Abstract

Oberwolfach Workshop:

Hilbert Complexes: Analysis, Applications, and Discretizations

Dates:

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Hilbert complexes arise throughout mathematical physics. The fundamental partial differential operators from which most models in continuum physics are built may be realized as unbounded operators mapping between Sobolev and related Hilbert spaces, and these spaces and operators assemble into chain complexes. The resulting structure is a Hilbert complex: a finite sequence of Hilbert spaces together with closed unbounded operators from one space to the next such that the composition of two consecutive operators vanishes. This is a rich structure which combines functional analysis and homological algebra. Hilbert complexes arise not only in physics - electromagnetics, fluid mechanics, elasticity, general relativity, etc. - but are also fundamental to parts of geometry, such as the Hodge theory of Riemannian manifolds.

In recent decades it has been discovered that the Hilbert space structure is crucial to numerical analysis as well. Stable discretization of the partial differential equations related to Hilbert complexes depends crucially on retaining the Hilbert space structure at the discrete level. This viewpoint has led to major advances in discretization for numerous problems, and holds great promise for other applications. The de Rham complex - whether presented on a domain in space in terms of vector calculus, or more generally, in terms of differential forms on Riemannian manifolds - is a canonical example of a Hilbert complex, and many aspects of its analysis, application, and discretization are wellunderstood. The situation is less developed for other Hilbert complexes which arise in applications such as elasticity, plate theory, and general relativity. In particular, a systematic approach to deriving discretizations for these applications is not yet at hand. In view of the many applications, this would be of immense utility.

Interest in Hilbert complexes blurs the lines between analysis, geometry, numerical analysis, and applications, between pure and applied mathematics. In this workshop, we aim to bring together researchers with an interest in Hilbert complexes and their implications coming from all these communities in order to establish new lines of communications and advance the state of the art.