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Komplexe Analysis

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Abstract. Complex Analysis is a very active branch of mathematics. The aim of this workshop was to discuss recent developments in several complex variables and complex geometry. Topics included singular Kähler–Einstein metrics, positivity of higher direct images, cycle spaces and extension theorems.

Mathematics Subject Classification (2010): 32xx, 14xx.

Introduction by the Organisers

The workshop *Komplexe Analysis*, organized by Philippe Eyssidieux (Grenoble), Jun-Muk Hwang (Seoul), Stefan Kebekus (Freiburg) and Mihai Păun (Seoul & Chicago), was held the week starting from the 28^{th} of August 2017. It was attended by over 50 participants from around the world, ranging from young post-doctoral researchers to senior leaders of the field.

The program featured twenty lectures, and allowed ample time for discussion and interaction; the discussion rooms were in fact constantly occupied. Among the "visible" results of these interactions the organizers are particularly glad to mention the article "Algebraically hyperbolic manifolds have finite automorphism groups" (arXiv 1709.09774) by Bogomolov, Kamenova and Verbitsky, whose final ideas were perceived during the workshop.

A large number of very interesting subjects were proposed for talks, which made it a rather difficult (though pleasant) task to choose the speakers. The organizers aimed for a balanced meeting, reflecting the current generation change in the subject: the program included many talks by younger colleagues, as well as talks by seniors, whose lectures were often full of interesting ideas and promising paths to follow.

The following list of talks and subjects is not exhaustive, but illustrates the diversity and the importance of recent contributions to the field.

Holonomy of singular foliations and Kähler–Einstein metrics. The main results of the lecture presented by Jorge Pereira were pointing towards a conjecture due to Dominique Cerveau and Alcides Lins-Neto concerning holomorphic foliations of codimension one. Given a projective manifold X, a holomorphic foliation $\mathcal{F}$ is a coherent subsheaf of the tangent bundle T_X which is invariant under the Lie bracket, and such that the quotient $T_X/\mathcal{F}$ is torsion-free. Now the problem is to show that any foliation of codimension one is either the pullback of a one-dimensional foliation on a surface or it is transversely projective (i.e. given by a flat meromorphic connection of a vector bundle). Pereira explained a few results extracted from his joint work with Benoît Claudon, Frank Loray and Frédéric Touzet providing a strong evidence in this direction.

Daniel Greb reported on joint work with Henri Guenancia and Stefan Kebekus on the structure of projective varieties with klt singularities and trivial first Chern class. In many respects, their main result can be seen as a singular analogue of the famous Beauville–Bogomolov decomposition theorem. The proof combines subtle techniques from algebraic and differential geometry, Monge–Ampère operators and others, adapted to the singular setting.

Nessim Sibony explained one of his recent contributions (joint with Tien-Cuong Dinh) to a very classical subject: the dynamics of foliations in the projective plane $\mathbb{P}^2$. One of the important tools in the proof of their surprising rigidity results is the theory of densities for currents. This can be seen as the wide generalization of Fulton's deformation to the normal cone in algebraic geometry, and it has become an indispensable technique in modern dynamics.

Higher direct images and positivity. This theme was very present in our workshop, as witnessed by the lectures by Yohan Brunebarbe, Ariyan Javanpeykar, Christophe Mourougane, Mark de Cataldo, Sai-Kee Yeung. Philipp Naumann, who only recently graduated, gave a very interesting lecture in which he presented his approach for no less than the famous and notoriously difficult Griffiths conjecture. His ideas have their origin in the positivity properties of direct images of twisted relative canonical bundles, combined with recent developments by Datar–Székelyhidi concerning the Kähler–Ricci flow. We are eagerly waiting for his article to appear!

Effective divisors and cycle spaces. Daniel Barlet, a long-time leader of the field, started his presentation with his view on generation change.

> My first participation at this meeting was in 1972, and looking in the audience now I am very surprised to see that many young people now turned 60 . . .

Barlet explained ideas for the proof of an old *folklore* conjecture which roughly states that the limit of a family of projective varieties is Moishezon. Important results concerning the cycle spaces which he has developed over many years are playing a crucial role.

The properties of the central fiber of families of hypersurfaces of smooth projective manifolds over the pointed unit disk were at the heart of Fabrizio Catanese's talk. He explained some of the main results obtained in collaboration with Yongnam Lee by sketching the proof and providing many instructive examples.

Martin Möller presented his joint work with Jan Bruinier, in which they are studying the structure of the cone of effective divisors on the moduli space of K3 surfaces. Their work has important connections with the theory of automorphic forms.

In the first part of his talk, Lawrence Ein explained some of the classical results concerning the Hilbert scheme parametrizing length d zero-dimensional subschemes of a fixed scheme X. In this way he set the stage for the new results he obtained recently in a joint work with Xudong Zheng. Finally, he shared generously his ideas about the future important questions in this field by proposing a few problems.

Extension theorems and classification results. A conjectural decomposition theorem for compact Kähler manifolds X with nef anticanonical bundle emerged from the fundamental work of Jean-Pierre Demailly, Thomas Peternell and Michael Schneider on this topic. Junyan Cao presented his recent progress in this direction. Together with Andreas Höring, he obtained a strong evidence for the conjecture by completely solving the case where X is projective. As always, the proof looks very natural, though technically demanding.

Our workshop ended with the impressive lecture by Jean-Pierre Demailly. He explained a version of the Ohsawa–Takegoshi extension theorem in singular setting, meaning that the section to be extended is defined over a possibly non-reduced scheme. Many participants felt that this was a "classical" Demailly talk, with crystal clear motivation and arguments of the –technically very demanding– proof. It is very safe to predict that his results will have important applications, given that they incorporate virtually all the known extension and injectivity theorems that we are aware of.

Acknowledgement: The MFO and the workshop organizers would like to thank the National Science Foundation for supporting the participation of junior researchers in the workshop by the grant DMS-1641185, "US Junior Oberwolfach Fellows". Moreover, the MFO and the workshop organizers would like to thank the Simons Foundation for supporting Mark A. De Cataldo in the "Simons Visiting Professors" program at the MFO.

Workshop: Komplexe Analysis

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Abstracts

Hilbert schemes of points of singular varieties

Lawrence Ein
(joint work with Xudong Zheng)

Grothendieck introduced Hilbert schemes to parametrize the closed subschemes of a given scheme X. In the following we will report on some recent progress in studying $\mathrm{Hilb}^d(X)$, which is the Hilbert scheme parametrizing length d zero-dimensional closed subschemes of X. In the following we will assume the base field is the complex numbers.

If X is a smooth irreducible curve, then $\mathrm{Hilb}^d(X)$ is just the d-th symmetric product of X. A famous theorem of Fogarty [6] says that if X is a smooth irreducible surface, then $\mathrm{Hilb}^d(X)$ is a smooth irreducible variety of dimension $2d$. Briançon [3] showed that if $p \in X$, then the punctual Hilbert scheme $\mathrm{Hilb}^d_p(X)$, which parametrizes closed subschemes of length d supported at p, is irreducible of dimension $d-1$. Moreover, Haigman [8] showed that $\mathrm{Hilb}^d_p(X)$ is reduced. If X is a smooth variety of dimension $n \geq 3$, then Iarrobino [9] showed that $\mathrm{Hilb}^d(X)$ is reducible for $d \gg 0$. For an irreducible reduced singular curve X, the classical theorem of Altman–Iarrobino–Kleiman [1] and Rego [10] says that $\mathrm{Hilb}^d(X)$ is irreducible for every d if and only if X has locally planar singularities. One sees that the inequality on the dimension of the tangent space at a singular point is obviously necessary. Otherwise $\mathrm{Hilb}^2_p(X)$, the punctual Hilbert scheme supported at a singular point p, would have dimension greater or equal to two, which implies that $\mathrm{Hilb}^2(X)$ is already reducible. From that one sees that $\mathrm{Hilb}^d(X)$ is reducible for all $d \geq 2$. Conversely if X is a curve in a smooth surface S, then $\mathrm{Hilb}_d(X)$ is defined as the zero scheme of rank d tautological vector bundles on $\mathrm{Hilb}^d(S)$. It follows that each irreducible component of $\mathrm{Hilb}^d(X)$ has dimension greater or equal to d. Using the fact that for a singular point p of X, $\mathrm{Hilb}^a_p(X) \subset \mathrm{Hilb}_p(S)$, so the dimension of $\mathrm{Hilb}^a_p(X)$ is less than or equal to $a-1$ by Briançon's theorem. One sees that the subvariety in $\mathrm{Hilb}^d(X)$ corresponding to those subschemes with nonempty intersection with the singular set of X has dimension at most $d-1$. We conclude that $\mathrm{Hilb}_d(X)$ is irreducible.

We see the only remaining case where the irreducibility of $\mathrm{Hilb}_d(X)$ for all d is undecided is the case when X is a singular surface. We can start with the following easy observations.

Theorem 1. *Let X be an irreducible reduced surface.*
(1) If there is a point $p \in X$ such that the dimension of the tangent space of X at p is ≥ 5, then $\mathrm{Hilb}^d(X)$ is reducible for all $d \geq 2$.
(2) If there is a point $q \in X$ such that $\mathrm{Mult}_q(X) \geq 5$, then $\mathrm{Hilb}^d(X)$ is reducible for $d \gg 0$.

The following nice theorem was proved by my student Xudong Zheng last year under my supervision.

Theorem 2. [12] *Let X be an irreducible surface with at worst rational double point singularities. Then* $\mathrm{Hilb}^d(X)$ *is irreducible for all d.*

The following are some of the key ingredients to the proof of the above theorem. Let Z be a zero-dimensional closed subscheme of X supported at a rational double point p in X. Let R be the local ring of X at p. Consider a resolution of the ideal I_Z of the following form:

$$0 \to F \to R^{\oplus r} \to I_z \to 0.$$

Observe that F is a direct sum of irreducible reflexive modules. There are only a finite number of irreducible modules on a rational double point and they are completely classified [2], [7]. Now we consider the morphism $\varphi\colon F \to R^{\oplus r}$. Using the various extensions between the reflexive modules, one shows that one can deform the module F and the morphism φ to a morphism $\varphi'\colon R^{\oplus r-1} \to R^{\oplus r}$. This will show that the closed subscheme Z is smoothable in X and it would follow that $\mathrm{Hilb}_d(X)$ is irreducible.

Cartwright and his coworkers [4] found the following very surprising behavior of $\mathrm{Hilb}^8(\mathbb{C}^4)$. Their theorem says that $\mathrm{Hilb}^8(\mathbb{C}^4)$ has exactly two irreducible components. The general point of the first component corresponds to eight distinct points and this component is of dimension 32. The second component is of dimension 25. Each closed subscheme Z corresponding to a point in the second component is supported at only one point p in $\mathbb{C}^4$. Let $\mathfrak{m}$ be the maximal ideal of p. Then I_Z is generated by $\mathfrak{m}^3$ plus a seven-dimensional subspace in $\mathfrak{m}^2/\mathfrak{m}^3$. Note that a closed subscheme Z corresponding to a point in the second component but not in the first is not smoothable in $\mathbb{C}^4$. Furthermore they show that at a general point q of the second component the tangent space at q of $\mathrm{Hilb}^8(\mathbb{C}^4)$ is of dimension 25. So $\mathrm{Hilb}^8(\mathbb{C}^4)$ is smooth at such a point q.

Suppose X is the cone over a twisted cubic curve. One can show that there is such a length 8 closed subscheme $Z \subset X$ which is only in the second component. This shows that the closed subscheme Z of length 8 in X will not be smoothable in X. Hence $\mathrm{Hilb}^8(X)$ is reducible. From this example, one can wonder whether the following is true: For a singular irreducible surface X, it holds that $\mathrm{Hilb}^d(X)$ is irreducible for every d if and only X has at worst rational double points as singularities.

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Holonomy representation of quasi-projective leaves

Jorge V. Pereira
(joint work with B. Claudon, F. Loray, F. Touzet)

1. Motivation

Singular holomorphic foliations of dimension one on projective manifolds abound. Since Frobenius integrability condition is automatically satisfied for rank one subsheaves of the tangent bundle, any rational vector field determines a foliation. It is known that for sufficiently general rational vector fields the corresponding foliation is dynamically interesting: according to [9, Theorem 1] every leaf is Zariski dense and it is not tangent to any other higher dimensional foliation (or web).

Foliations with leaves of higher dimensions are substantially harder to produce. One can of course start with a one dimensional foliation $\mathcal{F}$ on a projective manifold X, consider a surjective morphism $\pi\colon Y \to X$ and take the pull-back foliation $\pi^*\mathcal{F}$. If the general fiber of π is positive dimensional then we end up with a foliation of dimension greater than one.

