations

Monique Jeanblanc

(joint work with Anna Aksamit, Tahir Choulli, Jun Deng, Claudio Fontana, Shiqi Song)

Motivated by reduced form models of credit risk, we study the following problem. Let $\mathbb{F}$ be a given filtration and $\mathbb{G}$ the progressive enlargement of $\mathbb{F}$ with a default time τ. All asset prices are supposed to be $\mathbb{F}$-adapted. Assuming that there are no arbitrages of the first kind when trading with $\mathbb{F}$-adapted strategies, under which conditions on τ there are still no arbitrages of the first kind when trading with $\mathbb{G}$-adapted strategies? We give necessary and sufficient conditions before τ, based on the $\mathbb{F}$-supermartingales $Z_t = \mathbb{P}[\tau > t \,|\, \mathcal{F}_t]$ and $\widetilde{Z}_t = \mathbb{P}[\tau \geq t \,|\, \mathcal{F}_t]$. More precisely, we prove that there are no arbitrages of the first kind if and only if the set $\{\widetilde{Z} = 0 < Z_-\}$ is evanescent. For the part after τ, we restrict our attention to some specific times τ (i.e., honest times which satisfy $Z_\tau < 1$) for which we give also necessary and sufficient conditions, based on the two $\mathbb{F}$-supermartingales Z and $\widetilde{Z}$. We also present classical arbitrages for an honest time in a complete market.

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On traded local martingale deflators

Yuri Kabanov

(joint work with Kostas Kardaras, Shiqi Song)

In a very recent paper, Takaoka and Schweizer [7] (based on the Takaoka preprint of 2011) extended to the multi-asset case the following theorem, see [5]:

The NAA1 condition (called also BK, NUPBR) is equivalent to the existence of a local martingale deflator.

A local martingale deflator is a strictly positive stochastic process whose product with any portfolio process is a local martingale. This result is an important complement to the criterion due to Kostas Kardaras and Ioannis Karatzas [4]:

The NAA1 condition is equivalent to the existence of a supermartingale deflator.

Though Kardaras considered in [5] only a scalar risky process, his result is more precise:

In any neighborhood of the basic probability measure, one can find an equivalent probability measure under which there exists a tradable local martingale deflator, i.e., a portfolio process whose reciprocal is a local martingale deflator.

The original proof by Takaoka proof is based on the change of numéraire technique and a clever reduction to the FTAP of Delbaen and Schachermayer [1]. It is interesting to avoid the latter (which is one of the more complicated results of mathematical finance); as was shown by Kardaras [6], the NAA1 criterion may serve to get an alternative, purely probabilistic proof of the FTAP.

In the present talk, we show that the multidimensional version of the Kardaras theorem can be deduced from the Karatzas–Kardaras criterion (we provide a new, simpler proof of this important theorem using a LLN for stochastic integrals with truncated integrands). Our arguments are based on the Delbaen–Schachermayer theorem on the existence of an equivalent σ-martingale measure [2], which is a relatively simple probabilistic result. We dispose now of a proof of the latter which is even simpler than that in [3].

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Probabilistic representation of the parametrix method

Arturo Kohatsu-Higa
(joint work with Vlad Bally)

The parametrix method was discovered by E. Levi in 1907 in order to find solutions of elliptic partial differential equations. This method was later extended for parabolic differential equations with Hölder-continuous coefficients. In this talk, we develop a probabilistic interpretation of this method for evaluations of test functions of the marginals of a continuous diffusion process using the Euler–Maruyama (EM) scheme.

The method is based on a Taylor expansion-like method which expands the solution of the parabolic partial differential equation around a basic Gaussian density. We introduce two ways of carrying out this idea.

The first one applies directly a Taylor-like expansion on the semigroup of a continuous diffusion. This requires the regularity of the coefficients but leads to a first expression of the expectation of the marginal of the diffusion written using only the EM scheme defined at a random time partition given by an independent Poisson process.

The second method (see [4]), which relies on the expansion of the dual of the semigroup operator, leads to a similar expression based on a reversed time EM scheme which starts at a random initial value given by the test function. This method requires less regularity on the coefficients. Notably it only requires the diffusion coefficients to be Hölder-continuous and the drift coefficient bounded and measurable.

Both probabilistic representations have infinite variance and therefore some importance sampling methods have to be introduced if one wants to use these formulas for Monte Carlo simulation.

