Oberwolfach Seminar: Different mathematical perspectives on the description of unresolved scales in multiscale systems

Organizers
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Programme:
The mathematical modelling of multiscale phenomena poses a variety of challenges of very different nature. The seminar reviews these aspects and introduces mathematical techniques for addressing them.

- *Scale analysis and (multiple scales) asymptotic techniques* elucidate the mechanisms generating scale separations and multiscale dynamics in the setting of deterministic ODE/PDE problems.

- *Statistical mechanics combined with renormalization techniques* address systems with continuous hierarchies of scales lacking scale separations, such as turbulence in the atmosphere and oceans.

- *Variational methods* yield coarse-grained images of chaotic and dissipative dynamics. Key concepts are, e.g., controllability & observability and the GENERIC formalism for non-equilibrium systems.

- *Data-driven models of unresolved scales using information-theoretical concepts* invoke entropy principles for model discrimination. They will be discussed in the context of Machine Learning.

- *Data-driven stochastic time series analysis* is key in investigations of complex systems. We address parametric/nonparametric, stationary/nonstationary, statistical, machine-learning, and artificial intelligence approaches, and regularization of ill-posed problems.

Preparatory reading

[1] R. Klein. Scale-dependent models for atmospheric flows. *Annu. Rev. Fluid. Mech.* **42**:249–274, 2010.

[2] A.J. Majda and M. Branicki, Lessons in uncertainty quantification for turbulent dynamical systems, *Discrete Cont. Dyn. S.* **32**:3133-3221, 2012.

[3] P. Metzner, L. Putzig and I. Horenko, Analysis of persistent non-stationary time series and applications, *Comm. in Appl. Math. and Com. Sci.*, **7**:175–229, 2012.

[4] J.S. Frederiksen, T.J. O'Kane & M. Zidikheri, Stochastic closure based subgrid modeling for geophysical flows, Phil. Trans. Roy. Soc. A **371**: 20120166, 2013.