Computation and Learning in High Dimensions

The most challenging problems in science often involve the learning and accurate computation of high dimensional functions, i.e., functions which depend on many variables or parameters. High-dimensionality is a typical feature for a multitude of problems in various areas of science. Two highly visible examples are PDEs describing complex processes in computational chemistry and physics and stochastic/parameter-dependent PDEs arising in uncertainty quantification and optimal control. Other important examples require big data analysis including regression and classification which typically encounters high-dimensional data as input and/or output. Mathematical and statistical learning theory are primary mathematical gateways to the rapidly developing field of "Data Science" which has increasing influence, perhaps even *transformative*, on virtually all branches of science.

The so-called *curse of dimensionality* typically negates the use of traditional numerical techniques for the solution of high-dimensional problems. Instead, novel theoretical and computational approaches need to be developed to make them tractable and to capture fine resolutions and relevant features. Paradoxically, increasing computational power may even serve to heighten this demand, since the wealth of new computational data itself becomes a major obstruction. Extracting essential information from complex problem-inherent structures and developing rigorous models to quantify the quality of information in a high-dimensional setting pose challenging tasks from both theoretical and numerical perspective.

This has led to the emergence of several new computational methodologies for solving high-dimensional problems, accounting for the fact that by now well understood methods drawing on spatial localization and mesh-refinement are in their original form no longer viable. Common to these approaches is the nonlinearity of the solution method. For certain problem classes, these methods have drastically advanced the frontiers of computability.

The most visible of these new methods is *deep learning*. Although the use of deep neural networks has been extremely successful in certain application areas, their mathematical understanding is far from complete. Central theoretical questions concern: (i) the *optimization concepts* needed to successfully treat over-parametrized highly non-convex problems and (ii) the model classes that describe when these methods work well and why. Aside from purely data driven applications, the combination of these methods with physical modeling also opens wide open research directions of current interest.

This workshop proposes to deepen the understanding of the underlying mathematical concepts that drive this new evolution of computational methods and to promote the exchange of ideas emerging in various disciplines about how to treat multiscale and high-dimensional problems.