Another natural way to produce higher dimensional foliations is through flat meromorphic connections. Given a flat meromorphic connection ∇ on a vector bundle E over a projective manifold X, one has a natural foliation on the total space of E determined by the flat sections of ∇. Foliations produced in this way are invariant by the natural action of $\mathbb{C}^*$ on E and therefore determine foliations on $\mathbb{P}(E)$.

We can combine both constructions above to obtain the class of codimension one *transversely projective* foliations. A codimension one foliation $\mathcal{F}$ on a projective manifold X is called transversely projective if there exists a rank two vector bundle E endowed with a flat meromorphic connection ∇, and a rational section $\sigma\colon X \dashrightarrow \mathbb{P}(E)$ such that $\mathcal{F} = \sigma^*\mathcal{G}$, where $\mathcal{G}$ is the foliation on $\mathbb{P}(E)$ determined by ∇.

Every single known example of codimension one foliation on projective manifolds of dimension at least three fits into one of the descriptions above. The conjecture below due to Cerveau and Lins Neto predicts that every codimension one foliation on a projective manifold fits into this description.

Conjecture 1. *Let $\mathcal{F}$ be a codimension one holomorphic foliation on a projective manifold X. Then one of the following assertions holds true.*

(1) *The foliation $\mathcal{F}$ is transversely projective; or*
(2) *There exists a rational map $\pi\colon X \dashrightarrow Y$ to a projective surface Y and a foliation $\mathcal{G}$ on Y such that $\mathcal{F} = \pi^*\mathcal{G}$.*

2. Main Result

The main purpose of the talk is to discuss the result below, which provides some evidence in favor of the Cerveau–Lins Neto conjecture.

Theorem 2. *Let X be a quasi-projective manifold, and let $\rho\colon \pi_1(X,\cdot) \to \mathrm{Diff}(\mathbb{C},0)$ be a representation of the fundamental group of X in $\mathrm{Diff}(\mathbb{C},0)$, the group of formal biholomorphisms of $(\mathbb{C},0)$. Let also Γ be the image of ρ. If Γ is not virtually abelian then there exists a morphism $f\colon X \to C$ from X to an orbicurve C, and a representation $\varrho\colon \pi_1^{\mathrm{orb}}(C,\cdot) \to \Gamma/\mathrm{Center}(\Gamma)$ fitting into the commutative diagram*

$$\begin{array}{ccc} \pi_1(X,\cdot) & \xrightarrow{\quad\rho\quad} & \Gamma \\ \Big\downarrow{\scriptstyle f_*} & & \Big\downarrow \\ \pi_1^{\mathrm{orb}}(C,\cdot) & \xrightarrow{\quad\varrho\quad} & \dfrac{\Gamma}{\mathrm{Center}(\Gamma)} \end{array}$$

where the unlabeled arrow is the natural quotient morphism.

Representations like the one in the statement of the theorem above appear as the holonomy representation of quasi-projective leaves of codimension one foliations.

Theorem 2 was first proved for representations of fundamental groups of projective manifolds [4]. The case of representations of fundamental groups of quasi-projective manifolds was treated more than one year afterwards in [5].

3. Elements of the proof

The proof of Theorem 2 splits into three cases. In each of them, the proof makes use of a distinct criterion for the existence of fibrations on quasi-projective manifolds. Each of the cases is briefly discussed below under the label identifying the corresponding criterion.

3.1. **Cohomology jumping loci.** When the representation has infinite linear part, no matter if X is projective or quasi-projective, the fibration $f\colon X \to C$ is produced by applying results on cohomology jumping loci [1] to the first non-abelian truncation of ρ. The result for the original representation is deduced from Deligne's Theorem on the structure of the monodromy quasi-projective fibrations [6, Corollary 4.2.9].

3.2. **Castelnuovo–De Franchis–Catanese.** When the representation is tangent to the identity and the manifold X is projective, the fibration $f\colon X \to C$ is produced by means of the Castelnuovo–De Franchis–Catanese criterion for the existence of fibrations. Looking at the first non-abelian truncation of the image Γ of the representation, one is able to produce two linearly independent elements $a, b \in H^1(\Gamma, \mathbb{C})$ with $a \wedge b = 0$. The Castelnuovo–De Franchis–Catanese Theorem [3, Theorem 1.10] implies that the truncated representation factors through an orbicurve. As in the previous case, the factorization of the original representation is deduced from Deligne's Theorem.

3.3. **Three disjoint homologous divisors.** When the representation is tangent to the identity, the ambient manifold is quasi-projective, and the representation does not extend to a representation of a projective compactification, then the proof uses a result by Totaro [10, Theorem 2.1] which asserts the existence of a fibration on a give projective manifold whenever it carries three pairwise disjoint divisors with proportional Chern classes. Of course, such a fibration has (rational multiples of) the three disjoint divisors among its fibers. For alternative proofs of Totaro's result and variations on the statement see [8] and [2].

In order to produce the three disjoint divisors, the proof analyzes the representation in a neighborhood of infinity and uses the residual finiteness of Γ in order to produce finite coverings of X which compactify to projective manifolds having the sought three disjoint divisors with proportional Chern classes. The arguments are very similar to the ones carried out to investigate representations of fundamental groups of quasi-projective manifolds in $\mathrm{SL}(2, \mathbb{C})$ which are not quasi-unipotent at infinity, see [7, Theorem A].

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Volumes of subvarieties of compactifications of hermitian locally symmetric spaces

Yohan Brunebarbe

Let $\mathcal{D}$ be a bounded symmetric domain. If Γ is a torsion-free arithmetic lattice in $\mathrm{Aut}(\mathcal{D})$, then due to the work of Satake and Baily–Borel the quotient $X_\Gamma := \Gamma\backslash D$ is known to admit a unique structure of smooth quasi-projective complex variety compatible with its complex manifold structure. A complex algebraic variety obtained in this way is called an arithmetic locally symmetric variety. The variety X_Γ has numerous hyperbolicity properties: it is Brody hyperbolic (because its universal cover is a bounded domain), all its subvarieties are of log general type [7] and have a big logarithmic cotangent bundle [1]; see also [3]. However, its Satake–Baily–Borel compactification $\overline{X}^*_\Gamma$ might in general be very far from being hyperbolic. For example, the moduli space $\mathcal{A}_2(3)$ of principally polarized abelian surfaces with a level three structure, which is an arithmetic locally symmetric variety covered by the Siegel upper half space $\mathbb{H}^2$ of degree 2, is known to be birational to $\mathbb{P}^3$.

From the point of view of hyperbolicity, the situation becomes much improved if one allows oneself to take a finite index subgroup $\Gamma' \subset \Gamma$, or equivalently if one looks at the corresponding finite étale cover $X_{\Gamma'}$ of X_Γ. Indeed, Nadel [5] proved that there exists a finite index subgroup $\Gamma' \subset \Gamma$ such that the Satake–Baily–Borel compactification $\overline{X}^*_{\Gamma'}$ of $X_{\Gamma'}$ is Brody hyperbolic. Moreover, we proved in [2] that Γ' can be chosen such that in addition every subvariety of $\overline{X}^*_{\Gamma'}$ is of general type. In another direction, building on earlier work of Noguchi [6], Hwang and To [4] proved that for any $g \geq 1$ there exists a finite index subgroup $\Gamma_g \subset \Gamma$ such that every smooth projective curve admitting a non-constant map to $\overline{X}^*_{\Gamma_g}$ has genus at least g. Our main result is a common generalization of the results of [2] and [6, 4]:

Theorem 1. *Let $\mathcal{D}$ be a bounded symmetric domain and Γ be an arithmetic lattice in $\mathrm{Aut}(\mathcal{D})$. For any $v > 0$, there exists a finite index subgroup $\Gamma_v \subset \Gamma$ such that every smooth projective variety admitting a generically finite map to $\overline{X}^*_{\Gamma_v}$ satisfies $\mathrm{vol}(X) \geq v$.*

Recall that the volume $\mathrm{vol}(X) := \mathrm{vol}(\omega_X)$ of a smooth proper algebraic variety X is a non-negative real number which is a measure of the positivity of its canonical bundle ω_X. More generally, for any line bundle L on the n-dimensional variety X we set

$$\mathrm{vol}(L) := \limsup_{k\to\infty} \frac{n!}{k^n} \cdot h^0(X, L^{\otimes k}).$$

Note that two birational smooth proper algebraic varieties have the same volume because their canonical algebras are canonically isomorphic. Clearly, a smooth proper algebraic variety X is of general type exactly when $\mathrm{vol}(X) > 0$. If X is a smooth projective curve of genus g, then $\mathrm{vol}(X) = 2g2$.

The proof of Theorem 1 follows the strategy introduced in [2]. A key new input is provided by the following new result:

Theorem 2. *Let $\mathcal{D}$ be a bounded symmetric domain and Γ be an arithmetic lattice in* $\mathrm{Aut}(\mathcal{D})$. *For any $v > 0$, there exists a torsion-free subgroup $\Gamma_v \subset \Gamma$ of finite index such that for any smooth toroidal compactification X of X_{Γ_v} with boundary D and any smooth projective variety Y with a generically finite map $f\colon Y \to X$, we have* $\mathrm{vol}\left(f^*\omega_X(D)\right) \geq v$.

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Cones of Heegner divisors

Martin Möller

(joint work with Jan H. Bruinier)

The pseudo-effective cone $\mathrm{Eff}(X)$ of a projective algebraic variety X is an important invariant that is notoriously hard to compute. If X is a moduli space, the modular interpretation may provide insights to the structure of this cone. E.g. it was shown recently by Mullane ([5]) that the effective cone $\mathrm{Eff}(\overline{\mathcal{M}}_{g,n})$ of the moduli space of stable pointed curves is not finitely generated for $g \geq 2$ and $n \geq g+1$, contrasting the finite generatedness for $g = 0$ and small n.

Here we study the pseudo-effective cone on orthogonal Shimura varieties, in particular the case of the moduli space $\mathcal{F}_{2d}$ of polarized K3 surfaces, to which we restrict in this summary. This moduli space contains a class of natural divisors, the Noether–Lefschetz divisors, that may equivalently be described by lattice conditions and which are also called Heegner divisors. The question on the finite generatedness of $\mathrm{Eff}(\mathcal{F}_{2d})$ was raised in [6] in connection with computing Kodaira dimensions of $\mathcal{F}_{2d}$. In [3] we are able to determine the structure of the subcone generated by the irreducible components of the Heegner divisors.

Theorem 1. *The cone* $\mathrm{Eff}^{\mathrm{PH}}(\mathcal{F}_{2d})$ *generated by the primitive Heegner divisors is a finitely generated rational polyhedral cone.*

The question whether $\mathrm{Eff}^{\mathrm{PH}}(\mathcal{F}_{2d}) \subseteq \mathrm{Eff}(\mathcal{F}_{2d})$ is a strict subcone or whether the two cones coincide remains an interesting open problem.

By the work of Borcherds [4] and Bruinier [2], together with the recent result of Bergeron, Li, Millson and Moeglin [1], the rational Picard group of $\mathcal{F}_{2d}$ is isomorphic to the dual space of $M^0_{k,L}$, the vector space of vector-valued modular forms of weight k for the Weil representation that are zero at all but one cusp. The main theorem is thus a refinement (due to considerations of the irreducible components of Heegner divisors) of the following statement about modular forms. It contains, for L simply a hyperbolic plane, a statement about usual modular forms for $\mathrm{SL}_2(\mathbb{Z})$.

Theorem 2. *Let L be a lattice of signature (b^+, b^-) that splits off a hyperbolic plane. We suppose that $k \geq 2$ and $2k - b^+ + b^- \equiv 4 \pmod 8$. Then the cone $\mathcal{C}$ generated by the coefficient functionals $c_{m,\mu}$ on the space of weight k almost cusp forms $M^0_{k,L}$ for the lattice L (where $\mu \in L^\vee/L$ and $m \in (\mathbb{Z} + Q(\mu)) \cap \mathbb{Q}_{>0}$) is a finitely generated rational polyhedral cone.*

The proof is based on the geometric observation that if the generating rays of a cone converge to an interior ray of the cone, then the cone is in fact finitely generated. To verify this convergence we use estimates for the Fourier coefficients of elements in $M^0_{k,L}$ and to show the interior ray property we use pairings with appropriate weakly holomorphic modular forms of weight $2 - k$.

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An approach to the Griffiths conjecture

Philipp Naumann

In the talk we present a strategy to prove or to investigate the Griffiths conjecture, which asserts that any ample vector bundle over a compact complex manifold is Griffiths positive. We make use of curvature formulas for direct image metrics and recent results for the Kähler–Ricci flow on Kähler–Einstein Fano manifolds.