As a first financial application, we apply this formula to the price of a barrier-type option with constant volatility. Through a reflection principle, we see that this price is equal to the price of a European-type option based on a model with discontinuous drift coefficient.

Next, we claim that a similar formula to be applicable to stochastic volatility models will require the analysis of the parametrix method for discontinuous volatility coefficients. For this reason, we explain why the parametrix method is applicable to the basic case of a discontinuous diffusion coefficient of the type $\sigma^2(x) = 1 + I_{\{x\geq 0\}}$.

Time permitting, we may discuss the relation between these formulas with the multilevel Monte Carlo method, the non-Markovian setting and other possible extensions of these formulas.

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Monte Carlo method on pricing Bermudan derivatives

Shigeo Kusuoka

(joint work with Yusuke Morimoto, Bank of Tokyo Mitsubishi UFJ, University of Tokyo)

It is an interesting and practical problem to compute numericallly prices of American derivatives or Bermudan derivatives. Stochastic mesh methods and least square regression methods (so-called Longstaff–Schwarz methods) are well-known methods as Monte Carlo solution to this problem. However, each of them has some good points and some weak points.

As for the stochastic mesh method, the convergence to the true value is clear. But this method is available only when we know the explicit shape of the transition density function of the underlying Markov process, and so models to which we can apply this method are restricted.

As for the least square regression method, this method is available if we can simulate paths of the underlying Markov process, and so we can apply this method to a wide class of models. But the convergence to the true value is not clear and it depends on the choice of families of functions as approximating functions for value functions.

In this talk, we show our recent results for both methods on convergence when we take Hörmander-type diffusion processes as underlying processes. Also, we introduce a new class of random functions as approximating functions for value functions in least square regression methods.

The paper with the results for stochastic mesh methods will appear in Adv. Math. Econ. 18 (2014). The paper on the results for least square regression methods is in preparation.

Mean field games with common noise

Daniel Lacker
(joint work with René Carmona, François Delarue)

A general characterization is derived for the limits of approximate equilibria of large-population symmetric stochastic differential games as the number of agents tends to infinity. It is shown that the equilibrium empirical measures admit limits in distribution, and every limit is a weak solution of the mean field game (MFG). Conversely, every weak MFG solution can be obtained as the limit of a sequence of approximate equilibria in the finite-player games. In other words, the MFG precisely characterizes the possible limits of the finite-player games, formalizing the well-established intuition. The proofs use relaxed controls to provide the compactness needed to obtain limits under quite general assumptions, and it is then shown how to sharpen the characterization of the limit under various additional assumptions. In particular, under modest convexity assumptions, versions of the main theorems are stated with no mention of relaxed controls. Stronger assumptions yield uniqueness of the weak MFG solution and thus a full convergence result.

Streams, paths, signatures and the learning of functions

Terry Lyons

Streams of sequential data, where the order of distinct types of events is critical to understanding, occur widely, and certainly in finance. Signatures of these streams are transforms of the data into an infinite sequence of coefficients representing the path as an element of a tensor algebra.

The transformation is tractable and (up to tree-like components) faithful, leading to an alternative description of the data. The first few terms control the effects of the stream, while the later terms control the texture.

There is a strong theoretical basis for this transform (see e.g. Hambly and Lyons, Annals of Mathematics, 2010), but in this context we are interested in explaining how the way that polynomial functions of path signatures can be rewritten as linear functionals allows regression to become linear regression.

We apply this to examples of buckets of 500 paths of 30 steps in 4 dimensions, and use simple regression/least squares lasso to classify the data. This allows completely blind identification of differences in 2 of the buckets to the others which were indistinguishable (the data were normalised to remove the obvious volatility/volume differences).

The material is exposed in more detail in an arXiv paper with Gyurko, and in my talk for the ICM in Korea 2014. In fact it contains much mathematics.

Robust framework for pricing and hedging with trading restrictions and emergence of bubbles

Jan Obłój
(joint work with A.M.G. Cox, Zhaoxu Hou)

We consider a robust approach to the pricing and hedging of derivatives. Instead of pre-supposing a probabilistic model, we assume that options are available for trade at time zero for prices observed in the market. We are interested in implications this has for the pricing and hedging of an exotic derivative with payoff G, and we seek to establish a duality result equating the cheapest superhedging strategy for G to the supremum of expectations of G over all suitably calibrated market models. This framework, going back to the seminal contribution of Hobson [5], has been the subject of much recent interest and many papers.

