OBERWOLFACH SEMINAR: GROWTH IN FINITE AND INFINITE GROUPS 20–26 OCT 2019

LAURENT BARTHOLDI, GÖTTINGEN/LYON
HARALD HELFGOTT, GÖTTINGEN
MATTHEW TOINTON, CAMBRIDGE

How many numbers can one obtain by hitting n keys on a calculator? Or, to be more general, group-oriented and genteel: say we are given a group G and a finite subset A; write $A^r = \{x_1 x_2 \cdots x_r : x_i \in A\}$. How much larger are A^2 or A^3 than A? How does A^r grow as a function of r?

The week-long "Oberwolfach Seminar 1943a" will be devoted to such questions. The target audience is PhD students or post-doctoral researchers wishing to be quickly immersed in a modern, active research area. The number of participants is limited to 25; priority will be given to young, motivated researchers.

The theme of growth in groups is subdivided into two major areas, according to whether the emphasis is on varying A and fixed r, or varying r and fixed A. H. Helfgott will give an overview of current knowledge on the first case, including a full proof of the growth of generating subsets in $SL_2(\mathbb{K})$. He will also address contemporary developments relating growth in $\mathrm{Sym}(n)$ and SL_n.

Along the second vein, L. Bartholdi will focus on properties of infinite discrete groups. When A is a fixed generating set of G, the asymptotics of $|A^r|$ provide a powerful geometric invariant of G. The asymptotics may be polynomial, exponential, or intermediate, each time with (sometimes conjectural) consequences for the group's structure.

M. Tointon will describe the theory of sets A for which A^2 or A^3 is "not much larger" than A in the setting of abelian and nilpotent groups. This theory links to a large part of classical additive combinatorics and its more modern offshoot, approximate group theory. It also relates to harmonic analysis on finite abelian groups.

The courses will be given in the morning, with the afternoon reserved for discussion and problem sessions, as well as recent, more focused developments on, inter alia, the Poisson boundary of groups.

Recommended reading for Bartholdi's lectures

[1] L. Bartholdi, Growth of groups and wreath products, notes available at `https://arxiv.org/abs/1512.07044`
[2] L. Bartholdi, Amenability of groups and G-sets, notes available at `https://arxiv.org/abs/1705.04091`
[3] P. de la Harpe, *Topics in geometric group theory*, Chicago Lectures in Mathematics, University of Chicago Press, Chicago (2000).
[4] A. Mann, *How groups grow*, London Mathematical Society Lecture Note Series 395, Cambridge University Press (2011).

Recommended reading for Helfgott's lectures

[1] H. A. Helfgott, Growth in groups: ideas and perspectives, *Bull. Amer. Math. Soc.* **52** (2015), 357–413, available at `https://arxiv.org/abs/1303.0239`
[2] H. A. Helfgott, Growth and expansion in algebraic groups over finite fields, notes available at `https://arxiv.org/abs/1902.06308`

[3] T. Tao, Expansion in groups, course notes available at https://terrytao.wordpress.com/category/teaching/254b-expansion-in-groups/

Recommended reading for Tointon's lectures

[1] E. Breuillard, B. Green and T. Tao, Small doubling in groups, Erdős centennial, 129–151, Bolyai Soc. Math. Stud., 25, János Bolyai Math. Soc., Budapest (2013), available at https://arxiv.org/abs/1301.7718

[2] W. T. Gowers, A new way of proving sumset estimates, blog post available at https://gowers.wordpress.com/2011/02/10/a-new-way-of-proving-sumset-estimates/

[3] B. Green and I. Z. Ruzsa, Freiman's Theorem in an arbitrary abelian group, *J. Lond. Math. Soc.* **75**(1) (2007), 163–175, available at https://arxiv.org/abs/math/0505198

[4] M. Hall, *The theory of groups* (chapters 10 and 11), Amer. Math. Soc./Chelsea, Providence, RI (1999).

[5] M. C. H. Tointon. Freiman's theorem in an arbitrary nilpotent group, *Proc. London Math. Soc.* (3) **109** (2014), 318–352, available at https://arxiv.org/abs/math/0505198