Oberwolfach Seminar 2223a:

Taxis-type evolution systems: modeling and analysis

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Unlike lifeless particles, biological species do not just passively follow physical and chemical laws. They often adapt their motion in response to changes in their surroundings, such as of differences in concentration of a signal substance. This effect, termed taxis, is crucial for survival of many organisms, from bacteria to mammals, as they navigate through a complex environment. It also plays a fundamental coordinating role during processes occurring inside a multicellular organism through its life cycle, including embryogenesis, tissue regeneration, and tumor migration. The study of taxis mechanisms by means of partial differential equations began half a century ago with the by now classical Keller-Segel model of chemotaxis ([10]). This cross-diffusion system of two equations was originally introduced as a phenomenological description of slime mold aggregation due to an attractive chemical cue.

The continuously increasing comprehension of biological processes and therewith associated complexity calls for appropriate mathematical descriptions. Accordingly, a multitude of refined model variants has been developed during the last decades, accounting for a plethora of novel features such as further kinds of taxis, triggered by a variety of chemical and physical cues, degenerate diffusion mechanisms, nonlocality, flux limitation, larger systems and multiscality ([1], [3], [7], [11], [15]). While the ultimate goal is an effective description on the macroscopic scale of the whole population of motile agents, the derivation of a taxis system can begin on various scales. A clear hierarchy within and between the corresponding model classes is largely missing, even at non-rigorous levels; it involves challenges related, among others, to moment closure of the equations and to 'upscalings', i.e. approximations by macroscopic systems ([4], [21]).

From a perspective of mathematical analysis, the inclusion of taxis mechanisms in macroscopic evolutionary PDEs goes along with substantial challenges, mainly related to the core question how far such model elements may enhance the potential of the respectively resulting system to adequately describe processes of self-organization and pattern generation. Already at the level of simple models of Keller-Segel type, such structure-supporting features have been proved to be present even in the mathematically extreme sense of spontaneous singularity formation ([6], [9], [14], [18]); a noticeable history of preliminary and partial results in this regard, however, indicate a necessity of significant methodological differences in comparison to corresponding studies concerned with more standard types of reaction-diffusion equations and systems ([8], [12]). Accordingly, the development of appropriate analytical approaches has been the objective of a considerable literature during the past few years, partially focusing on techniques capable of addressing more complex models ([2], [17], [5], [19]; cf. also the survey [1]), and partially concerned with a refined understanding of simpler model classes (see [13], [20] and [16] for some examples).

The aim of this seminar is to expose talented doctoral students and postdoctoral researchers to current developments in this thriving research area, attempting to showcase the lively interplay between application-oriented considerations on the one hand, and scrutinies focused on mathematical aspects on the other. Correspondingly, the planned lectures intend to address questions from modeling of taxis-influenced processes in biology, to discuss topics from basic solution theories for taxis systems at various levels of complexity, and to finally also dwell on some recent developments in the analytical literature which allow for a refined description of qualitative mathematical facets genuinely due to taxis-type cross-diffusion. Particular subjects to be brought up are

- model classes involving various kinds of taxis in the context of cell migration (C. Surulescu);
- haptotaxis models for cancer invasion with degenerate diffusion (A. Zhigun);
- the role of boundary conditions in chemotaxis problems (J. Lankeit);
- taxis-driven singularity formation (M. Winkler).

From Monday to Thursday, the morning sessions will be comprised of lectures. For the afternoons, smaller groups will be formed that will discuss and clarify the material of the morning lectures and, beyond this, apply this to open mathematical questions of interest. It is thereby intended to facilitate collaborations between the participants for an even longer-lasting impact of this Oberwolfach seminar. Possible core themes for the discussion groups include the analysis of various taxis systems, with a special emphasis on the detection of blow-up, analysis in settings of generalized solution concepts, and modeling of biological processes involving taxis, specifically in cancer. Concrete topics will be chosen in accordance with the interest of the participants. The morning of Friday will be dedicated to a presentation and the discussion of questions and first results having arisen from the afternoon workshops. During the week, we also plan to offer the participants the possibility to present some results or formulate open questions.

Recommended reading

As an optional but not mandatory preparation for the lectures to be held during the seminar, prospective participants might consult the following.

1. Bellomo, N., Bellouquid, A., Tao, Y., Winkler, M.: *Toward a Mathematical Theory of Keller-Segel Models of Pattern Formation in Biological Tissues.* Math. Mod. Meth. Appl. Sci. **25**, 1663-1763 (2015)
2. Hillen, T., Painter, K.: *A user's guide to PDE models for chemotaxis.* J. Math. Biol. **58**, 183-217 (2009)
3. Jäger, W., Luckhaus, S.: *On explosions of solutions to a system of partial differential equations modelling chemotaxis.* Trans. Amer. Math. Soc. **329**, 819-824 (1992)
4. Lankeit, J., Winkler, M.: *Facing low regularity in chemotaxis systems.* Jahresber. DMV 2019, `https://doi.org/10.1365/ s13291-019-00210-z` (2019)
5. Nagai, T.: *Blowup of Nonradial Solutions to Parabolic-Elliptic Systems Modeling Chemotaxis in Two-Dimensional Domains.* J. Inequal. Appl. **6**, 37-55 (2001)

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[11] Kolbe, N., Sfakianakis, N., Stinner, C., Surulescu, C., Lenz, J.: *Modeling multiple taxis: tumor invasion with phenotypic heterogeneity, haptotaxis, and unilateral interspecies repellence.* Discrete Contin. Dyn. Syst. Ser. B **26**, 443-481 (2021)

[12] Lankeit, J., Winkler, M.: *Facing low regularity in chemotaxis systems.* Jahresber. DMV 2019, `https://doi.org/10.1365/ s13291-019-00210-z` (2019)

[13] Luckhaus, S., Sugiyama, Y., Velázquez, J.J.L.: *Measure valued solutions of the 2D Keller-Segel system.* Arch. Ration. Mech. Anal. **206**, 31-80 (2012)

[14] Nagai, T.: *Blowup of Nonradial Solutions to Parabolic-Elliptic Systems Modeling Chemotaxis in Two-Dimensional Domains.* J. Inequal. Appl. **6**, 37-55 (2001)

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[21] Zhigun, A., Surulescu, C.: *A novel derivation of rigorous macroscopic limits from a micro-meso description of signal-triggered cell migration in fibrous environments,* `arXiv:2010.04148`.