We first consider a discrete-time framework and the case when call or put options, for all strikes and possibly multiple maturities, are traded. Such a combination of prices encodes probability measures (μ_i) which correspond to marginal distributions of a calibrated market model. Our focus is on the case when the means of μ_i, which correspond to the forward prices implicit in the prices of options, are strictly decreasing. This would normally lead to an obvious arbitrage. However, we argue that by imposing no-short-selling constraints, such market inputs can be accommodated within the robust framework. Further, we incorporate a parsimonious description of modelling beliefs by allowing to specify a set $\mathfrak{P}$ of feasible paths. This has the effect that the superhedging only holds on $\mathfrak{P}$ and the calibrated models have to be supported on $\mathfrak{P}$. We show that no arbitrage (in the sense of no weak free lunch with vanishing risk of Cox and Obłój [3]) is equivalent to the existence of a calibrated market model. We then study the pricing-hedging duality. When call options are traded, we establish a general duality result inspired by, and using the methods of, Beiglböck, Henry-Labordère and Penkner [1]. In contrast, when put options trade, the trading restrictions may lead to a duality gap. In a simple one-period model, the gap is equal to

$$\gamma(s_0 - f_0), \qquad \text{where } \gamma = \limsup_{x\to\infty,\, x\in\mathfrak{P}} \frac{G(x)}{x},$$

and s_0 is the spot price, and f_0 the forward implicit in the put options. Such a setting has a natural interpretation as a *speculative bubble* in which the market (superhedging) price is strictly larger than the fundamental price, defined as supremum of expectations of G over calibrated market models. It also shows that supermartingales arise as natural discrete-time probabilistic models for such bubbles.

Finally, we consider a continuous-time setting. By embedding the discrete-time setting in continuous time, we obtain analogues of the results above. In particular, we show that when put options trade and imply a mispricing of the forward, a bubble arises and market (superhedging) prices may be strictly larger than the fundamental prices. An arbitrage opportunity does not arise because of no-short-selling constraints combined with a pathwise superhedging requirement. We observe that the latter encodes a collateral requirement, akin to the one in Cox and Hobson [2]. The set of calibrated market models is given by strict local martingale measures with fixed marginal(s), or more generally and in function of the class of admissible strategies, the set of supermartingale measures with fixed marginal(s). In mathematical finance, the modelling of financial bubbles using local martingale models can be traced back to [4], with subsequent contributions including [2, 6]. We believe that an important consequence of our work is the emphasis that local martingale models are intrinsically models which arise due to trading constraints.

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Density analysis of BSDEs

Dylan Possamaï
(joint work with Thibaut Mastrolia, Anthony Réveillac)

In recent years the field of backward stochastic differential equations (BSDEs) has been a subject of growing interest in stochastic calculus as these equations naturally arise in stochastic control problems in finance, and as they provide Feynman–Kac type formulae for semilinear PDEs. Before going further, let us recall that a solution to a BSDE is a pair of *regular enough* (in a sense to be made precise) predictable processes (Y, Z) such that

$$Y_t = \xi + \int_t^T h(s, Y_s, Z_s)\, ds - \int_t^T Z_s\, dW_s, \qquad t \in [0, T], \tag{1}$$