Let $E \to S$ be a holomorphic vector bundle of rank r over a compact complex manifold. The projectivized bundle $\mathcal{X} := \mathbb{P}(E^*)$ carries the tautological line bundle $\mathcal{O}_E(1)$. There is an isomorphism

$$f_*(\mathcal{O}_E(1)) \cong E \quad \text{where} \quad f\colon \mathbb{P}(E^*) \to S.$$

We start with a hermitian metric H on E, which induces a metric h on $\mathcal{O}_E(1)$. Its curvature form is denoted by

$$\omega_{\mathcal{X}} := -\sqrt{-1}\partial\bar{\partial}\log h.$$

We choose local holomorphic coordinates (s^k) on the base S and denote the horizontal lift of a tangent vector ∂_k by v_k (see [7]). We obtain the Kodaira–Spencer form

$$A_k := \bar{\partial}(v_k)|_{X_s}$$

and define the geodesic curvature in the direction of k, ℓ by

$$\varphi_{k\bar{\ell}} = \langle v_k, v_\ell \rangle_{\omega_{\mathcal{X}}}.$$

By using the local expression for the curvature form for the indued metric h on $\mathcal{O}_E(1)$, we see that the matrix $(\varphi_{k\bar{\ell}})$ is positive definite iff the hermitian bundle (E, H) is Griffiths positive. We have the following results:

Proposition 1. *The equation*

$$(\Box - r)\varphi_{k\bar{\ell}} = A_k \cdot A_{\bar{\ell}} - f^*(\operatorname{tr} R^E_H)_{k\bar{\ell}},$$

holds, where R^E_H is the curvature of (E, H).

Proposition 2. *The Kodaira–Spencer forms A_k are harmonic, hence zero.*

Together with a curvature formula for L^2 metrics on general direct images due to To and Weng ([8]), we show by a computation that the curvature of the direct image metric on $f_*\mathcal{O}_E(1) = E$ gives back the original curvature of (E, H) up to a constant factor.

Now we forget about the metric H on E and start with a positive hermitian metric h on $\mathcal{O}_E(1)$. The forms $-\sqrt{-1}\partial\bar{\partial}\log h$ on the fibers X_s induce a metric on $K^{-1}_{\mathcal{X}/S}$ which we denote by $(-\sqrt{-1}\partial\bar{\partial}\log h)^n$. Furthermore, the metric h on $\mathcal{O}_E(1)$ induces a direct image metric on $\det E \cong f_*(K_{\mathcal{X}/S} \otimes \mathcal{O}_E(r))$ (see [1]) and hence also a hermitian metric on $f^*\det(E)^{-1}$, which we normalize in an appropriate way. By a careful analysis of the previous computation, we obtain the following result:

Theorem 3. *If the canonical isomorphism*

$$K^{-1}_{\mathcal{X}/S} \cong \mathcal{O}_E(r) \otimes f^*\det(E)^{-1} \tag{1}$$

is an isometry, then the direct image metric on $f_*(\mathcal{O}(1)) = E$ is Griffiths positive.

The relation stated in the theorem implies that the curvature form of h defines Kähler–Einstein metrics on the fibers, which are projective spaces. But the Fubini–Study metrics on $\mathbb{P}(E^*_s)$ are in one-to-one correspondence with hermitian structures on E^*_s, hence the theorem should actually give a characterisation of those metrics on $\mathcal{O}_E(1)$ which are induced by metrics on E. Furthermore it gives a link between the Kähler–Einstein problem on projective spaces and the Griffiths conjecture. Therefore we propose to study the relative Kähler–Ricci flow on the bundle $\mathcal{O}_E(r)$ instead of $K^{-1}_{\mathcal{X}/S}$ (cf. [3]).

For this purpose we start with a hermitian metric h on $\mathcal{O}_E(1)$ which is positive along the fibers. We use the notation $h^r = e^{-\varphi}$ by which we mean the pointwise norm squared of a local trivializing section of $\mathcal{O}_E(r)$. The associated L^2 norm of a section $u_s \in H^0(X_s, K_{X_s} \otimes \mathcal{O}_E(r)) \cong \det E$ can the be written as

$$||u_s||^2 := \int_{X_s} |u_s|^2 e^{-\varphi} := \int_{X_s} c_n |s|^2 e^{-\varphi} dz \wedge d\overline{z},$$

where dz is a local trivializing section of K_{X_s}. Here all (local) sections are chosen to be compatible with the isomorphism (1). Now $e^{-\psi} := ||u_s||^2$ is a metric on $f^* \det E$. Hence

$$e^{-\varphi} := e^{-\varphi} e^{\psi}$$

is a metric on $K^{-1}_{\mathcal{X}/S}$ which is the pointwise norm squared of the trivializing section $(dz)^{-1}$. But more globally this is a volume form on X_s, in other words

$$\mu_\varphi := e^{-\varphi} \bigg/ \int_{X_s} |u_s|^2 e^{-\varphi}$$

is a volume form on each fiber X_s, which does not depend on the choice of the local trivializing section u. Moreover we have

Proposition 4. *μ_φ is a probability measure.*

Thus we define the flow of hermitian metrics φ_t on $\mathcal{O}_E(r)$ which are all positive along the fibers X_s by

$$\dot{\varphi}_t = \log\left(\frac{(dd^c\varphi_t)^n/n!}{\mu_\varphi}\right), \quad \varphi_0 = \varphi. \tag{2}$$

For the convergence we would like to argue like in [3], where the fiberwise flow is formulated for the bundle $K^{-1}_{\mathcal{X}/S}$ for a family of Kähler–Einstein Fano manifolds with fiberwise discrete automorphism group. But instead we have to invoke the recent developments concerning the Kähler–Ricci flow on Fano Kähler–Einstein manifolds ([9, 5, 6, 4]) that study the case of a non discrete automorphism group. Using these results, the argument is roughly as follows: For the normalized flow (2) the time derivative $\dot{\varphi}_t$ coincides with the normalized Ricci potential for which we have Perelman's estimates. The convergence of $\dot{\varphi}_t$ to zero with exponential rate can be integrated to show that the sequence (φ_t) is Cauchy. Thus the flow converges in C^∞ to a hermitian metric φ_∞ on $\mathcal{O}_E(r)$ which is positive along the fibers.

Remark 5. As an observation we remark that the hermitian metric on $K^{-1}_{\mathcal{X}/S}$ given by the Monge–Ampère measure $(dd^c\varphi_t)^n/n!$ splits in the limit into the product metric

$$e^{-\varphi_\infty} \cdot \left(\int |u_s|^2 e^{-\varphi_\infty}\right)^{-1},$$

which is of course what we expect in the Kähler–Einstein limit.

Remark 6. The construction works for any holomorphic vector bundle $E \to S$. In particular if $\det E$ is trivial, then the bundle $K^{-1}_{\mathcal{X}/S}$ is canonically isomorphic to $\mathcal{O}_E(r)$ and the flow (2) coincides with the normalized flow in [3].

Now we turn back to the Griffiths problem and start with a positive metric h on $\mathcal{O}_E(1)$. Then we run the flow (2) for the positive initial metric $h^r = e^{-\varphi_0}$. In order to apply Theorem 3 to the limit metric, we are left to prove that the positivity of φ is preserved under the flow (2). The obstruction to apply the usual maximum principles for the heat equation is the term

$$\left(\int |u_s|^2 e^{-\varphi_\infty} \right)^{-1},$$

whose curvature is negative if $e^{-\varphi}$ is positive (see [2]). Thus this question remains open.

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Chern's conjecture for special affine manifolds

Bruno Klingler

An affine manifold X in the sense of differential geometry is a differentiable manifold admitting an atlas of charts with values in an affine space, with locally constant affine change of coordinates. Equivalently, it is a manifold whose tangent bundle admits a flat torsion free connection. Around 1955, Chern conjectured that the Euler characteristic of any compact affine manifold has to vanish. Kostant and Sullivan [5] proved this conjecture in the case where X is a compact quotient $\Gamma\backslash\mathbb{R}^n$ of $\mathbb{R}^n$ by a discrete subgroup Γ of the affine group, acting properly discontinuously on $\mathbb{R}^n$. In this talk we explained our proof [4] of Chern's conjecture in the case where X admits a parallel volume form (such a manifold is called *special affine*):

Theorem 1. *If X is a connected closed special affine manifold then $\chi(X) = 0$.*

Let us now describe the strategy of the proof of Theorem 1. We start with the classical:

Proposition 2. *Let X be a connected oriented closed n-manifold and E an oriented real vector bundle on X of rank $r > 0$, with total space $\mathcal{E}$. The Euler class $e_{\mathbb{R}}(E) \in H^r(X, \mathbb{R})$ vanishes if and only if the natural morphism between cohomology with compact support and usual cohomology*

$$\mathbb{R} \simeq H^r_c(\mathcal{E}, \mathbb{R}) \longrightarrow H^r(\mathcal{E}, \mathbb{R}) \simeq H^r(X, \mathbb{R})$$

vanishes.

We study differential forms on $\mathcal{E}$ using the geometry of $\mathcal{E}$. When E is a mere bundle, the only natural geometric structure on $\mathcal{E}$ is the foliated structure given by the projection $\pi \colon \mathcal{E} \longrightarrow X$. If in addition we assume that the bundle E is endowed with a flat connection ∇, the total space $\mathcal{E}$ has a natural *local product structure.* For any manifold M endowed with a local product structure, the de Rham complex of sheaves of real differential forms $(\Omega^\bullet_M, d)$ is enriched with a natural bigrading $(\Omega^{\bullet,\bullet}_{\mathcal{E}}, d', d'')$, d' being the differential in the "horizontal" direction and d'' the one in the "vertical" direction. This bigrading defines two filtrations ${}_{d'}F^\bullet$ and ${}_{d''}F^\bullet$, on $H^\bullet_c(\mathcal{E}, \mathbb{R})$ and also on $H^\bullet(\mathcal{E}, \mathbb{R})$. As usual the graded pieces of these filtrations are computed by spectral sequences ${}_{d'}E^{\bullet,\bullet}_\bullet$ and ${}_{d''}E^{\bullet,\bullet}_\bullet$ (both in the compact support case and the usual one). I don't know how to compute these filtrations for a general local product structure.

On the other hand, when M is the total space $\mathcal{E}$ of a flat bundle E on X, one can compute these filtrations, with the exception of ${}_{d''}F^\bullet$ on $H^\bullet_c(\mathcal{E}, \mathbb{R})$.

The morphism $H^\bullet_c(\mathcal{E}, \mathbb{R}) \longrightarrow H^\bullet(\mathcal{E}, \mathbb{R})$ we want to study is induced by a morphism of spectral sequences $\varphi^{\bullet,\bullet}_\bullet \colon {}_{d''}E^{\bullet,\bullet}_{c,\bullet} \longrightarrow {}_{d''}E^{\bullet,\bullet}_\bullet$ and the relation between the local product structure on $\mathcal{E}$ and the vanishing of $e_{\mathbb{R}}(E)$ is given by the following refinement of Proposition 2:

Proposition 3. *Let X be a connected oriented closed n-manifold. Let E be an oriented flat real vector bundle on X of rank $r > 0$ with total space $\mathcal{E}$ and projection $\pi : \mathcal{E} \longrightarrow X$. The Euler class $e_{\mathbb{R}}(E) \in H^r(X, \mathbb{R})$ vanishes if and only if the map*

$$\varphi^{0,r}_\infty : \mathrm{Gr}^0_{{}_{d''}F^\bullet} H^r_c(\mathcal{E}, \mathbb{R}) = {}_{d''}E^{0,r}_{c,\infty} \longrightarrow {}_{d''}E^{0,r}_\infty = \mathrm{Gr}^0_{{}_{d''}F^\bullet} H^r(\mathcal{E}, \mathbb{R}) = \mathbb{R}$$

vanishes.

We are mainly interested in the case $n = r$. In this case the local product structure on $\mathcal{E}$ is called a *para-complex structure* on $\mathcal{E}$. The bigrading $(\Omega^{\bullet,\bullet}_{\mathcal{E}}, d', d'')$ is formally similar to the bigrading of the complex analytic de Rham complex on a complex manifold, *except that there is no involution of $(\Omega^\bullet_{\mathcal{E}}, d)$ exchanging d' and d'' (like conjugation in the complex setting).*

Suppose now that the vector bundle E is the tangent bundle TX. Any linear connection ∇ on TX defines a natural almost complex structure I on $\mathcal{E}$. Moreover Dombrowski [2] proved that I is a complex structure if and only if ∇ is flat *and* *torsion-free*, i.e. X is an affine manifold. This complex structure was further studied by Cheng and Yau [1].

