Topology of arrangements and representation stability

Abstract

The theories of homological and representation stability on sequences of spaces or groups endowed with compatible group actions grew out of work on configuration spaces, mapping class groups, and automorphism groups of free groups. The language and machinery that makes up the notion of FI-modules and related constructions is designed to help understand stability patterns of sequences of representations V_n of groups G_n; some of its successes have been in the study of sequences of configuration spaces of ordered points on an arbitrary manifold, moduli spaces of n-pointed curves, hyperplane complements and flag varieties, spaces of polynomials, congruence subgroups, and "pure braid like" groups. The study of representation stability has had interplay with algebraic combinatorics and commutative algebra, as well as an impact on its motivating questions of homological stability.

Arrangement theory, on the other hand, generalizes the study of the topology of configuration spaces, and provides a combinatorial framework for understanding some of the homological questions that arise in representation stability. Common points of origin include the characterization of the action of the symmetric group on the cohomology of the pure braid groups, as well as point-count formulas from étale cohomology for the complements of subspace arrangements. One focus of the workshop is expected to be the application of arithmetic aspects of representation stability to questions about the topology of spaces of polynomials with specified root multiplicities, via the Grothendieck–Lefschetz principle and recent work on statistics of polynomials defined over finite fields. Another objective is to try to bring techniques from representation stability to bear on the many open problems regarding Milnor fibres and cohomology jump loci of reflection arrangements. The former carry commuting reflection group and monodromy group actions. In spite of this rich structure, questions about cohomology, even in characteristic zero, appear to be quite difficult, but may greatly benefit from new approaches.