Arbeitsgemeinschaft mit aktuellem Thema:
Thin Groups and Super-approximation
Mathematisches Forschungsinstitut Oberwolfach
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Organizers:

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Talks:

1. Overview of random walk on compact groups and super-approximation.

In this lecture we say what it means for a random-walk on a compact group to have spectral gap property. Then super-approximation property will be formulated. The connection between super-approximation and expanders will be pointed out. The main results of [BG08, BGS10, Var12, BV12, SGV12, SG17, SG19, BG08, BG12, BdS16, HdS] will be surveyed. There are at least three related survey articles that can be extremely useful [Lub11, BL18, Sar14].

2. The Bourgain-Gamburd machine.

Proving a random walk has the spectral gap property is equivalent to giving a quantitative measurement of how fast the random walk is getting close to equidistribution. How well a measure is distributed can be captured with various forms of entropy. Bourgain and Gamburd in their seminal work [BG08] showed that if the Rényi-entropy of a random variable does not get substantially larger after two steps random walk, then either

Rényi-entropy is *very small*, or it is *very large*, or the random variable is concentrated on an *approximate subgroup*.

This result in conjunction with the groundbreaking results of [Hel08, Hel11, BGT11, PS16] where it is showed that an approximate subgroup of a finite simple group of Lie type is *close* to a subgroup reduced the spectral gap problem to three subproblems: (1) finding an initial Rényi-entropy, (2) proving a Diophantine property (escaping proper subgroups), and (3) understanding high Rényi-entropy case; (at least for random walks on a finite simple group of Lie type).

In this talk the Bourgain-Gamburd theorem will be explained and we see how Gowers's quasi-randomness [Gow08] (or the Sarnak-Xue trick) can help to deal with the high Rényi-entropy case. We will indicate how one can use these to get super-approximation for subgroups of $\mathrm{SL}_2(\mathbb{Q})$ and prime moduli.

The following references can be useful to prepare for this lecture [BG08], [Var12, Lemma 15], and [Tao13, §1.4].

3. Approximate subgroup.

As it was mentioned in the previous paragraph, the study of approximate subgroups played an important role in the recent advances on random walks on various compact groups. In this lecture, we go over the main ideas of the proof of Helfgott's product theorem in $\mathrm{SL}_2(\mathbb{F}_p)$ (see [Hel08]); moreover we will discuss basic properties of approximate subgroups of compact groups that are discussed in [Tao08].

4. Multi-scale setting, I: the Archimedean case.

A sum-product in a finite field proved in [BKT04] was a key result in understanding approximate subgroups of finite simple groups of Lie types. Such sum-product results and similarly understanding of approximate subgroups in multi-scale settings are much more subtle. Bourgain's proof of Katz-Tao's discretized ring conjecture is a manifestation of this difficulty. Bourgain and Gamburd in their works on random walk on $\mathrm{SU}_d(\mathbb{C})$ ([BG08, BG12]) use this result to study approximate subgroups in a given *scale*. They also introduce a *Littlewood-Payley decomposition* which gives a way to reduce the problem of proving a spectral gap property to the study of functions that live in arbitrarily small scales. Later de Saxcé [dS] extended this method to show a product result in simple Lie groups; and based on that Benoist and de Saxcé [BdS16] extended results of Bourgain and Gamburd to all compact simple Lie groups.

In this talk the main ideas of the work of Bourgain and Gamburd on random walks on $\mathrm{SU}_d(\mathbb{C})$ will be mentioned.

5. Multi-scale setting, II: square-free case.

In [Var12], Varjú introduced a method of showing that an approximate subgroup of $\prod_{i=1}^{\infty} G_i$ at a given scale is very close to a subgroup for instance when G_i's are non-isomorphic finite simple groups of bounded rank. Starting with two sequences of random variables, one with local Diophantine property and the other with initial entropy, Varjú produced a new random variable that in average has both properties. Varjú's result was later used in [SGV12] to prove the square-free case of the super-approximation conjecture; and his method was used in [LSG] to get the best known case of super-approximation in the positive characteristic case.

In this talk, we go over Varjú's technique, and discuss a bit what more is needed to get the super-approximation property.

6. Multi-scale setting, III: the p-adic case.

Following the ideas of the Archimedean case, Bourgain [BG09] proved a sum-product result for a fixed non-Archimedean local field, and Bourgain and Gamburd used this result in order to study approximate subgroups in $\mathrm{SL}_n(\mathbb{Z}_p)$. They used random matrix theory in order to prove the desired Diophantine property, and overall proved the p-adic case of super-approximation property for Zariski-dense subgroups of $\mathrm{SL}_n(\mathbb{Z})$.