The interplay of this complex structure on $\mathcal{E}$ and the natural para-complex structure on $\mathcal{E}$ is our main tool for studying the vanishing of $e_{\mathbb{R}}(TX)$: the total space $\mathcal{E}$ of the tangent bundle of an affine manifold acquires a very rich *para-hypercomplex structure*, a notion analogous to a hypercomplex structure in complex geometry. In particular *the standard para-complex structure on* $\mathcal{E}$ *is the value at* $\theta = 0 \in [0, 2\pi[$ *of an* S^1*-family of para-complex structures, induced by a canonical* $SO(2)$*-action on* $T\mathcal{E}$. Such an S^1-family simply does not exist if ∇ is flat but has non-trivial torsion. Notice moreover that for $\theta \neq 0 \mod \pi/2$, the para-complex structure on $\mathcal{E}$ corresponding to θ does not come from a flat bundle structure on TX.

For each $\theta \in S^1$ the corresponding para-complex structure defines, as above, a filtration ${}_{d''_\theta}F^\bullet$, on $H^\bullet_c(\mathcal{E}, \mathbb{R})$ and on $H^\bullet(\mathcal{E}, \mathbb{R})$. It satisfies ${}_{d''_{\theta=0}}F^\bullet = {}_{d''}F^\bullet$ and ${}_{d''_{\theta=\pi/2}}F^\bullet = {}_{d'}F^\bullet$. The main idea in the proof of Theorem 1 is that while the filtrations ${}_{d'}F^\bullet$ and ${}_{d''}F^\bullet$ are unrelated when $\mathcal{E}$ is the total space of a mere flat bundle, the S^1-family of para-complex structures on the total space $\mathcal{E}$ of the tangent bundle of an affine manifold induces an S^1-family of filtrations interpolating between them. Technically speaking, we construct a spectral sequence in the category of sheaves over S^1, obtaining a morphism

$$\varphi^{0,n}_{\infty, S^1} \colon {}_{d''}\mathcal{E}^{0,n}_{c,\infty} \longrightarrow {}_{d''}\mathcal{E}^{0,n}_{\infty}$$

of sheaves over S^1. The subtle relation between this spectral sequence of sheaves, and the spectral sequence we are interested in at the point $\theta = 0$, lies in the existence of a canonical factorisation of the morphism $\varphi^{0,n}_\infty$ as

$$(1) \quad \varphi^{0,n}_\infty \colon {}_{d''}E^{0,n}_{c,\infty} \xrightarrow{\sim} \left({}_{d''}\mathcal{E}^{0,n}_{c,\infty}\right)_{\theta=0} \xrightarrow{\left(\varphi^{0,n}_{\infty,S^1}\right)_{\theta=0}} \left({}_{d''}\mathcal{E}^{0,n}_{\infty}\right)_{\theta=0} \longrightarrow {}_{d''}E^{0,n}_{\infty} \ .$$

A crucial feature of the sheaves ${}_{d''}\mathcal{E}^{0,n}_{c,\infty}$ and ${}_{d''}\mathcal{E}^{0,n}_{\infty}$ on S^1 is their constructibility, as they are quotients of the constant sheaf $\mathbb{R}_{S^1}$. Suppose now that X is *special* affine. We use this constructibility, the fact that ${}_{d''_{\theta=\pi/2}}F^\bullet = {}_{d'}F^\bullet$ and the existence of an affine volume form on X to show that the sheaf ${}_{d''}\mathcal{E}^{0,n}_{\infty}$ is identically zero. It follows from (1) that the morphism $\varphi^{0,n}_\infty$ vanishes. By Proposition 3, this finishes the proof of Theorem 1.

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Borel's theorem for the moduli of canonically polarized varieties

Ariyan Javanpeykar
(joint work with Robert Kucharczyk, Ruiran Sun, Kang Zuo)

In [1] Borel showed that, for a finite type reduced scheme S over $\mathbb{C}$ and arithmetic locally symmetric variety X, every holomorphic map $S^{\mathrm{an}} \to X^{\mathrm{an}}$ is algebraic. The first thing we do in this talk is formalizing this property: a locally finite type scheme over $\mathbb{C}$ is *Borel hyperbolic* if, for all finite type reduced schemes S over $\mathbb{C}$, every holomorphic map $S^{\mathrm{an}} \to X^{\mathrm{an}}$ is algebraic. In this terminology, an arithmetic locally symmetric variety is Borel hyperbolic (by the aforementioned theorem of Borel). Moreover, by a theorem of Kobayashi and Kwack, if X is hyperbolically embedded in a proper scheme Y, then X is Borel hyperbolic.

We show that there are Borel hyperbolic varieties which are not Kobayashi hyperbolic (and therefore not hyperbolically embeddable). This motivates studying Borel hyperbolicity independently from Kobayashi hyperbolicity.

We use algebraic arguments to show that a finite type scheme X over $\mathbb{C}$ is Borel hyperbolic if and only if, for all smooth affine curves C over $\mathbb{C}$, every holomorphic map $C^{\mathrm{an}} \to X^{\mathrm{an}}$ is algebraic. In particular, by GAGA, if X is a dense open subscheme of a proper scheme Y, and every holomorphic map $\mathbb{D}^* \to X^{\mathrm{an}}$ extends to a holomorphic map $\mathbb{D} \to Y^{\mathrm{an}}$, then X is Borel hyperbolic. Here $\mathbb{D}$ is the open unit disk in $\mathbb{C}$ and $\mathbb{D}^* = \mathbb{D} \setminus \{0\}$.

Our motivation for establishing such a result is the question of whether the moduli stack of smooth canonically polarized varieties is Borel hyperbolic.

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Supports for Hitchin fibrations

Mark Andrea de Cataldo
(joint work with Jochen Heinloth, Luca Migliorini)

Let $f\colon X \to Y$ be a proper morphism of complex algebraic varieties, with X nonsingular. The decomposition theorem of Beilinson–Bernstein–Deligne–Gabber yields an isomorphism

$$Rf_*\mathbb{Q}_X \cong \bigoplus_{q\geq 0} \bigoplus_{(S,L)} \mathcal{IC}_S(L)[-q],$$

where, for each q, (S,L) ranges in a finite set of pairs, where S is a closed integral subvariety of Y, and L is a semisimple local system defined on some Zariski-dense open subset of S^{reg}. For each q, the set of such pairs is uniquely determined (provided we insist that S is not repeated).

One defines the supports of the morphism f to be the set of varieties $S \subseteq Y$ appearing in the decomposition theorem for $Rf_*\mathbb{Q}_X$.

The "problem of supports" is the problem of determining these varieties.

In general, this is a difficult and subtle problem. For example, Ngô's proof of the fundamental lemma for a complex reductive group G required him to determine the supports of the Hitchin fibration associated with G and a compact Riemann surface G of genus ≥ 2, "with high degree poles" (i.e. the Higgs fields are sections of the adjoint bundles twisted by the canonical bundle tensor a divisor D with $D \gg 0$) over a rather large open subvariety (elliptic locus) of the target of the Hitchin fibration.

In the case when $G = GL(n)$ with $D > 0$, Chaudouard–Laumon proved that the only support is the target of the Hitchin fibration (no extra supports). In the case when $G = SL(n)$ with $D > 0$, I proved that the only supports are the ones already existing over the elliptic loci, namely the target of the Hitchin fibration together with a finite collection of subvarieties naturally associated with the endoscopy data for $SL(n)$ (which can be made explicit).

The case of the Hitchin fibration without poles ($D = 0$) seems to be much more subtle and geometrically more appealing. For example the Hitchin fibration is an algebraically completely integrable system and it appears in the context of non-abelian Hodge theory.

In joint work in progress with Jochen Heinloth and Luca Migliorini, we determine the supports for the Hitchin fibration without poles ($D = 0$) for $G = GL(n)$ over the open subset A^{red} of the target of the Hitchin fibration determined by reduced spectral curves. In this case, A^{ell} is strictly contained in A^{red}, as it corresponds to integral spectral curves.

In order to state our main result, let me introduce a disjoint union decomposition $A^{\text{red}} = \coprod_{n_*} S_{n_*}$, where $n_* = (n_1, \ldots, n_s)$ ranges over the partitions of n and S_{n_*} is the locally closed subvariety corresponding to those spectral curves that can be written as a union of s spectral curves, each for $GL(n_i)$, $i = 1, \ldots, s$.

Theorem 1 (d.–Heinloth–Migliorini)**.** *The supports of the Hitchin fibration that meet A^{red} are exactly the $\overline{S_{n_*}}$.*

Holonomy of singular Kähler–Einstein metrics on klt varieties with trivial canonical divisor

Daniel Greb

(joint work with Henri Guenancia and Stefan Kebekus)

1. Towards a singular version of the Decomposition Theorem

One of the cornerstones of the theory of compact Kähler manifolds with vanishing first real Chern class is the following *Decomposition Theorem*: every such manifold admits a finite, étale cover that splits as a product of a complex torus, a couple of simply-connected *Calabi–Yau (CY) manifolds* (that is, compact Kähler manifolds with trivial canonical bundle and $H^0\big(X, \Omega_X^p\big) = \{0\}$ for all $1 < p < \dim X$), and a couple of *irreducible holomorphic-symplectic (IHS) manifolds* (that is, simply-connected compact Kähler manifolds whose algebra of holomorphic differential forms is generated by an everywhere non-degenerate holomorphic 2-form).

Motivated by the Minimal Model Program as well as by the much higher availability of meaningful examples, it is natural to investigate singular varieties with trivial canonical sheaf. For reasons of availability of certain technical results, we will restrict to the case of projective varieties in the following, where a first step in the direction of a singular version of the Decomposition Theorem is done by the following result, which can also be deduced from the more analytic approach taken by Guenancia in [5].

Theorem 1 (G.–Kebekus–Peternell '11, [4])**.** *Let X be a normal projective variety with at worst canonical singularities, defined over the complex numbers. Assume that the canonical divisor of X is numerically trivial: $K_X \equiv 0$. Then there exists an Abelian variety A as well as a projective variety $\widetilde{X}$ with at worst canonical singularities, a finite cover $f\colon A \times \widetilde{X} \to X$, étale in codimension one, and a decomposition $\mathscr{T}_{\widetilde{X}} \cong \bigoplus \mathscr{E}_i$ such that the following holds.*

- *The $\mathscr{E}_i$ are integrable saturated subsheaves of $\mathscr{T}_{\widetilde{X}}$ with* $\det \mathscr{E}_i \cong \mathscr{O}_{\widetilde{X}}$.

Further, if $g\colon \widehat{X} \to \widetilde{X}$ is any finite cover, étale in codimension one, then the following properties hold in addition.

- *The reflexive pull-back sheaves $(g^*\mathscr{E}_i)^{\vee\vee}$ are slope-stable with respect to any ample polarisation on $\widehat{X}$.*
- *The irregularity of $\widehat{X}$ is zero; i.e., $h^1\big(\widehat{X}, \mathscr{O}_{\widehat{X}}\big) = 0$.*

In the following, we will call reflexive sheaves enjoying the property that they don't destabilise in any finite cover that is étale over the smooth locus (such as the $\mathscr{E}_i$) *strongly stable.*

Recently, it was shown by Druel [1] that in dimension less than or equal to five, the foliations $\mathscr{E}_i$ can be integrated algebraically and in fact lead to a splitting of a further cover of $\widetilde{X}$. We are thus lead to study varieties with canonical singularities, trivial canonical bundle and strongly stable tangent sheaf.

2. CY and IHS varieties

By looking at examples such as singular Kummer surfaces, it becomes clear relatively quickly that the condition "simply-connected" is not the right one when trying to define the building blocks of the singular version of the Decomposition Theorem. At the same time, Theorem 1 indicates that looking at finite covers $\eta\colon \widehat{X} \to X$, where $\widehat{X}$ is connected, normal and where η is étale over the smooth locus, but potentially branched in the singularities, is crucial; we will call such covers *quasi-étale* in the following discussion.

Definition 2 (CY and IHS varieties)**.** Let X be a normal projective variety with $\omega_X \cong \mathscr{O}_X$ and with at worst canonical singularities. We say that

- X is a *Calabi–Yau variety* if for every quasi-étale cover $\widehat{X} \to X$ and every $0 < p < \dim X$, we have $H^0\big(\widehat{X}, \Omega^{[p]}_{\widehat{X}}\big) = \{0\}$.[1]
- X is an *irreducible holomorphic-symplectic variety* if for every quasi-étale cover $\eta\colon \widehat{X} \to X$ we have that $\bigoplus_p H^0\big(\widehat{X}, \Omega^{[p]}_{\widehat{X}}\big) = \mathbb{C}[\eta^*\sigma]$, where $\sigma \in H^0\big(\widehat{X}, \Omega^{[2]}_{\widehat{X}}\big)$ is everywhere non-degenerate on X_{reg}.