where W is a one-dimensional Brownian motion, h is a predictable process and ξ is an $\mathcal{F}_T$-measurable random variable (with $(\mathcal{F}_t)_{t\in[0,T]}$ the natural completed and right-continuous filtration generated by W). Since it is not possible to provide an explicit solution to (1) (except when h is a linear mapping of (y, z)), one of the main issues especially regarding the applications is to provide a numerical analysis for the solution of a BSDE. This calls for a deep understanding of the regularity of the solution processes Y and Z. Here, we focus on the marginal laws of the random variables Y_t, Z_t at a given time t in $(0, T)$. More precisely, we are interested in providing sufficient conditions ensuring the existence of a density (with respect to Lebesgue measure) for these marginals on the one hand, and in deriving some estimates on these densities on the other hand. This type of information on the solution is of theoretical and of practical interest since the description of the tails of the (possible) density of Z_t would provide more accurate estimates on the convergence rates of numerical schemes for quadratic growth BSDEs. This issue has been rarely studied in the literature, since up to our knowledge only references [2, 1] address this question. The first results about this problem have been derived in [2], where the authors provide existence of densities for the marginals of the Y component only and when the driver h is Lipschitz-continuous in (y, z), and some smoothness properties of this density. Concerning the Z component, much less is known since existence of a density for Z has been established in [1] only under the condition that the driver is linear in z. We revisit and extend the results of [2, 1] by providing sufficient conditions for the existence of densities for the marginal laws of the solution Y_t, Z_t (with t an arbitrary time in $(0, T)$) of a qgBSDE with a terminal condition ξ in (1) given as a deterministic mapping of the value at time T of the solution to a one-dimensional SDE, together with estimates on these densities. En route to these results, we provide new conditions for the Malliavin differentiability of solutions of Lipschitz or quadratic BSDEs. These results rely on the interpretation of the Malliavin derivative as a Gâteaux derivative in the directions of the Cameron–Martin space. Incidentally, we provide a new formulation for the characterization of the Malliavin–Sobolev type spaces $\mathbb{D}^{1,p}$.

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Hedging, arbitrage and optimality under superlinear frictions

Miklós Rásonyi

(joint work with Paolo Guasoni)

In a continuous-time model with multiple assets described by càdlàg processes, we characterize superhedging prices, absence of arbitrage, and utility maximizing strategies, under general frictions that make execution prices arbitrarily unfavorable for high trading intensity. Such frictions induce a duality between feasible trading strategies and shadow execution prices with a martingale measure. Utility maximizing strategies exist even if arbitrage is present, because it is not scalable at will. The talk is based on the manuscript [1].

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Estimate nothing

Chris Rogers
(joint work with Moritz Duembgen)

In the econometrics of financial time series, it is customary to take some parametric model for the data, and then estimate the parameters from historical data. This approach suffers from several problems. Firstly, how is estimation error to be quantified, and then taken into account when making statements about the future behaviour of the observed time series? Secondly, decisions may be taken today committing to future actions over some quite long horizon, as in the trading of derivatives; if the model is re-estimated at some intermediate time, our earlier decisions would need to be revised — but the derivative has already been traded at the earlier price. Thirdly, the exact form of the parametric model to be used is generally taken as given at the outset; other competitor models might possibly work better in some circumstances, but the methodology does not allow them to be factored into the inference. What we propose here is a very simple (Bayesian) alternative approach to inference and action in financial econometrics which deals decisively with all these issues. The key feature is that nothing is being estimated.

Limit theorems for nearly unstable Hawkes processes

Mathieu Rosenbaum
(joint work with Thibault Jaisson)

Because of their tractability and their natural interpretation in terms of market quantities, Hawkes processes are nowadays widely used in high frequency finance. However, in practice, the statistical estimation results seem to show that very often, only *nearly unstable Hawkes processes* are able to fit the data properly. By nearly unstable, we mean that the L^1-norm of their kernel is close to unity. We study in this work such processes for which the stability condition is almost violated. Our main result states that after suitable rescaling, they asymptotically behave like integrated Cox/Ingersoll/Ross models. Thus, modelling financial order flows as nearly unstable Hawkes processes may be a good way to reproduce both their high and low frequency stylized facts. We then extend this result to Hawkes based high frequency price models. We show that under a similar criticality condition, these processes converge to Heston models. Again, we recover well-known stylized facts of prices, both at the microstructure level and at the macroscopic scale.

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Martingale optimal transport in the Skorokhod space

H. Mete Soner

(joint work with Yan Dolinsky of Hebrew University)

In this paper, we use the discretization approach to martingale optimal duality to extend our previous results [1, 2] to the setting of càdlàg processes. The financial market consists of a savings account which is normalized to unity $B_t \equiv 1$ by discounting and of d risky assets with price process $\mathbb{S}_t \in \mathbb{R}^d_+$, $t \in [0,T]$, with initial value $\mathbb{S}_0 = (1,\ldots,1)$. We assume that each component of the price process is right-continuous with left limits. $\mathbb{D}$ denotes the set of all such trajectories.

