Abstract

Oberwolfach Workshop:

Incidence Problems in Harmonic Analysis, Geometric Measure Theory, and Ergodic Theory

Dates:

4 – 10 June 2023 (Code: 2323)

Organizers:

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The goal of the workshop is to unite mathematicians working on harmonic analysis, geometric measure theory, and ergodic theory. While working on these areas, we are often confronted with problems which involve counting incidences between geometric objects, such as tubes, narrow annuli, and fractals, in complex geometric configurations.

Many conjectures in geometric measure theory can be directly classified as incidence counting problems: famous examples include the Kakeya conjecture, Falconer's distance set conjecture, and Wolff's conjecture on Furstenberg sets. These problems are, however, relevant beyond the realm of geometric measure theory: a large, expanding, body of literature links them to the heart of contemporary harmonic analysis, and ergodic theory. In their first year of PhD, if not earlier, aspiring harmonic analysts learn that Stein's restriction conjecture implies the Kakeya conjecture. On a more advanced class, they may study Bourgain and Demeter's decoupling theory, which played a key role in recent progress on Falconer's distance set conjecture. On the ergodic theory side, a circle of problems around Furstenberg's x 2, x 3 conjecture is connected to inverse theorems in additive combinatorics, and the sum-product phenomenon. These topics link back to geometric measure theory via, for example, projection problems, and Wolff's conjecture on Furstenberg sets.

In summary, a number of key topics in geometric measure theory, harmonic analysis, and ergodic theory are so interlinked that a graph depicting their connections would be nearly complete. Since few researchers can master all the nodes of this graph simultaneously, constant interaction between the fields is indispensable. The aim of our workshop is to contribute to this discussion, and advance the unity of the fields we represent.