3. Holonomy of varieties with strongly stable tangent sheaf

As in the smooth case, the classification of the irreducible pieces appearing in the Decomposition Theorem is built upon the availability of special metrics on varieties with trivial canonical class. In fact, the relevant generalisation of Yau's solution of the Calabi conjecture was shown by Eyssidieux–Guedj–Zeriahi in [2]: every ample class $[H] \in H^2\big(X, \mathbb{R}\big)$ can be represented by a singular Kähler–Einstein metric whose restriction to X_{reg} is smooth, though non-complete.

This non-completeness makes it impossible to apply classical results regarding Ricci-flat manifolds such as the Cheeger–Gromoll Theorem or the de Rham Decomposition Theorem. In particular, the fundamental group of the smooth part X_{reg} of X could be infinite. Nevertheless, by carefully analysing the differential-geometric holonomy of the EGZ metric and by relying on some powerful technical results obtained by Druel in [1] we are able to prove the following result, see [3].

Theorem 3 (G.–Guenancia–Kebekus '17)**.** *Let X be a klt variety with numerically trivial canonical divisor. Fix a point $x \in X_{\mathrm{reg}}$. Then, the following holds.*

- *The holonomy group Hol_x of the EGZ metric on X_{reg} at x has only finitely many connected components.*
- *The tangent sheaf $\mathscr{T}_X$ is strongly stable if and only if the identity component of Hol_x acts irreducibly on T_xX.*
- *If $\mathscr{T}_X$ is strongly stable, then either X is Calabi–Yau or there exists a quasi-étale cover of X that is irreducible holomorphic-symplectic.*[2]
- *X is Calabi–Yau if and only if $Hol_x = \mathrm{SU}(\dim X)$ and X is irreducible holomorphic-symplectic if and only if $Hol_x = \mathrm{Sp}(\dim X/2)$.*

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[1]Here and in the following, we denote by $\Omega^{[p]}_X = \big(\Omega^p_X\big)^{\vee\vee}$ the sheaf of reflexive p-forms on X.
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Hyperbolicity problems on some family of polarized manifolds

Sai-Kee Yeung

The purpose of the talk is to report on a joint project with Wing-Keung To on hyperbolicity properties of some families of polarized manifolds.

It is well-known classically that moduli spaces of elliptic curves, Riemann surfaces of genus $g \geq 2$, and hyperbolic punctured Riemann surfaces all share hyperbolicity properties in the sense that there exists a Hermitian metric with holomorphic sectional curvature and Ricci curvature bounded from above by a negative constant. As a result, such moduli spaces are Kobayashi hyperbolic and of log general type.

There has been a lot of interest in generalizing the above classical results to families of higher dimensional polarized manifolds. As a result, we consider the following three families of polarized manifolds: (a) canonically polarized manifolds, (b) polarized Kähler–Ricci flat manifolds or orbifolds, and (c) log-canonically polarized manifolds. We consider the following two problems. The first one is the existence of a metric with holomorphic curvature bounded from above by a negative constant, from which Kobayashi hyperbolicity of the base manifold follows. The second one is the conjecture of Viehweg that the base space of the family is of general type. Here are the results that we obtained.

Theorem 1. *Let $\pi\colon \mathcal{X} \to S$ be an effectively parametrized holomorphic family of polarized complex manifolds of type (a), (b) or (c) over a complex manifold S. Then S admits a C^∞ $Aut(\pi)$-invariant Finsler metric whose holomorphic sectional curvature is bounded above by a negative constant. As a consequence, S is Kobayashi hyperbolic.*

Theorem 2. *Let $\pi\colon \mathcal{X} \to S$ be an effectively parametrized family of n-dimensional manifolds of any one of the types (a), (b) or (c) over an m-dimensional quasi-projective manifold S. Denote by $\overline{S}$ a smooth projective compactification of S in which $D := \overline{S}\backslash S$ is a simple normal crossing divisor. Then the following statements hold:*

(i) *There exist a positive integer $1 \leq \ell \leq m$, an $Aut(\pi)$-invariant torsion-free coherent subsheaf V of $S^\ell(\Omega^1_{\overline{S}}(\log D))$ and an $Aut(\pi)$-invariant singular Hermitian metric h on V of positive curvature in the sense of Griffiths.*
(ii) *There exists a big invertible subsheaf L of $(\Omega^1_{\overline{S}}(\log D))^{\otimes k}$ over $\overline{S}$ for some positive integer k.*
(iii) *The log canonical line bundle $K_{\overline{S}} + D$ is big on $\overline{S}$.*

As a consequence of the above result, we get the following theorem as well.

Theorem 3. *Let $\pi\colon \mathcal{X} \to S$ be a holomorphic family of polarized manifolds of type (a), (b) or (c) over a quasi-projective manifold S. Then the following statements hold.*

(i) *Suppose π has maximal variation. Then S is of log general type.*
(ii) *Suppose S is special. Then π is isotrivial.*

Theorem 3(ii) in case (a) originally was a conjecture of Campana.

There are many results in the literature about the above problems. We refer the reader to our papers [4]–[7] for details. Here we would just like to point out that the approach here begins with a paper by Siu [2]. Brody hyperbolicity for a family in case (a) was first proved by Viehweg–Zuo [8]. The sheaf satisfying the conditions in Theorem 2(ii) is usually called a Viehweg–Zuo sheaf. Existence of a Viehweg–Zuo sheaf in case (a) was first proved in [9]. The first proof of Theorem 3(i) and Theorem 3(ii) in case (a) was given by Campana–Paun [1] and Taji [3], respectively.

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Properness of the space of relative divisors and semi-continuity of the algebraic dimension

Daniel Barlet

We show that in a holomorphic family of compact complex connected manifolds parametrized by an irreducible complex space S, assuming that on a dense Zariski open set S^* in S the fibres satisfy the $\partial\bar{\partial}$-lemma, the algebraic dimension of each fibre in this family is at least equal to the minimal algebraic dimension of the fibres in S^*. For instance, if each fibre in S^* is Moishezon, then all fibres are Moishezon.

A decomposition theorem for projective manifolds with nef anticanonical bundle

Junyan Cao
(joint work with Andreas Höring)

Let X be a compact Kähler manifold with nef anticanonical bundle. Initiated by the fundamental papers [2, 7, 8], we can study the structure of X by looking at the natural maps attached to X, e.g. the Albanese map or the MRC fibration. Thanks to [13, 14, 11], we know that the Albanese map is semistable and the Kodaira dimension of the base of the MRC fibration is 0. Conjecturally, we expect that these two natural maps are locally isotrivial. More precisely, we have the following conjecture.

Conjecture 1. *Let X be a compact Kähler manifold with nef anticanonical class. Then the universal cover $\widetilde{X}$ of X decomposes as a product*

$$\widetilde{X} \simeq \mathbb{C}^q \times \prod Y_j \times \prod S_k \times Z,$$

where Y_j are irreducible Calabi-Yau manifolds, S_k are irreducible hyperkähler manifolds, and Z is a rationally connected manifold.

This conjecture has been proven under the stronger assumption that T_X is nef [2, 7], $-K_X$ is hermitian semipositive [8, 3], 3-dimensional projective case [12, 1] or the general fibre of the Albanese map is weak Fano [5].

In this joint work with A. Höring [6], we focus on the case where X is a projective manifold. Thanks to [4], for any projective manifold X with nef anticanonical bundle the Albanese map $X \to \mathrm{Alb}(X)$ is a locally trivial fibration. By [10], after a finite étale cover, the generic fiber of the Albanese map is simply connected. So the next step is to study X when it is simply connected. We show that the structure of X is as simple as possible:

Theorem 2. [6] *Let X be a projective manifold such that $-K_X$ is nef and $\pi_1(X) = 1$. Then $X \simeq Y \times F$ such that $K_Y \sim 0$ and F is a rationally connected manifold.*

As a consequence, we obtain a precise description of the MRC-fibration:

Theorem 3. [6] *Let X be a projective manifold such that $-K_X$ is nef. Then there exists a locally trivial fibration $X \to B$ such that the fibre F is rationally connected and $K_B \equiv 0$. Moreover the following holds:*

(i) *There exists a finite étale cover $X' \to X$ such that $X' \simeq Y \times Z$ where $K_Y \simeq \mathcal{O}_Y$ and Z is a locally trivial fibre bundle $Z \to \mathrm{Alb}\,(Z)$ with fibre F.*

(ii) *If $H^0(F, T_F) = 0$, there exists a finite étale cover $X' \to X$ such that $X' \simeq B' \times F$ where $K_{B'} \simeq \mathcal{O}_{B'}$ and F is rationally connected.*

In particular, Conjecture 1 holds if X is projective.

Let us explain briefly the proof of Theorem 2.

Sketch of the proof of Theorem 2. Thanks to the seminal work of Q. Zhang [14], the base of the MRC fibration $p : X \dashrightarrow Y$ has Kodaira dimension zero, so the situation of Theorem 2 looks similar to the case of the Albanese fibration studied in [4]. However, as the MRC fibration is only an almost holomorphic map, we have to proceed in a less direct way.

Let $\varphi : \Gamma \to Y$ be a resolution of the MRC fibration. The first step is to describe the structure of the fibre space $\varphi : \Gamma \to Y$. By using the positivity of direct images as well as the arguments in [14], we construct a φ-big line bundle L_0 on Γ such that for all p sufficiently divisible, the direct image sheaves

$$\varphi_\star(\mathcal{O}_\Gamma(pL_0))$$

are trivial vector bundles over a certain open subset $Y_0 \subset Y$ which is simply connected and has only constant holomorphic functions. Following an argument going back to [7] this implies that we have a birational map

$$\varphi^{-1}(Y_0) \dashrightarrow Y_0 \times F,$$

where F is a general fibre of the MRC fibration.

The second step is to see how the product structure of some birational model yields a product structure on X. Let E be the exceptional locus of the birational map $\pi : \Gamma \to X$. In our case we can prove that the product structure on $Y_0 \times F$ induces a splitting of the tangent bundle of $\varphi^{-1}(Y_0) \setminus E$. Since the complement of $\varphi^{-1}(Y_0) \setminus E \subset X$ has codimension at least two (by the construction of Y_0), we obtain a splitting of the tangent bundle T_X defining two algebraically integrable foliations. Then the standard arguments for manifolds with split tangent bundle (cf. [9]) yield the theorem. □

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Foliations in $\mathbb{P}^2$ with invariant curves

Nessim Sibony

Consider the following polynomial differential equation in $\mathbb{C}^2$:

$$\frac{\mathrm{d}z}{\mathrm{d}t} = P(z,w), \qquad \frac{\mathrm{d}w}{\mathrm{d}t} = Q(z,w).$$

The polynomials P and Q are holomorphic, the time is complex. In order to study the global behavior of the solutions, it is convenient to consider the extension to a foliation in the projective plane $\mathbb{P}^2$. There are however singular points. When the line at infinity is invariant, Khudai Verenov has shown that generically, except for the line at infinity, leaves are dense. This follows from the study of the holonomy on the invariant line. But generically on the vector field, more precisely, generically on the foliation of geometric degree d, there is no invariant line and even no invariant algebraic curve as shown by Jouanolou. This example is a special case of a lamination (with singularities) by Riemann surfaces. In particular, one can consider similar questions in any number of dimensions.

In order to understand their dynamics, we need some analysis on such objects. We will discuss harmonic currents directed by the lamination, the heat equation with respect to a harmonic measure and geometric ergodic theorems for laminations with singularities. They are the analogues in this context of the classical Birkhoff ergodicity theorem.

There are quite surprising rigidity theorems: Foliations in $\mathbb{P}^2$, with no invariant algebraic curve and with all singularities hyperbolic, are uniquely ergodic.

I will discuss mostly the proof of a recent result with T. C. Dinh which asserts the following: Consider a foliation in the projective plane admitting an invariant line, which is the unique invariant algebraic curve. Assume that the foliation is generic in the sense that its singular points are hyperbolic. Then, there is a unique positive harmonic $(1,1)$-current of mass 1 which is directed by the foliation and this is the current of integration on the invariant line. In a point of view from Nevanlinna's theory, every leaf of the foliation is concentrated near the invariant line. However every leaf, except the line at infinity, is dense. The result uses an extension of our theory of densities for currents. One obtains similar results for foliations on compact Kähler surfaces.

