MFO WORKSHOP ID 2321: HYPOELLIPTIC OPERATORS IN GEOMETRY

ABSTRACT.

Hypoelliptic operators are a natural generalization of, and share similar analytic properties to, elliptic operators. These were originally studied in the PDE and microlocal analysis literature in classical works of Hörmander [6], Boutet de Monvel [3] and Rothschild-Stein [8] among others. Hypoelliptic operators have since increasingly appeared and are now pervasive in several branches of geometry. Some examples of these are the following:

(1) Cauchy Riemann (CR) geometry: the central example is the Kohn Laplacian [7]. Further examples include the CR Paneitz operator, the sub-Laplacian of Jerison-Lee and its CR invariant powers defined by Graham-Gover [4].
(2) Conformal geometry: conformally invariant analogues of hypoelliptic operators in CR geometry include the Paneitz operator, conformal Laplacian and GJMS operators [5].
(3) Sub-Riemannian (sR) geometry: the main example here is the sub-Riemannian Laplacian. Another example in the Laplacian of Rumin's contact complex [9].
(4) Index theory: A central program here is around the geometric hypoelliptic Laplacian and Dirac operator of Bismut relating to analysis on loop spaces [2]. Another example includes index theory of operators in the Heisenberg calculus [1].

More specific topics of research within these include the study of Bergman-Szegő kernel asymptotics, applications of CR and conformally invariant operators to Sobolev and isoperimetric type inequalities, spectral theory of the sub-Riemannian Laplacian, hypoelliptic index theorems and application of Bismut's hypoelliptic Laplacian to analysis of loop spaces. There have been recent advances in each of these. The purpose of this workshop will be to bring together researchers from different geometric fields working on hypoelliptic operators to explore common themes.

References

[1] P. F. Baum and E. van Erp, *K-homology and index theory on contact manifolds*, Acta Math., 213 (2014), pp. 1–48.

[2] J.-M. Bismut, *Loop spaces and the hypoelliptic Laplacian*, Comm. Pure Appl. Math., 61 (2008), pp. 559–593.

[3] L. Boutet de Monvel, *Hypoelliptic operators with double characteristics and related pseudo-differential operators*, Comm. Pure Appl. Math., 27 (1974), pp. 585–639.

[4] A. R. Gover and C. R. Graham, *CR invariant powers of the sub-Laplacian*, J. Reine Angew. Math., 583 (2005), pp. 1–27.

[5] C. R. Graham, R. Jenne, L. J. Mason, and G. A. J. Sparling, *Conformally invariant powers of the Laplacian. I. Existence*, J. London Math. Soc. (2), 46 (1992), pp. 557–565.

[6] L. Hörmander, *Hypoelliptic second order differential equations*, Acta Math., 119 (1967), pp. 147–171.

[7] J. J. Kohn, *Boundaries of complex manifolds*, in Proc. Conf. Complex Analysis (Minneapolis, 1964), Springer, Berlin, 1965, pp. 81–94.

[8] L. P. Rothschild and E. M. Stein, *Hypoelliptic differential operators and nilpotent groups*, Acta Math., 137 (1976), pp. 247–320.

[9] M. Rumin, *Formes différentielles sur les variétés de contact*, J. Differential Geom., 39 (1994), pp. 281–330.