OBERWOLFACH SEMINAR:
Free Boundary problems in fluid dynamics

23 - 29 October 2022

Organizers:

Thomas Alazard, ENS Paris-Saclay
Mihaela Ifrim, University of Wisconsin, Madison
Daniel Tataru, University of California, Berkeley

Additional invited lecture: Albert Ai, University of Wisconsin, Madison

The purpose of this seminar is to introduce young participants, primarily PhD students and postdoctoral fellows, to state-of-the-art methods and results in the study of free boundary problems which are arising from compressible as well as from incompressible Euler's equations in general. A particular set of such problems is given by gaseous stars considered in a vacuum (modeled via the compressible Euler equations), water waves in their full generality (seen as recasts of incompressible Euler equations), as well as parabolic counterparts of these free boundary problems (examples are Hele-Shaw and Muskat equations).

This is a broad research area which is highly relevant to many real life problems, and in which substantial progress has been made in the last decade. Our goal is to present several selected current research directions, with the ultimate goal of fostering interactions, stimulating research in this area, as well as training the next generation of researchers.

Thomas Alazard will give three lectures on some parabolic free boundary problems, modeling the dynamics of a free surface carried by the flow of an incompressible fluid evolving according to Darcy's law. The goal is to illustrate the role of tools at the interface of harmonic analysis and nonlinear partial differential equations to study the Cauchy problem.

Mihaela Ifrim, in a series of three lectures, will present the physical derivation of the water wave equations, and introduce the most recent analytic tools developed in their study. The goal will be two-fold, namely (i) to understand the local solvability of the Cauchy problem for water waves in a low regularity setting, as well as (ii) to describe the long time behavior of solutions when the data is assumed to be small and localized.

Another tool in the study of the low regularity local well-posedness theory for gravity water waves is given by the so called *Strichartz estimates.* Albert Ai will give one lecture on this topic.

Daniel Tataru will talk about yet another free boundary problem, which describes the compressible Euler evolution of a gas with a vacuum boundary. The defining characteristic of the gas (as opposed to a fluid) is that its density decays to zero at the vacuum boundary. The three lectures will describe a new, simpler method to construct solutions in the context of free boundary problems, which should prove useful for a broader class of problems.

Recommended reading for T. Alazard's lectures:

[1.] Thomas Alazard and Quoc-Hung Nguyen.Quasilinearization of the 3D Muskat equation, and applications to the critical Cauchy problem. Preprint arXiv:2103.02474.

[2.] Thomas Alazard and Quoc-Hung Nguyen. On the Cauchy problem for the Muskat equation. II: Critical initial data. Annals of PDE (2021), no. 7, Art. 7.

[3.] Thomas Alazard, Nicolas Meunier and Didier Smets. Lyapunov functions, Identities and the Cauchy problem for the Hele-Shaw equation. Commun. Math. Phys. 377, no. 2, 1421-1459 (2020).

Recommended reading for D. Tataru's lectures:

[1.] M. Ifrim and D. Tataru, Local well-posedness for quasilinear problems: a primer, https://arxiv.org/abs/2008.05684,

[2.] M. Ifrim and D. Tataru, The compressible Euler equations in a physical vacuum: a comprehensive Eulerian approach, https://arxiv.org/abs/2007.05668,

[3.] M. Disconzi, M. Ifrim and D. Tataru, The relativistic Euler equations with a physical vacuum boundary: Hadamard local well-posedness, rough solutions, and continuation criterion, https://arxiv.org/abs/2007.05787

Recommended reading for M. Ifrim's lectures:

[1.] A. Ai, Low regularity solutions for gravity water waves, Water Waves, 1(1):145-215, 2019 https://arxiv.org/abs/1712.07821

[2.] A. Ai, Low regularity solutions for gravity water waves II: The 2D case, Ann. PDE, 6,4, 2020 https://doi.org/10.1007/s40818-020-00081-z

[3.] A. Ai, M. Ifrim, and D. Tataru, Two dimensional gravity waves at low regularity I: Energy estimates, https://arxiv.org/abs/1910.05323

[4.] A. Ai, M. Ifrim, and D. Tataru, Two dimensional gravity waves at low regularity II: Global solutions, https://arxiv.org/abs/2009.11513,

[5.] J. K. Hunter, M. Ifrim, and D. Tataru, Two dimensional water waves in holomorphic coordinates, *Comm. Math. Phys.*, 346, no. 2, 483-552, 2016 arXiv:1401.1252

[6.] M. Ifrim and D. Tataru, Two dimensional water waves in holomorphic coordinates II: global solutions, Bull. Soc. Math. France, 144, no. 2, 369-394, 2016 https://arxiv.org/abs/1401.1252

[7.] M. Ifrim, D. Tataru, D. T. Wang, A modified energy method proving enhanced lifespan of smooth solutions of a Burgers-Hilbert equation, J. K. Hunter, *Proceedings of the AMS*, Vol. 143(8), pp. 3407-3412, 2015, http://arxiv.org/abs/1301.1947

[8.] Daniel Tataru. Strichartz estimates for operators with nonsmooth coefficients and the nonlinear wave equation. American Journal of Mathematics, 122(2):349–376, 2000

[9.] Hart F Smith and Daniel Tataru Sharp local well-posedness results for the nonlinear wave equation. Annals of Mathematics, pages 291–366, 2005.