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Asymptotics of L^2 and Quillen metrics for degenerations of Calabi–Yau manifolds

Christophe Mourougane
(joint work with Dennis Eriksson, Gerard Freixas i Montplet)

Our first motivation was to generalize Kodaira's bundle formula for elliptic surfaces $\pi\colon S \to C$ (a proper surjective relatively minimal Kähler morphism with connected fibres between a smooth complex surface S and a smooth complex curve C with general fibers of genus one) to families of higher dimensional Calabi–Yau manifolds, using a metric point of view. The original formulation of Kodaira [5], giving the contributions of the singular fibres in topological terms,

$$\begin{aligned} K_{S/C} &= \pi^\star\pi_\star(K_{S/C}) \otimes \mathcal{O}_S\left(\sum_{\substack{\pi^{-1}(p)\text{ of}\\ \text{type } m\mathrm{I}_n}} \frac{m-1}{m}\pi^{-1}(p)\right), \\ (\pi_\star K_{S/C})^{\otimes 12} &= \mathcal{O}_S\left(\sum_{p\in C} \chi\big(\pi^{-1}(p)_{\mathrm{red}}\big)p\right), \end{aligned}$$

turns out to be related to Quillen-type metrics, whereas the reformulation by Kawamata in terms of the modular map $J\colon C \to \mathbb{P}^1$ for elliptic fibers and the log canonical threshold $\mathrm{lct}(S, \pi^{-1}(p))$ of the singular fibres (see [6])

$$(K_{S/C})^{12} = \pi^\star J^\star \mathcal{O}_{\mathbb{P}^1}(\infty) \otimes \pi^\star\mathcal{O}_C\left(12\sum_{p\in C}\big(1-\mathrm{lct}(S,\pi^{-1}(p))\big)p\right)$$

turns out to be related to L^2 metrics.

We consider a Calabi–Yau family $\pi\colon \mathcal{X} \to C$, that is a proper surjective Kähler morphism with connected fibres between a connected smooth complex manifold $\mathcal{X}$ and a connected smooth complex curve C, whose smooth fibers have trivial canonical bundle.

By computations of Tian [8] and Todorov [9], the curvature of the L^2 metric on the direct image $\pi_\star(K_{\mathcal{X}/C})$ of the relative canonical bundle on the smooth part of the morphism is given by the Weil–Petersson metric through the classifying map ι to the Kuranishi family

$$\mathrm{c}_1\big(\pi_\star(K_{\mathcal{X}/C}), h_{L^2}\big) = \iota^*\omega_{WP}.$$

This accounts for the non-negativity of the modular part. The asymptotic of this metric around a singular fibre can be estimated by fibre integrals and displays as the coefficient of the dominant term (a log-term) one minus the log canonical threshold of the singular fibre (that vanishes for semi-stable degenerations) and as the coefficient of the sub-dominant term (a log log-term) an integer computed from the weighted incidence graph of the singular fibre when arranged with normal crossings (that reflects the strength of the degeneration). Those two terms can also be given in terms of the limit Hodge structure [7]. Explicit computations can be made for semi-stable degenerations of an elliptic curve : the L^2 norm of the form $\mathrm{d}z$ in the model $\mathbb{C}/(\mathbb{Z} \oplus \tau\mathbb{Z})$ is equal to the volume of the elliptic curve and to the imaginary part $\mathrm{Im}(\tau)$ of the period τ whereas the parameter $q \coloneqq \exp(2\pi i\tau)$ behaves like the regular discriminant function when the period τ tends to $i\infty$. This leads to the asymptotic

$$-\log \|\mathrm{d}z\|_{L^2} \sim -\log|\log|t||$$

for a local regular parameter t on the base C centred at a singular value of π.

Turning to Quillen type metrics, we consider the BCOV bundle, named after Bershadsky–Cecotti–Ooguri–Vafa [1], together with the BCOV metric defined by Fang–Lu–Yoshikawa [4]. By results of Bismut–Gillet–Soulé [2], its curvature on the smooth part is also related to the Weil–Petersson metric

$$\mathrm{c}_1\big(\lambda_{\mathrm{BCOV}}(\Omega^{\bullet}_{\mathcal{X}/S}), h_{\mathrm{BCOV}}\big) = \frac{\chi(X_\infty)}{12}\iota^*\omega_{WP}.$$

where X_∞ is any smooth fibre. Building on works of Yoshikawa [10], we show that the dominant term in the asymptotic of this metric displays the vanishing cycles when the relative minimality assumption $K_{\mathcal{X}} = \mathcal{O}_{\mathcal{X}}$ is made: for a local frame σ of the BCOV bundle and a local frame η of the direct image $\pi_\star(K_{\mathcal{X}/C})$, we find

$$-\log\|\widetilde{\sigma}\|^2_{\mathrm{BCOV}} \sim \frac{9n^2+11n+2}{24}\big(\chi(X_\infty) - \chi(X_0)\big)\log|t|^2 - \frac{\chi(X_\infty)}{12}\log\|\eta\|^2_{L^2}.$$

Details are written in [3].

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Non-hyperbolicity of hyperkähler manifolds

Ljudmila Kamenova
(joint work with Steven Lu, Misha Verbitsky)

The Kobayashi pseudometric d_M on a complex manifold M is the maximal pseudometric such that any holomorphic map from the Poincaré disk to M is distance-decreasing. Kobayashi conjectured that the pseudometric d_M vanishes on compact Calabi–Yau manifolds, and in particular, Calabi–Yau manifolds are Kobayashi non-hyperbolic and contain entire curves.

Using ergodicity of complex structures, together with S. Lu and M. Verbitsky in [1] we proved this conjecture for all K3 surfaces and for many classes of hyperkähler manifolds. The proof relies on a very careful understanding of the Teichmüller space of hyperkähler complex structures and the action of the mapping class group Γ. This action is ergodic, as M. Verbitsky proved in [3]. In a recent erratum [4], M. Verbitsky found an extra orbit type with respect to the action of the mapping class group Γ. In this talk we discussed that the presence of the extra orbit does not affect our proof of the Kobayashi conjecture.

In the talk I also gave an algebraic version of hyperbolicity. Together with M. Verbitsky we proved that projective hyperkähler manifolds with Picard rank at least two are algebraically non-hyperbolic, [2].

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Deformation of a generically finite map to a hypersurface embedding and generalized Inoue-type manifolds

Fabrizio Catanese
(joint work with Yongnam Lee)

Motivated by the theory of Inoue-type manifolds (these are the quotients $M = W/G$ of a hypersurface W in a projective classifying space Z by the free action of a finite group G), begun in previous joint work with Ingrid Bauer [1], we give a structure theorem for projective manifolds W_0 with the property of admitting a 1-parameter deformation where W_t is a hypersurface in a projective smooth manifold Z_t.

Their structure is elucidated as being the one of special iterated univariate coverings which we call of normal type, which essentially means that the line bundles where the univariate coverings live are tensor powers of the normal bundle to the image X of W_0. A converse is also shown.

We give applications to the case where Z_t is projective space, respectively an Abelian variety.

The set-up. We consider **a 1-parameter deformation to hypersurface embedding,** i.e. the following situation:

(1) a one dimensional family of smooth projective varieties of dimension n (i.e., a smooth projective holomorphic map $p\colon \mathcal{W} \to T$ where T is a smooth holomorphic curve) mapping to another family $\pi\colon \mathcal{Z} \to T$ of smooth projective varieties of dimension $n+1$ via a relative map $\Phi\colon \mathcal{W} \to \mathcal{Z}$ such that $\pi \circ \Phi = p$ (hence we have the following commutative diagram)

$$\begin{array}{ccc} \mathcal{W} & \xrightarrow{\ \Phi\ } & \mathcal{Z} \\ & {\scriptstyle p}\searrow \quad \swarrow{\scriptstyle \pi} & \\ & T, & \end{array}$$

such that moreover

(2) for $t \neq 0$ in T, Φ_t is an embedding,

(3) the restriction of the map Φ on W_0 is a generically finite morphism of degree m, so that the image of $\Phi|_{W_0}$ is the cycle $\Sigma_0 \coloneqq mX$ where X is a reduced hypersurface in Z_0, defined by an equation $X = \{\sigma = 0\}$.

Question 1. What can we say about W_0? Can we fully describe such a situation?

Examples. The typical example that everybody should know is the deformation of canonical maps of hyperelliptic curves of genus 3 to canonical embeddings of plane quartic curves (a double cover of a plane conic deforms to a smooth quartic).

This example fits into a series of examples, where the image of W_0 is the smooth hypersurface $\{\sigma = 0\} \subset \mathbb{P}^{n+1}$, σ being a homogeneous polynomial of degree d. We let then W_0 be the complete intersection in the weighted projective space $\mathbb{P}(1, 1, \dots, 1, d)$ defined by the equations

$$W_0 = \Bigg\{ (z_0, z_1, \dots, z_{n+1}, w) \;\Bigg|\; \sigma(z_0, z_1, \dots, z_{n+1}) = 0,$$

$$P(z_0, z_1, \dots, z_{n+1}, w) \coloneqq w^m + \sum_{i=1}^{m} w^{m-i} a_i(z_0, z_1, \dots, z_{n+1}) = 0 \Bigg\}.$$

We can easily deform the complete intersection by deforming the degree d equation adding a constant times the variable w, hence obtaining the following complete intersection:

$$P(z_0, z_1, \dots, z_{n+1}, w) = 0, \; tw - \sigma(z_0, z_1, \dots, z_{n+1}) = 0, \; t \in \mathbb{C}.$$

Clearly, for $t = 0$ we obtain the previous W_0, a degree m covering of the hypersurface $\{\sigma = 0\}$, whereas for $t \neq 0$ we can eliminate the variable w and obtain a hypersurface W_t in $\mathbb{P}^{n+1}$ with equation (of degree md)

$$P(z_0, z_1, \dots, z_{n+1}, \sigma(z)/t) = 0.$$

Example 1. (Iterated weighted deformations). One can iterate this process, and consider, in the weighted projective space

$$\mathbb{P}(1, 1, \dots, 1, d, dm_1, \dots, dm_k), \text{ where } m_1|m_2| \dots |m_k|m =: m_{k+1}$$

a complete intersection W of multidegrees $(d, dm_1, \dots, dm_k, dm)$.

Then, necessarily, there exist constants $t_0, t_1, \dots, t_k$ such that W is defined by the following equations, where the Q_j's are weighted homogeneous of degree $= dm_j$:

$$\begin{cases} \sigma(z) = w_0 t_0 \\ Q_1(w_0, z) = w_1 t_1 \\ \cdots \quad \cdots \\ Q_k(w_0, \dots, w_{k-1}, z) = w_k t_k \\ Q_{k+1}(w_0, \dots, w_k, z) = 0. \end{cases} \tag{1}$$

Again, if all the t_j's are $\neq 0$, we can eliminate the variables w_j, and we obtain a hypersurface $\{F(z) = 0\}$ in $\mathbb{P}^{n+1}$.

The above description generalizes, and the main idea of the following main theorem is that one can replace weighted projective space

$$\mathbb{P}(1, 1, \dots, 1, d, dm_1, \dots, dm_k), \ m_1|m_2| \dots |m_k,$$

by the total space of a direct sum of line bundles over some projective variety X, Z_0, or over a family $\mathcal{Z}$ of projective varieties.

The main theorem. We need the following definition: i) Given a complex space (or a scheme) X, a **univariate covering** of X is a hypersurface Y, contained in a line bundle L over X, and defined there as the zero set of a monic polynomial

$$P = w^m + a_1(x)w^{m-1} + a_2(x)w^{m-2} + \cdots + a_m(x) = 0,$$

where $a_j \in H^0(X, \mathcal{L}^j)$.

ii) The univariate covering is said to be **smooth** if both X and Y are smooth.

iii) An **iterated univariate covering** $W \to X$ is a composition of univariate coverings

$$f_{k+1} \colon W \to X_k, \ f_k \colon X_k \to X_{k-1}, \ \dots, \ f_1 \colon X_1 \to X,$$

whose associated line bundles are denoted $\mathcal{L}_k, \mathcal{L}_{k-1}, \dots, \mathcal{L}_1, \mathcal{L}_0$. It is said to be of **normal type** if all the line bundles $\mathcal{L}_j$ are pull back from X of a line bundle of the form $\mathcal{O}_X(m_j X)$, $m_1|m_2| \dots |m_k$, and the degree of f_j equals $\frac{m_j}{m_{j-1}}$.

