Oberwolfach Seminar

Stochastic Geophysical Fluid Dynamics

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The mathematical study of geophysical fluid dynamics is at the foundation of many models in meteorology, climate science and geophysics. In these models stochastic influences arise naturally when taking into account the uncertainty inherent in measurements and the estimation of physical parameters, and a stochastic point of view is often more practical and closer to the real-world applications than a deterministic one. A special focus of the mathematical research in this field is laid on the derivation of physically meaningful stochastic differential equations, the properties of these equations and the question if and in which sense solutions exist, improvements in the solution theory due to the stochastic influences compared to the deterministic models, and the possibility to use stochastic terms to simplify complex dynamics.

In the lectures we would like to give an introduction into several directions of the recent research on stochastic geophysical fluid dynamics and discuss different geophysical stochastic partial differential equations. In the lectures by F. Flandoli, some ideas of stochastic model reduction are illustrated. This theory aims to identify simplified stochastic models associated to complex dynamics. Models with transport-type noise appear and their mixing and dissipation properties are investigated. D. Holm will discuss derivations and properties of stochastic versions of standard models, such as quasi-geostrophic (QG) equations, thermal quasi-geostrophic balance (TQG) equations, rotating shallow water (RSW) equations, Euler-Boussinesq (EB) equations, and also of wave-current interaction on free surfaces (WCI FS) of fluid flows, all as direct implications of stochastic variational principles from a unified viewpoint. A. Hussein will give an introduction to the primitive equations of the ocean as a fundamental geophysical model. An approach via random point vortices is presented by M. Saal for the generalized inviscid SQG equations. The random point vortices are used to construct solutions with white noise initial conditions and to illustrate the regularization by stochastic terms by considering a multiplicative noise.