Abstract

Oberwolfach Workshop:

Structure-Preserving Discretizations for Nonlinear Systems of Hyperbolic, Involution-Constrained Partial Differential Equations on Manifolds

Dates:

10 Apr - 16 Apr 2022 (Code: 2215a)

Organizers:

Manuel Castro, Malaga
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The topic of the half-size workshop is the progress in mathematical and numerical analysis for involution-constrained hyperbolic partial differential equations on manifolds. Many mathematical models of important physical phenomena are contained in this class of problems, where the involution restricts the space of allowable solutions, such as the well-known condition $\nabla \cdot B = 0$ in the Maxwell and MHD equations. Also the equations of large-strain nonlinear elasticity are subject to linear involutions, while the involutions of the Einstein field equations of general relativity are even nonlinear. Conservation laws on manifolds are not only of interest in astrophysics, where they are an intrinsic part of the theory of general relativity, but they also appear naturally in geophysics, when considering the shallow water equations on the sphere, for example. While the underlying mathematical theory of many involution-constrained PDEs has been studied, there remain relatively few numerical methods that can preserve the known mathematical structure exactly also on the discrete level. Even less is known about how to achieve these properties on general manifolds or for nonlinear involutions. Recent advances in structure preserving schemes in Euclidean space suggest that a broad class of important problems may benefit from a more focused attention of mathematical specialists coming from different fields also in the more general case of involution-constrained conservation laws on manifolds. The proposed half-size workshop will bring together an international group of pure, applied, and computational mathematicians as well as theoretical physicists who are all actively working on various aspects of the same topic, hence providing a unique opportunity for interdisciplinary advances and mathematical breakthroughs.

Further related topics on structure-preserving schemes are novel well-balanced steady-state preserving methods for conservation laws on manifolds, schemes that respect a discrete entropy inequality, positivity-preserving schemes, as well as Galilean- and rotation-invariant and angular momentum preserving methods on moving meshes.