Theorem 2. *(A) Suppose we have a 1-parameter deformation to hypersurface embedding and assume that K_{W_0} is ample. Then we have:*

(A1) X is smooth,

(A2) There are line bundles $\mathcal{L}_0, \dots, \mathcal{L}_k$ on $\mathcal{Z}$, such that $\mathcal{L}_j|_{Z_0} = \mathcal{O}_{Z_0}(m_j X)$ for $j = 0, \dots, k$, such that $1 = m_0|m_1|m_2 \dots |m_k|m_{k+1} := m$ and such that W_0 is a

complete intersection in $\mathcal{L}_0 \oplus \cdots \oplus \mathcal{L}_k|_{Z_0}$, given by a smooth iterated univariate covering of normal type.

(A3) $\mathcal{W}$ is obtained from Σ by a finite sequence of blow-ups and the smooth iterated univariate covering $W_0 \to X$ is normally induced.

(B1) A converse holds, ensuring the existence of deformations to hypersurface embedding for a smooth iterated univariate covering of normal type

$$\varphi_0 : W_0 \to X$$

and a 1-parameter family $\mathcal{Z}$ of deformations of Z_0, see [4] *for details.*

The hypothesis of ampleness is needed, and used via a simple adjunction calculation implying the finiteness of a certain intermediate map.

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Restricted volumes of big cohomology classes and degenerations of Kähler manifolds

David Witt Nyström

Let (X, ω) be a compact Kähler manifold. Let us first recall some definitions regarding positivity of cohomology classes due to Demailly.

Definition 1. We then say that a class $\alpha \in H^{1,1}(X, \mathbb{R})$ is *big* if there is a closed positive $(1,1)$-current T in α such that $T \geq \varepsilon\omega$ for some $\varepsilon > 0$.

Definition 2. A point $x \in X$ is said to lie in the *Kähler locus* $K(\alpha)$ of α if there exists a closed positive current $T \in \alpha$ which is smooth near x. The complement $E_{nK}(\alpha) \coloneqq X \setminus K(\alpha)$ is called the *non-Kähler locus* of α.

It follows from the deep regularization theorem of Demailly [4] that when α is big $E_{nK}(\alpha)$ is a proper analytic subset of X.

Definition 3. The *volume* of a big class α is defined as

$$\mathrm{vol}(\alpha) \coloneqq \sup \left\{ \int_{\mathrm{reg}(T)} T^n : T \in \alpha, T \geq 0 \right\},$$

where $n = \dim_{\mathbb{C}} X$ and $\mathrm{reg}(T)$ denotes the smooth locus of T. Similarly, if Y is a subvariety of dimension m, then the *restricted volume* of α along Y is defined as

$$\mathrm{vol}_{X|Y}(\alpha) := \sup \left\{ \int_{Y \cap \mathrm{reg}(T)} T^m : T \in \alpha, T \geq 0 \right\}.$$

Remark 4. When X is projective and $\alpha = c_1(L)$ for some holomorphic line bundle L then it was shown by Demailly [3] that α being big is equivalent to L being big, i.e. that

$$\mathrm{vol}(L) := \limsup_{k \to \infty} \frac{h^0(X, L^k)}{k^n/n!} > 0.$$

Furthermore Boucksom [1] proved that in this case

$$\mathrm{vol}(\alpha) = \mathrm{vol}(L),$$

while Hisamoto [5] proved that

$$\mathrm{vol}_{X|Y}(\alpha) = \mathrm{vol}_{X|Y}(L) := \limsup_{k \to \infty} \frac{h^0(X|Y, L^k)}{k^m/m!},$$

where $h^0(X|Y, L^k)$ denotes the dimension of the image of the restriction map from $H^0(X, L^k)$ to $H^0(Y, L^k_{|Y})$.

In the talk I presented the following theorem:

Theorem 5. *If X is compact Kähler, $\alpha \in H^{1,1}(X, \mathbb{R})$ is big and Y is a prime divisor intersecting $K(\alpha)$, then*

$$\left.\frac{\mathrm{d}}{\mathrm{d}t}\right|_{t=0} \mathrm{vol}\big(\alpha + t \cdot c_1(\mathcal{O}(Y))\big) = n \cdot \mathrm{vol}_{X|Y}(\alpha).$$

In particular the restricted volume $\mathrm{vol}_{X|Y}(\alpha)$ only depends on α and $c_1(\mathcal{O}(Y))$.

Remark 6. The case of Theorem 5 when X is assumed to be projective and $\alpha = c_1(L)$ for some holomorphic line bundle L was proved by Boucksom–Favre–Jonsson [2] and independently by Lazarsfeld–Mustata [6].

In my talk I discussed the proof of Theorem 5 and hinted at why this is relevant to the study of degenerations of Kähler manifolds.

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K-stability of Fano spherical varieties

Thibaut Delcroix

The proof by Datar and Székelyhidi of an equivariant version of the Yau–Tian–Donaldson conjecture in the Fano case [2] shows that existence of a Kähler–Einstein metric is equivalent to K-stability with respect to equivariant special test configurations. Though it is very impractical to establish K-stability with respect to general test configurations of a given example of polarized variey, it becomes possible for very symmetric varieties as soon as we can restrict to special test configurations, equivariant under a big enough group of automorphisms. For example, generalized flag manifolds do not admit non-trivial special test configurations that are equivariant under their automorphism group, or a toric manifold $X \circlearrowleft (\mathbb{C}^*)^n$ admits only product special $(\mathbb{C}^*)^n$-equivariant test configurations. Let us recall the definition of K-stability.

Definition 1.

- A *special test configuration* for a Fano manifold (or more generally $\mathbb{Q}$-Fano variety) X is a family $\pi\colon \mathcal{X} \to \mathbb{C}$ of $\mathbb{Q}$-Fano varieties $X_t = \pi^{-1}(t)$, indexed by $\mathbb{C}$, equipped with a $\mathbb{C}^*$-action on $\mathcal{X}$ that makes π equivariant with respect to the standard action of $\mathbb{C}^*$ on $\mathbb{C}$, and such that X_1 is isomorphic to X.
- Whenever X is equipped with an action of a reductive group G, we may additionally require that $\mathcal{X}$ is equipped with an action of G on the fibers, such that the isomorphism between X_1 and X is G-equivariant. Such a test configuration is called *G-equivariant.*
- The *Donaldson–Futaki invariant* $\mathrm{DF}(\pi)$ of the test configuration may be defined as the classical Futaki invariant of the central fiber X_0 with respect to the action of $\mathbb{C}^*$ induced by the action on $\mathcal{X}$.
- The variety X is *K-stable* (with respect to special, G-equivariant test configurations) if and only if $\mathrm{DF}(\pi) \geq 0$ for any special, G-equivariant test configuration, with equality if and only if $X_0 \simeq X$.

We obtained a combinatorial criterion for K-stability (with respect to special, G-equivariant test configurations) of a $\mathbb{Q}$-Fano spherical variety $X \circlearrowleft G$, which translates to a combinatorial criterion for existence of Kähler–Einstein metrics on a Fano spherical manifold, thanks to Datar and Székelyhidi's theorem.

Definition 2. A normal algebraic G-variety $X \circlearrowleft G$ is *spherical* if any Borel subgroup B of G acts with an open orbit in X.

The class of spherical varieties has two remarkable properties. The first is that it is very rich in examples and generalizes the classes of toric varieties, of biequivariant reductive group compactifications or of homogeneous toric bundles. The second is that spherical varieties are classified by combinatorial data in a way that generalizes the case of toric varieties [5], allowing also for example a combinatorial description of line bundles [1], of morphisms between spherical varieties, etc.

It should be first remarked that a spherical variety is almost homogeneous under the action of G, that is, G acts with an open dense orbit $G/H \subset X$. One possible combinatorial datum classifying $\mathbb{Q}$-Fano spherical varieties with fixed open dense orbit G/H is their moment polytope. Fix a Borel subgroup B of G and consider the set of all λ/k where λ is in the group $\mathfrak{X}(B)$ of characters of B, and is the eigenvalue of a B-eigenvector in the linear G-representation given by the space of sections $H^0(X, K_X^{-k})$ of tensor powers of the anticanonical line bundle of X. Then the closure Δ^+ of this set in $\mathfrak{X}(B) \otimes \mathbb{R}$ turns out to be a convex polytope with rational vertices, completely encoding X [4], called the *moment polytope* of X.

We already record additional data extracted from the moment polytope to use in the statements. Let Φ^+ denote the set of positive roots of G with respect to B. Denote by Φ_P^+ the subset of all roots that are not orthogonal to Δ^+, set $2\rho_P = \sum_{\alpha\in\Phi_P^+} \alpha$ and let $\mathrm{bar}_{DH}(\Delta^+)$ denote the barycenter of Δ^+ with respect to the measure $\prod_{\alpha\in\Phi_P^+} \langle \alpha, p\rangle \, dp$ where dp is a fixed Lebesgue measure on the affine space generated by Δ^+.

Another application of the combinatorial classification of spherical varieties allowed us to prove that special G-equivariant test configurations for a $\mathbb{Q}$-Fano spherical variety $X \curvearrowleft G$, modulo base change of the form $z \mapsto z^d$, are encoded by rays in the *valuation cone* $\mathcal{V}_-$ of X. This valuation cone here is a convex rational polyhedral cone depending only on the open orbit G/H of G in X. Furthermore, the central fibers of such test configurations remain $\mathbb{Q}$-Fano varieties, spherical under the action of G, with the same moment polytope in $\mathfrak{X}(B) \otimes \mathbb{R}$, and there are only a finite number of possibilities. More precisely, the central fiber depends only on the faces of the valuation cone in which the ray lies, and we can obtain all combinatorial data associated to it from the data associated to X and the ray. As a consequence, a G-equivariant special test configuration for X is a product if and only if the corresponding ray is in the linear part $\mathcal{V}_- \cap -\mathcal{V}_-$ of the valuation cone.

Another remarkable case is when the ray is in the relative interior of the valuation cone. In this case, the central fiber is the (polarized) horospherical degeneration X_0 of X. It is a horospherical variety, which may be defined by the fact that its open dense G-orbit is a homogeneous fibration over a generalized flag manifold G/P, with fiber a torus $(\mathbb{C}^*)^r$.

In fact the horospherical degeneration of X is also the horospherical degeneration of any of the central fibers of G-equivariant special test configurations for X. Using this remark, we prove that we may compute any of the Donaldson–Futaki invariant of such a test configuration as a classical Futaki invariant on the horospherical degeneration X_0. Finally, on horospherical varieties, it is possible to deal with Kähler metrics in a way that is very similar to the case of toric varieties.

Theorem 3. [3] *Let $X \curvearrowleft G$ be a $\mathbb{Q}$-Fano horospherical variety, denote by Δ^+ its moment polytope, identify the direction of Δ^+ with $\mathbb{R}^r$, denote by K a maximal compact subgroup of G. Then*

- *a K-invariant Kähler form ω in $c_1(X)$ is encoded by a convex function $u : \mathbb{R}^r \to \mathbb{R}$ such that $\nabla u(\mathbb{R}^r) = \mathrm{Int}(\Delta^+ - 2\rho_P)$,*
- *and the Futaki invariant for the vector field induced by an element ξ of $\mathbb{R}^r$ on the fibers of the open orbit is given up to a normalizing positive constant by*

$$\mathrm{Fut}_X(\xi) = \left\langle \mathrm{bar}_{DH}(\Delta^+) - 2\rho_P, \xi \right\rangle.$$

Combining this last result with the description of test configurations up to base change by rays in the valuation cone, which encodes at the same time the central fiber and the induced action of $\mathbb{C}^*$, we obtain the main result.

Theorem 4. [3] *A Fano spherical manifold $X \curvearrowleft G$ admits a Kähler–Einstein metric if and only if*

$$\mathrm{bar}_{DH}(\Delta^+) - 2\rho_P \in \mathrm{Relint}(\mathcal{V}_-^\vee).$$

More generally, a $\mathbb{Q}$-Fano spherical variety $X \curvearrowleft G$ is K-stable with respect to G-equivariant special test configurations if and only if the condition above is satisfied.

The theorem is in particular a generalization of the criterion obtained by Wang and Zhu for toric manifolds [6].

Example 5. Consider the space of all non-degenerate conics in $\mathbb{P}^2$. It is a spherical homogeneous space under the natural action of $\mathrm{SL}_3(\mathbb{C})$. Embed this space equivariantly in $\mathbb{P}^5 \times \mathbb{P}^5$ by sending a conic to the pair formed by its equation and the equation of its dual conic, that is, the conic defined by its set of tangents. The closure of the image of this embedding turns out to be a normal variety called the *variety of complete conics*, which is in addition smooth and Fano. Our criterion shows that the variety of complete conics admits Kähler–Einstein metrics.

