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HOMOTOPIC & GEOMETRIC GALOIS THEORY

1/2-**Workshop (Id: 2110a)**

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ABSTRACT

A fundamental idea in studying the absolute Galois group of a field is to make it act on geometric objects such as Galois covers, étale cohomology groups and fundamental groups. The following research topics emphasize this seminal idea: (a) Galois covers, G-torsors and their parametrizing families, (b) motivic Galois representations, (c) anabelian towers of fundamental groups.

Striking advances have recently shed new light on the whole topic:

(a) in *Galois Covers*: the success for supersolvable groups of the Colliot-Thélène approach to the Noether program (Harpaz-Wittenberg), a realizing-lifting-parametrizing program pushing further the Hilbert specialization method (Dèbes, Fried, et al.), Tannakian considerations within $\mathbb{Q}_\ell$-perverse sheaves (Dettweiler et al.), the use of patching methods for local-global issues (Harbater et al.);

(b) in *Motivic Representations*: motivic constructions of given Tannaka groups using automorphic and perverse techniques (Yun, Patrikis, Katz), the use of p-adic or ultraproduct techniques (Cadoret, Ambrosi) to study ℓ-adic motives, the characterization of geometric representations in deformation rings (Litt), the proof of a Deligne conjectured fixed-point counting formula (Yu);

(c) in *Anabelian Geometry*: the successful introduction of methods from étale homotopy theory (Schmidt-Stix) and from motivic $\mathbb{A}^1$-homotopy theory for moduli stacks of curves (Collas), the import of operads (Fresse-Horel) which echo the Galois techniques of Pop and Hoshi-Mochizuki-Minamide, the construction of arithmetic operads for Hurwitz moduli spaces (Westerland-Wickelgren).

Essential crossbridging principles connect these advances: homotopic methods, higher stacks, Tannakian symmetries. Based on the recent results and their promising connections, and on the 2018 MFO mini-workshop in a similar spirit, this workshop aims to crystallize these innovative approaches and to strengthen fruitful desire paths in homotopic and geometric Galois theory.

Subject Classification (2010).
Primary (Main Aims): 12F12, 14G32, 14H30, 14H45.
Secondary (Imported Techniques): 14F05, 55Pxx, 14F22, 14G05, 14F42.

Keywords: Inverse Galois Problem, Motivic Galois Representations, Hurwitz Moduli Spaces, Anabelian Geometry, Specialization, Lifting Problems, Simplicial Homotopical Geometry.