Programme

Almost one century ago Birkhoff proved the existence of at least one closed geodesic in any 2-dimensional sphere endowed with an arbitrary Riemannian metric. His argument is probably the first "min-max" argument in Riemannian geometry: if sweep out the sphere with closed curves, then some curves must be relatively long; if we optimize the length of the maximal curve in the sweepout, we then find a geodesic which gives, in some sense, the width of the sphere.

Generalizing this idea to generate minimal surfaces has proved to be a rather challenging task and in fact a satisfactory theory is only available for hypersurfaces and it is due to Almgren and Pitts. The tools developed by Almgren and Pitts have been used very recently by Marquez and Neves to settle two famous long-standing questions: the Willmore Conjecture and a conjecture of Yau on the existence of infinitely many minimal surfaces in arbitrary Riemannian 3-manifold with positive Ricci curvature .

In this seminar we will present the main points of the min-max method of Almgren and Pitts and its variants and of the theorems of Marques and Neves. Not everything will be proved in full: the emphasis will be on understanding the underlying ideas and exploring the implications.

Preparatory readings

- Standard facts in the calculus of variations and a good knowledge of Riemannian geometry (any standard textbook).
- Birkhoff's argument, cf. Colding & Minicozzi, A course in mininimal surfaces, Chapter 5, Section 1 to Section 3.
- Some classical things about minimal surfaces, for instance Chapter 1, Sections 1 to 9 in Colding & Minicozzi. -
- A basic knowledge of the main statements in the theory of varifolds: Section 1 in Chapter 1 of Colding & Minicozzi gives a very brief summary; a more advanced reading is Simon, "Lecture notes in geometric measure theory". You are not expected to work through the proofs in the Chapters 3 and 4 of Simon's book: it will suffice to understand the notation, the main definitions and the main statements.
- A basic knowledge of the main statements in the theory of integral currents, cf. Chapter 6 of Simon. As above, it will suffice to understand the notation, the main definitions and the main statements.