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Lyapunov exponents of the Brownian motion over a compact Kähler manifold

Jeremy Daniel
(joint work with Bertrand Deroin)

Let E be a flat bundle of rank r over a compact Kähler manifold X. I define the Lyapunov spectrum of E: a set of r numbers controlling the growth of flat sections of E, along Brownian trajectories. I explain how to compute these numbers, by using harmonic measures on the foliated space $\mathbb{P}(E)$. Then, I explain a general inequality relating the Lyapunov exponents and the degrees of holomorphic subbundles of E; I also discuss the equality case.

L^2 extension theorems of Ohsawa–Takegoshi type

Jean-Pierre Demailly
(joint work with Junyan Cao, Shin-ichi Matsumura)

The main goal of our work is to prove general extension theorems with weak semi-positivity curvature assumptions. This can be useful e.g. for the study of the Minimal Model Program for algebraic varieties that are not necessarily of general type; one potential use is to construct sections via induction on dimension. The technique, however, relies on analytic tools, and also applies to transcendental varieties (in absolute or relative situations). The main statement is as follows.

Theorem 1 (joint with Junyan Cao and Shin-ichi Matsumura, [CDM17]). *Let (X,ω) be a Kähler manifold, and assume that X is holomorphically convex, i.e. that X admits a proper morphism $X \to S$ onto a Stein base. Let (L,h_L), with $h_L = e^{-\varphi_L}$, be a hermitian holomorphic line bundle on X, ψ a quasi-psh function with analytic singularities, $\mathcal{I}(e^{-\psi})$ the associated multiplier ideal sheaf, defined by*

$$\mathcal{I}(e^{-\psi})_{x_0} = \left\{ f \in \mathcal{O}_{X,x_0}\,;\ \exists U \ni x_0\,,\ \int_U |f|^2 e^{-\psi} d\lambda < +\infty \right\},$$

$Y = V(\mathcal{I}(e^{-\psi}))$ its zero subscheme, and $\mathcal{O}_Y = \mathcal{O}_X/\mathcal{I}(e^{-\psi})$ the corresponding structure sheaf. Assume that there exists a positive continuous function $\delta > 0$ on X such that

$$\Theta_{L,h_L} + (1+\alpha\delta)i\partial\overline{\partial}\psi \geq 0 \quad \textit{in the sense of currents, for } \alpha = 0,1. \tag{1}$$

If φ_L is smooth, the restriction morphism

$$H^q(X, \mathcal{O}(K_X \otimes L)) \to H^q(Y, \mathcal{O}(K_X \otimes L)_{|Y}) \tag{2}$$

is surjective for all $q \geq 0$, or equivalently,

$$H^q(X, \mathcal{O}(K_X \otimes L) \otimes \mathcal{I}(e^{-\psi})) \to H^q(X, \mathcal{O}(K_X \otimes L)) \tag{3}$$

is injective. More generally, if φ_L is possibly singular, the morphism induced by the natural ideal sheaf inclusion $\mathcal{I}(h_L e^{-\psi}) \to \mathcal{I}(h_L)$, namely

$$H^q(X, \mathcal{O}(K_X \otimes L) \otimes \mathcal{I}(h_L e^{-\psi})) \to H^q(X, \mathcal{O}(K_X \otimes L) \otimes \mathcal{I}(h_L)) \tag{4}$$

is injective for every $q \geq 0$.

A typical case of application is the situation where

$$\psi(z) = c\log|s|^2_{h_E}, \quad c > 0,$$

where $s \in H^0(X, E)$ and (E, h_E) is a holomorphic hermitian vector bundle over X. In fact, by using Hironaka's desingularization theorem and a suitable composition of blow-ups, the proof can always be reduced to the divisorial case where $\mathcal{O}(E) = \mathcal{O}(\sum a_j D_j)$ is an invertible sheaf associated with a simple normal crossing divisor $D = \sum a_j D_j$. Then $\mathcal{I}(\psi) = \mathcal{O}_X(-\sum\lfloor ca_j\rfloor D_j)$, and if the metrics h_L, h_E are smooth, the curvature condition (1) is equivalent to

$$(1') \qquad \Theta_{L,h_L} - (1+\alpha\delta)c\,\Theta_{E,h_E} \geq 0 \quad \text{for } \alpha = 0, 1.$$

The special case where $\mathcal{I}_Y = (\mathcal{I}_{Y_{\mathrm{red}}})^m$ was observed by D. Popovici [Pop05]; it corresponds to the situation where $\psi(z) = c\log|s|^2_{h_E}$, where $s \in H^0(X, E)$ is transverse to the zero section, $Y_{\mathrm{red}} = s^{-1}(0)$ has codimension r and $c = m + r$. Here, we allow essentially all possible multiplier ideals $\mathcal{I}(\psi)$, so this is considerably more general, and the hypotheses become also more natural.

Our proof is based on the observation that for a holomorphically convex space X, the cohomology groups $H^q(X, \mathcal{F}) = H^0(S, R^q\pi_*\mathcal{F})$ are always Hausdorff. Therefore the coboundary spaces are closed and it is sufficient to solve the relevant $\overline{\partial}$ equations only approximately. Specifically, if λ, η are positive functions on X and

$$B = B^{n,q}_{L,h_L,\omega,\eta,\lambda} = [\eta\,\Theta_{L,h_L} - i\,\partial\overline{\partial}\eta - i\lambda^{-1}\partial\eta\wedge\overline{\partial}\eta, \Lambda_\omega] \in C^\infty(X, \mathrm{Herm}(\Lambda^{n,q}T^*_X\otimes L)),$$

the equation $\overline{\partial}u_\varepsilon = v + w_\varepsilon$ with error term w_ε can be solved in bidegree (n, q) with an estimate

$$\int_X (\eta+\lambda)^{-1}|u_\varepsilon|^2\,dV_{X,\omega} + \frac{1}{\varepsilon}\int_X |w_\varepsilon|^2\,dV_{X,\omega} \leq M(\varepsilon) := \int_X \langle (B+\varepsilon I)^{-1}v, v\rangle\,dV_{X,\omega},$$

assuming that $B + \varepsilon I > 0$ and $M(\varepsilon)$ is finite. Now, any cohomology class in

$$H^q(Y, \mathcal{O}(K_X\otimes L)\otimes\mathcal{I}(h_L)/\mathcal{I}(h_L e^{-\psi}))$$

can be represented by a holomorphic Čech q-cocycle with respect to a Stein covering $\mathcal{U} = (U_i)$, say

$$(c_{i_0\ldots i_q}), \qquad c_{i_0\ldots i_q} \in H^0\big(U_{i_0}\cap\ldots\cap U_{i_q}, \mathcal{O}_X(K_X\otimes L)\otimes\mathcal{I}(h_L)/\mathcal{I}(h_L e^{-\psi})\big).$$

By the Dolbeault isomorphism, this class is represented by a smooth (n, q)-form

$$f = \sum_{i_0,\ldots,i_q} c_{i_0\ldots i_q}\rho_{i_0}\overline{\partial}\rho_{i_1}\wedge\ldots\wedge\overline{\partial}\rho_{i_q}$$

by means of a partition of unity (ρ_i) subordinate to (U_i). This form is to be interpreted as a form on the (non reduced) analytic subvariety Y associated with the ideal sheaf $\mathcal{J} = \mathcal{I}(h_L e^{-\psi}) : \mathcal{I}(h_L)$ and the structure sheaf $\mathcal{O}_Y = \mathcal{O}_X/\mathcal{J}$. We get an extension as a smooth (no longer $\overline{\partial}$-closed) (n,q)-form on X by taking

$$\widetilde{f} = \sum_{i_0,\ldots,i_q} \widetilde{c}_{i_0\ldots i_q} \rho_{i_0} \overline{\partial}\rho_{i_1} \wedge \ldots \wedge \overline{\partial}\rho_{i_q}$$

where $\widetilde{c}_{i_0\ldots i_q}$ is an extension of $c_{i_0\ldots i_q}$ from $U_{i_0} \cap \ldots \cap U_{i_q} \cap Y$ to $U_{i_0} \cap \ldots \cap U_{i_q}$. The strategy of the proof consists in solving approximately a $\overline{\partial}$ equation of the form

$$\overline{\partial} u_\varepsilon = v_\varepsilon + w_\varepsilon, \quad \text{where } v_\varepsilon = \overline{\partial}(\theta(\psi - t_\varepsilon) \cdot \widetilde{f}),$$

θ is a cut-off function and $t_\varepsilon \to -\infty$. The main point is then to show that in our situation $t_\varepsilon \in \mathbb{R}_-$, $\lambda = \lambda_\varepsilon$ and $\eta = \eta_\varepsilon$ can be chosen so that the error term w_ε converges to 0 (while the solution u_ε need not be under control !).

Getting L^2 estimates for the extension in the case $q = 0$ of holomorphic sections. When the singularities of ψ are *log canonical*, namely when $\mathcal{I}(e^{-(1-\varepsilon)\psi}) = \mathcal{O}_X$ for $\varepsilon > 0$, one can introduce an intrinsic *residue measure* $dV_{Y^\circ,\omega}[\psi]$ on the set $Y^\circ = Y_{\rm reg}$ of regular points of Y, following Ohsawa [Ohs01]: if $g \in \mathcal{C}_c(Y^\circ)$ is a compactly supported continuous function on Y° and $\widetilde{g}$ a compactly supported extension of g to X, one sets

$$\int_{Y^\circ} g \, dV_{Y^\circ,\omega}[\psi] = \lim_{t\to-\infty} \int_{\{x\in X\,,\ t<\psi(x)<t+1\}} \widetilde{g} e^{-\psi} \, dV_{X,\omega}. \tag{5}$$

Then, assuming $\psi \le 0$ and h_L smooth for the simplicity of the statement, any section $f \in H^0(Y, \mathcal{O}(K_X \otimes L)_{|Y})$ admits an extension $F \in H^0(X, \mathcal{O}(K_X \otimes L))$ such that

$$\int_X (1+\delta^2\psi^2)^{-1} \, |F|^2_{\omega,h_L} e^{-\psi} dV_{X,\omega} \le \frac{C}{\delta} \int_{Y^\circ} |f|^2_{\omega,h} dV_{Y^\circ,\omega}[\psi]. \tag{6}$$

In the non-log canonical case, there is a discrete sequence of jumps

$$0 = m_0 < m_1 < \ldots < m_p < \ldots$$

such that $\mathcal{I}(m\psi) = \mathcal{I}(m_{p-1}\psi)$ whenever $m \in \,]m_{p-1}, m_p]$, and $\mathcal{I}(m_p\psi) \subsetneq \mathcal{I}(m_{p-1}\psi)$ (this is formally always true if X is compact, and can be true only locally over X otherwise). Let $\mathcal{J} = \mathcal{I}(m_p\psi) : \mathcal{I}(m_{p-1}\psi)$ be the conductor ideal and $Y = V(\mathcal{J})$ its zero variety. Given a section

$$f \in H^0(Y, \mathcal{O}(K_X \otimes L)_{|Y} \otimes \mathcal{I}(m_{p-1}\psi)/\mathcal{I}(m_p\psi))$$

vanishing according to the ideal sheaf $\mathcal{I}(e^{-m_{p-1}\psi})$, one can define a "higher level" residue measure $|f|^2 \, dV^{m_p}_{Y^\circ,\omega}[\psi]$ by putting formally

$$\int_{Y^\circ} g|f|^2 \, dV^{m_p}_{Y^\circ,\omega}[\psi] = \lim_{t\to-\infty} \int_{\{x\in X\,,\ t<\psi(x)<t+1\}} \widetilde{g}|f|^2 \, e^{-m_p\psi} \, dV_{X,\omega}. \tag{7}$$

Then, as we showed in[Dem15b], one gets an extension

$$F \in H^0(X, \mathcal{O}(K_X \otimes L) \otimes \mathcal{I}(m_{p-1}\psi))$$

satisfying the L^2 estimate

$$(8) \qquad \int_X (1+\delta^2 m_p^2 \psi^2)^{-1} \, |F|^2_{\omega,h_L} e^{-m_p\psi} dV_{X,\omega} \leq \frac{C}{m_p \delta} \int_{Y^\circ} |f|^2_{\omega,h} dV^{m_p}_{Y^\circ,\omega}[\psi]$$

under the curvature assumption

$$(9) \qquad \Theta_{L,h_L} + m_p(1+\alpha\delta) i\partial\overline{\partial}\psi \geq 0 \quad \text{for } \alpha = 0, 1.$$

Without assuming that f is a nilpotent section associated with the ideal sheaf of the previous jump m_{p-1}, the existence of a reasonably intrinsic L^2 estimate is an open problem. The definition of the L^2 norm of f on Y is even unclear, as we do not know how to define a "multistep" residue measure in that case. The difficulty of getting the estimate arises already when X has complex dimension one and when $Y \subset X$ is a point with some multiplicity attached.

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