For a probability measure μ, let

$$\mathcal{H} = \{h(\mathbb{S}) = g(\mathbb{S}_T) : g \in \mathbb{L}^1(\mathbb{R}^d_+,\mu)\}, \qquad \mathcal{L}(g) = \int_{\mathbb{R}^d_+} g\, d\mu.$$

On $\Omega := \mathbb{D}$, let $\mathbb{S}$ be the canonical process and $\mathcal{F}$ the canonical σ-field. A probability measure $\mathbb{Q}$ on $(\Omega,\mathcal{F})$ is a *martingale measure* if the canonical process is a $\mathbb{Q}$-martingale with respect to the canonical filtration. Further let $\mathbb{M}_\mu$ be the set of all martingale measures $\mathbb{Q}$ such that the probability distribution of $\mathbb{S}_T$ under $\mathbb{Q}$ is equal to μ.

Definition. A *semi-static portfolio* is a pair $\phi := (g,\gamma)$, where $g \in \mathbb{L}^1(\mathbb{R}^d_+,\mu)$ and $\gamma : [0,T] \times \mathbb{D} \to \mathbb{R}^d$ is left-continuous and progressively measurable, where $\gamma_t(\mathbb{S})$ denotes the number of shares in the portfolio ϕ at time t, before a transfer is made at this time. A semi-static portfolio is *admissible* if for every $\mathbb{Q} \in \mathbb{M}_\mu$, the stochastic integral $\int \gamma_u \, d\mathbb{S}_u$ is a $\mathbb{Q}$-supermartingale. An admissible semi-static portfolio is called *superreplicating* if

$$g(\mathbb{S}_T) + \int_0^T \gamma_u(\mathbb{S}) \, d\mathbb{S}_u \geq G(\mathbb{S}), \qquad \forall \mathbb{S} \in \mathbb{D}.$$

The (minimal) *superhedging cost* of G is defined by

$$V(G) := \inf \left\{ \int g \, d\mu : \exists \gamma \text{ such that } \phi := (g, \gamma) \text{ is superreplicating} \right\}.$$

□

Theorem. *For any exotic option*

$$G : \mathbb{D}([0,T]; \mathbb{R}^d) \to \mathbb{R}$$

that is bounded and uniformly continuous in the Skorokhod metric, we have for the minimal superreplication cost the dual representation

$$V(G) = \sup_{\mathbb{Q} \in \mathbb{M}_\mu} \mathbb{E}_{\mathbb{Q}}[G(\mathbb{S})],$$

where $\mathbb{E}_{\mathbb{Q}}$ *denotes the expectation with respect to the probability measure* $\mathbb{Q}$.

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Martingale transport, Skorokhod embedding and peacocks

Xiaolu Tan

(joint work with Pierre Henry-Labordère, Nizar Touzi, Gaoyue Guo, Sigrid Källblad)

A peacock is a stochastic process $X = (X_t)_{t \geq 0}$ increasing in the convex ordering, i.e. $t \mapsto \mathbb{E}[\phi(X_t)]$ is increasing for every convex function ϕ. It follows by Kellerer's theorem that X is a peacock if and only if there is an associated martingale M, which is not unique in general, with the same one-dimensional marginals, i.e. $X_t \sim M_t$ in law for every $t \geq 0$. Then given a peacock, we consider the class of all associated martingales and look for the optimal one with respect to a reward function. The problem is related to the so-called martingale optimal transport problem and the optimal Skorokhod embedding problem (SEP). We study the problem using the SEP formulation, and obtain some duality results. As an application in finance, the dual problem is related to the minimum superhedging cost of exotic options, using dynamic and static strategies. When the reward function is given by the expectation of an increasing functional of the running maximum of the Brownian motion up to the last stopping time in SEP, we give a study to the optimal SEP as well as to its dual problem.

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A convergence result in the Emery topology and another proof of the fundamental theorem of asset pricing (FTAP)

Josef Teichmann

(joint work with C. Cuchiero)

We work in the general setting of admissible portfolio wealth processes as introduced e.g. by Y. Kabanov [2]. We show that in this setting, the property "no unbounded profit with bounded risk" (NUPBR), see [2], implies the so called (P-UT) property, a boundedness property in the Emery topology which has been introduced by C. Stricker [4]. Combining this insight with well-known results from Mémin and Słominski [3] leads to a short variant of the proof of the fundamental theorem of asset pricing initially proved by Delbaen and Schachermayer [1].

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