Extending Bourgain's result, a sum-product phenomenon where the implied constants just depend on the degree of the given non-Archimedean local field was proved in [SG], and it was used to study random walks in a compact open subgroup of a semisimple p-adic analytic group.

To extend this result to all perfect groups, one needs to study *approximate submodules* of a semisimple p-adic analytic group. This is done in [SG19] where the super-approximation conjecture for powers of primes is proved.

In this lecture, we see how one can prove the super-approximation property for Zariski-dense subgroups of $\mathrm{SL}_n(\mathbb{Z}) \ltimes V(\mathbb{Z})$ and powers of primes where $V(\overline{\mathbb{Q}})$ is a completely reducible $\mathrm{SL}_n(\overline{\mathbb{Q}})$ module.

7. Preliminaries on dynamics on geometrically finite hyperbolic manifolds.

This lecture will discuss basic definitions and properties of the Patterson-Sullivan density, convex cocompact groups, geometrically finite groups, the

Bowen-Margulis-Sullivan measure and the Burger-Roblin measure on $\Gamma\backslash \mathrm{SO}(d+1,1)$ ([Pa], [Su1],[Su2], [Bu],[Ro], [OS], [MO]). Examples of geometrically finite groups should be given (e.g., Schottky groups, Apollonian groups). If time permits, the characterizations that the Bowen-Margulis-Sullivan measure is the unique measure of maximal entropy ([Su1],[Su2], [OP]) and that the Burger-Roblin measure is the unique horospherical invariant locally finite measure (not supported on a single horosphere) may be stated ([Bu], [Ro], [Wi1]).

8. Mixing and the decay of matrix coefficients.

This lecture will begin by stating of the mixing of the geodesic/frame flow for the Bowen-Margulis-Sullivan measure of a geometrically finite hyperbolic manifold $M = \Gamma\backslash\mathbb{H}^{d+1}$, due to Babillot [Ba] and Winter [Wi1]. It should explain the equivalence of the (resp. exponential) mixing of the BMS measure and the (resp. exponential) local mixing for the Haar measure ([Ro], [OS], [MO], [KeO]).

9. Exponential decay of matrix coefficients: the case of the critical exponent bigger than $d/2$.

In this lecture, precise asymptotic expansion of the matrix coefficients for the geodesic flow on the unit tangent bundle of a geometrically finite hyperbolic d-manifold M will be discussed, when the critical exponent of M exceeds $d/2$, where the main term is identified in terms of Burger-Roblin measures ([MO], [EO]). The spectral gap result of Lax-Phillips [LP], classification of unitary dual of $\mathrm{SO}(d+1,1)$ and asymptotic behavior of the matrix coefficients of complementary series representation should be discussed. It should also be recalled (from previous lectures) that this gives the exponential mixing of the Bowen-Margulis-Sullivan measure when the critical exponent of M exceeds $d/2$ ([MO], [KeO]).

10. Uniform exponential decay of matrix coefficients: the case of the critical exponent bigger than $d/2$.

This lecture concerns geometrically finite hyperbolic $d+1$ manifolds with critical exponent at least $d/2$. It should begin by recalling Selberg's eigenvalue conjecture as well as his 3/16 theorem. The work of Bourgain-Gamburd-Sarnak on the transfer property of combinatorial spectral gap to archimedean spectral gap will be explained ([BGS], see also [Kim]). Combining the work [GV] and the precise asymptotic expansion of matrix coefficients

[EO], this gives the uniform exponential decay of matrix coefficients for congruence covers $\Gamma(q)\backslash\mathbb{H}^{d+1}$ when Γ is a thin subgroup and the combinatorial spectral gap of the family of finite groups $\{\Gamma(q)\backslash\Gamma\}$ is known.

11. Exponential mixing: the case of convex cocompact manifolds.

This lecture is devoted to explaining the exponential mixing of the Bowen-Margulis-Sullivan measure for convex cocompact hyperbolic manifolds; this uses thermodynamic formalism and Dolgopyat operators. This is due to Stoyanov for the geodesic flow, and to Sarkar-Winter for the frame flow ([Do], [St],[SW]).

