

Oberwolfach - RIMS 2023
Arithmetic Homotopy & Galois Theory
Oberwolfach-RIMS Tandem Workshop
On the 24-30 September 2023

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ABSTRACT

Recent progress in the arithmetic-geometry theory of Galois and homotopy groups have shown to crystallize into a new geometry of Galois symmetries of spaces. While still relying on Grothendieck's original pillars – the resolution of the discrete and the continuous, the quintessential intersection of arithmetic and geometry, and the local-to-global thinking by generisation-specialisation – it results in *a new geometrification of the original insight* that goes beyond its classical group-theoretic legacy.

Principles and techniques of this new geometry can be read more explicitly within the various classical avatars of the field: **(1)** in the *arithmetic of covers and their Hurwitz moduli spaces*, local-to-global patching and Hilbert specialisation techniques are applied *beyond their original geometric frontier*; **(2)** in the *theory of Galois representations*, continuous properties from Langlands geometry, Tannaka symmetries and derived methods *enhance the original structures*; **(3)** in *arithmetic anabelian geometry*, the pursuit of a minimalistic program leads to close-to-anabelian birational constructions and to \mathbb{A}^1 -geometry in-between étale and motivic considerations. Absolute reconstructions beyond the ring structure brings the Diophantine world one step closer, as well as new combinatorial Galois models.

In this panorama, the last twenty-year progress of the Japanese arithmetic-geometry school (e.g. Y. Ihara's programme, S. Mochizuki's anabelian approach) offers some complementary decisive insights in terms of algorithmic and absolute anabelian techniques that still have to be reported. The original pillars, the specificities of higher dimension and stack symmetries will also act as guiding beacons to develop our program.

The proposed workshop, with the inclusion of satellite topics and by relying on explicit geometric properties, will exploit these cross-bridging principles for a mutual and consolidated enrichment of the theories. At this stage, it is expected for the new geometrification of our field to result in concrete objectives and collaborations and to a renewed international program in arithmetic homotopy and Galois geometry.

Subject Classification (2010).

Primary (Main Aims): 12F12, 14G32, 14H30, 14H45.

Secondary (Imported Techniques): 14F05, 55Pxx, 14F22, 14G05, 14D15, 11F70.

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