Oberwolfach Seminar: Singularity analysis for geometric flows

Date
September 6 Sep - 12 Sep 2015

Organizers
Simon Brendle, Stanford
Gerhard Huisken, Tübingen/MFO

ID
1537a

Programme
Study of the Ricci-flow of Riemannian metrics and the mean curvature flow of hypersurfaces in an ambient manifold as prominent examples of parabolic systems of second order partial differential equations that can be used for the smoothing and uniformisation of a given geometric object. In view of the nonlinear nature of these flows the curvature of the deforming objects follows a non-linear reaction-diffusion law causing singularities in the flow that can be overcome by "surgeries".

The Seminar will discuss the analytical and geometrical tools necessary to understand and characterize singularities of the evolution equations above with a view towards enabling surgery constructions for the flows due to Hamilton, Perelman in case of Ricci-flow and Huisken-Sinestrari, Brendle-Huisken in case of mean curvature flow. Specific techniques to be covered in the seminar include
1) Invariant curvature conditions
2) Non-collapsing estimates
2) Monotonicity formulae
3) Pseudolocality concepts
4) Convexity estimate, positive curvature estimates due to Hamilton-Ivey
4) Singularity classification
In each of these cases there are phenomena in Ricci- flow and mean curvature flow that appear similar at first sight but require different geometrical and analytical techniques. The course hopes to explain key techniques while describing similarities and differences between the two flows.

Preparatory Reading
S. Brendle, Ricci-flow and the sphere theorem, Graduate Studies in Math. 111 (2010), Amer. Math. Soc, chapters 2 and 3.
K. Ecker, Regularity theory for mean curvature flow, Progress in nonlinear differential equations and its applications, Birkhäuser (2004), chapters 2 and 3.
G. Huisken and C. Sinestrari, Mean curvature flow with surgeries of two-convex hypersurfaces, Invent. Math.175 (2009) 137-221, introduction.
S. Brendle and G. Huisken, Mean curvature flow with surgery of mean convex surfaces in R^3, preprint (2013) arxiv:1309.1461, introduction.
R.S. Hamilton, Formation of singularities in the Ricci flow, Surveys in Differential Geometry 2 (1995) 7-136, International Press, Boston, sections 1 to 4.