12. Uniform exponential mixing: the case of convex cocompact manifolds.

This talk is devoted to the proof of the uniform exponential mixing of the Bowen-Margulis-Sullivan measure for congruence covers of convex cocompact hyperbolic manifolds; this is based on the study of congruence transfer operators, by combining Dolgopyat operators and expanders ([Do], [BGS],[BGS10], [GV]). If time permits, its application to resonance-free regions for Laplacian will be discussed as well. The main references are ([OW], [Sar]).

13. Application to Equidistribution and counting.

Using the decay of matrix coefficients, one can describe the asymptotic behavior of translates of horospheres and symmetric subgroup orbits in the setting of geometrically finite hyperbolic manifolds, generalizing the work of Duke-Rudnick-Sarnak [DRS], and of Eskin-McMullen [EM] (see [BO] for effective approach, also see [Oh1], [Oh2] for survey). This in turn gives a way to understand the asymptotic distribution of a discrete orbit of a geometrically finite group in an affine symmetric space of the orthogonal group $\mathrm{SO}(d+1,1)$ with respect to a well-rounded family of bounded subsets ([OS], [MO]). When the exponential decay of matrix coefficients is known, the equidistribution and counting can also be obtained with exponential error term. When exponential decay of matrix coefficients is known to be uniform for congruence covers, one can get an estimate on the asymptotic number of *almost prime* points in the counting problem.

Counting circles in Apollonian circle packings can be understood as a special case of these more general counting problems ([Kon09], [KO], [LO], [Vi]).

14. Overview of Thin Groups and Applications.

This lecture will introduce basic definitions of thin groups and applications, with material from [KLLR19, Sar14]. Without proof, some applications relevant to the workshop which could be mentioned are: local-global problems [Kon13, Fuc13, Bou14], sieve methods [BGS10, Kon14, LM12], and continued fractions [Kon16, BK14a], as well as rich sources of thin groups such as monodromy groups [FMS14, BT14], and superintegral sphere packings [KN19].

15. Local-Global I: Apollonian Packings.

Setting up the local-global problem for Apollonian packings, and attacks on it (see the exposition in [Kon13, §3,§5]): "elementary" [GLM+03, Sar07]; by "quadratic forms," getting positive density [BF11], and the "circle method," giving density one [BK14b]. (Time permitting, discuss the proof of the full local-global conjecture in higher dimensional sphere packings [Kon19]; or generalizations of the density-one method in [Zha15, FSZ19].)

16. Local-Global II: Zaremba's Conjecture.

Formulation of Zaremba's conjecture and background on its applications to pseudorandom numbers, numerical integration [Kon13, §2,§5]. Setting up and executing the circle method in this context to prove density-one version [BK14a]. Time permitting (likely without proofs), extensions of the method [FK13, Hua15] and power savings error [Bou12, MOW19, BKM19].

17. Beyond Expansion Lecture I: Einsiedler-Lindenstrauss-Michel-Venkatesh Problem.

Formulation and sieve method solution [BK17] to ELMV problem [ELMV09] on low lying fundamental geodesics on the modular surface. Relevant background on hyperbolic geometry, geodesic flow, binary quadratic forms, and Duke's theorem is collected in [Kon16, §1]. Basic sieving techniques are described in [Kon14, §3.3].

18. Beyond Expansion Lecture II: Achieving or Exceeding the Ramanujan-quality Exponent of Distribution.

The affine sieve (see introduction in [Kon14, §3.3]) has a natural barrier in its exponent of distribution, corresponding to what would be an analogue of the Ramanujan-quality spectral gap. In rare cases, one can achieve this exponent unconditionally in sieve problems, for example in the trace problem related to McMullen's Arithmetic Chaos problem (see [Kon16, §2] and [BK18]). Rarer still is to go beyond even what Ramanujan can give, as is done in the Pythagorean Almost-Prime Triples problem (see [Kon13, §4] and [BK15]).

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Participation:

The idea of the Arbeitsgemeinschaft is to learn by giving one of the lectures in the program.

If you intend to participate, please send your full name and full postal address to

ag@mfo.de

by MAY 31, 2020 at the latest.

You should also indicate which talk you are willing to give:
First choice: talk no. ...
Second choice: talk no. ...
Third choice: talk no. ...

You will be informed shortly after the deadline if your participation is possible and whether you have been chosen to give one of the lectures.

The Arbeitsgemeinschaft will take place at Mathematisches Forschungsinstitut Oberwolfach, Lorenzenhof, 77709 Oberwolfach-Walke, Germany. The institute offers accomodation free of charge to the participants. Travel expenses cannot be covered. Further information will be given to the participants after the deadline.