Abstract

Oberwolfach Workshop:

Homogeneous Structures:
Model Theory meets Universal Algebra

Dates:

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Many fundamental mathematical first-order structures, such as the order of the rational numbers or the random graph, are homogeneous in the sense that isomorphisms between finite or finitely generated substructures can be extended to automorphisms of the entire structure. Homogeneous structures can be built systematically as limits of classes of finite structures. That way, a class of finite structures is stored in a single homogeneous structure, and can be investigated via that structure from the viewpoint of model theory or, more recently, universal algebra. This perspective has found many applications in computer science, for example in constraint satisfaction, automata theory, and verification.

Because of their high degree of symmetry, homogeneous structures tend to have large automorphism groups. Their abstract and topological properties make these groups an extremely interesting topic in topological dynamics with many recent developments, for example the connection between extreme amenability and the Ramsey property of classes of finite structures due to Kechris, Pestov, and Todorčević.

For some of the applications of homogeneous structures in computer science, the automorphism group does not store enough information about the homogenous structure, and one has to instead study the richer polymorphism clone of the structure. This is the place where universal algebra enters the picture.

Universal algebra has achieved a series of milestone results in the past years regarding the equational structure of polymorphism clones of finite structures. This development culminated in 2017 in a proof of the Feder-Vardi dichotomy conjecture from theoretical computer science, obtained independently by Bulatov and Zhuk. Their result states that every finite structure either has a constraint satisfaction problem that can be solved in polynomial time, or is NP-complete. A similar conjecture exists in the context of homogeneous structures, but is open in spite of emerging results on the structure of their polymorphism clones.

The aim of the workshop is to carry the recent successful structural advances of universal algebra from finite structures to infinite structures with large automorphism groups. Such an enterprise requires a joint effort of model theory and universal algebra.