Oberwolfach Seminar 2223a:

# Control of PDEs Models for Living Systems
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Describing the individual and collective dynamics of living organisms by mathematical models is a subject which probably takes his roots in the 13th century, when Fibonacci introduced his famous sequence to predict growing of a population of rabbits. Major characteristics of living systems are that they undergo phenomena like birth, deaths, size modifications, pattern formation or spatial spreading. A natural framework to describe these phenomena is provided by partial differential equations (PDEs) and integral equations, dating back to seminal works by Sharpe and Lotka in 1911 [8], McKendrick in 1925 [6] or by Fisher in 1937 [1]. The existing literature on PDEs describing living organisms is by now overwhelming and it studies a variety of questions, including wellposedness, large time behavior, or wave propagation. An important PDE based literature is devoted to the mathematical understanding of the adaptation of the motion of some living organisms in response to changes in their surroundings, such as of differences in concentration of a signal substance. This effect, termed taxis, is crucial for survival of many organisms, from bacteria to mammals, as they navigate through a complex environment. The study of taxis mechanisms by means of partial differential equations began half a century ago with the by now classical Keller-Segel model of chemotaxis ([2]) and still is an extremely active research field.
Much less is known on the control problems for systems describing living organisms. The term "control" designs here the capacity of acting on so called *input functions* (such as boundary condition, birth rates, ...) in order to steer the considered system exactly or close to a target state. Control and management for systems involving living organisms is one of the main objectives of recent control theory applications. We refer to the control of pest species, e.g. in agriculture, epidemics, invasions or of genetically modified organisms, causing possibly catastrophic ecosystem-breakdowns, economic damage or threats on health, but also to the control of cell growing with application in the cancer study. The prediction of optimal harvesting rates, for instance in fishery or forestry, is intended to guarantee a sustainable development and optimal yields. Controlling irregularly fluctuating populations with threshold mechanisms can also be used to model basic ecological processes such as migration. Formulating relevant control theoretic questions and proposing implementable inputs is a difficult question, namely due to the fact that both the control function and the state trajectory are constrained (e.g., by positivity conditions, see, for instance, [4]). The continuously increasing comprehension of wellposedness issues for PDEs describing living organisms is an essential ingredient in approaching control theoretic issues. Indeed, to give a precise

mathematical meaning to relevant control problems, the first step consists in identifying appropriate function spaces such that the maps from the initial state and the input function to the current state and to the output functions are continuous and possibly differentiable. From a control theoretic perspective, a favourable situation is when the governing equations determine a time invariant well posed system (with input and output), see, for instance, Tucsnak and Weiss [11].
The aim of this seminar is to expose talented doctoral students and postdoctoral researchers to current developments in this emerging area, attempting to stress the interaction of traditionally different mathematical fields (PDEs, functional analysis, control theory) in order to advance towards a new understanding of the analysis and of the control of systems describing living organisms. Correspondingly, the planned lectures intend to address questions from the analysis of linear and nonlinear PDE systems describing complex processes in biology (in particular chemotaxis), to introduce the basic concepts of control theory for infinite dimensional systems and to discuss more specialized topics such as constrained controllability in population dynamics or control of systems with incomplete data.
Particular subjects to be brought up are

- Introduction to the analysis of PDEs for living systems (J. Lankeit).
- Controllability in population dynamics with age structuring and diffusion (D. Maity, see [4], [5]);
- Control of models with incomplete data, with focus on less and low regret control for population dynamics systems with missing data. (G. Mophou, see [3], [7]).
- Introductory topics on infinite dimensional systems, with focus on controllability issues (M. Tucsnak, see [10]);
- Existence, stability and finite-time blow-up of solutions for PDE systems describing chemotaxis (M. Winkler, see [9], [12]);

From Monday to Thursday, the morning sessions will be comprised of lectures. For the afternoons, smaller groups will be formed that will discuss and clarify the material of the morning lectures and, beyond this, apply this to open mathematical questions of interest. It is thereby intended to facilitate collaborations between the participants for an even longer-lasting impact of this Oberwolfach seminar. Possible core themes for the discussion groups include existence and blow-up of solutions for particular systems coming from mathematical biology, controllability properties of various linear or semi-linear parabolic systems. Concrete topics will be chosen in accordance with the interest of the participants or they will be driven towards real-life applications, namely to ecology. The morning of Friday will be dedicated to a presentation and the discussion of questions and first results having arisen from the afternoon workshops. During the week, we also plan to offer the participants the possibility to present some results or formulate open questions.

# Recommended reading

As an optional but not mandatory preparation for the lectures to be held during the seminar, prospective participants might consult the following.


1. Hillen, T., Painter, K.: *A user's guide to PDE models for chemotaxis.* J. Math. Biol. **58**, 183-217 (2009)
2. J.-L. Lions *Sentinelles pour les systèmes distribués à données incomplètes.* Masson, Paris,(1992).
3. Lankeit, J., Winkler, M.: *Facing low regularity in chemotaxis systems.* Jahresber. DMV 2019, `https://doi.org/10.1365/ s13291-019-00210-z` (2019)

4. Tucsnak, M., Weiss, G. : *Observation and control for operator semigroups.* Springer Science & Business Media (2009).

5. Webb, G. F.: *Population models structured by age, size, and spatial position.* Structured population models in biology and epidemiology, 1-49, Lecture Notes in Math., 1936, Math. Biosci. Subser., Springer, Berlin, 2008.

# References

[1] R. A. Fisher, *The wave of advance of advantageous genes*, Annals of eugenics, 7 (1937), pp. 355–369.

[2] E. F. Keller and L. A. Segel, *Initiation of slime mold aggregation viewed as an instability*, Journal of theoretical biology, 26 (1970), pp. 399–415.

[3] C. Kenne, G. Leugering, and G. Mophou, *Optimal control of a population dynamics model with missing birth rate*, SIAM Journal on Control and Optimization, 58 (2020), pp. 1289–1313.

[4] D. Maity, M. Tucsnak, and E. Zuazua, *Controllability and positivity constraints in population dynamics with age structuring and diffusion*, Journal de Mathématiques Pures et Appliquées, 129 (2019), pp. 153–179.

[5] ———, *Controllability of a class of infinite dimensional systems with age structure*, Control and Cybernetics, 48 (2019).

[6] A. M'kendrick, *Applications of mathematics to medical problems*, Proceedings of the Edinburgh Mathematical Society, 44 (1925), pp. 98–130.

[7] G. M. Mophou and O. Nakoulima, *Sentinels with given sensitivity*, European Journal of Applied Mathematics, 19 (2008), pp. 21–40.

[8] F. R. Sharpe and A. J. Lotka, *A problem in age-distribution*, The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, 21 (1911), pp. 435–438.

[9] P. Souplet and M. Winkler, *Blow-up profiles for the Parabolic–Elliptic Keller–Segel system in dimensions* $n \geq 3$, Communications in Mathematical Physics, 367 (2019), pp. 665–681.

[10] M. Tucsnak and G. Weiss, *Observation and control for operator semigroups*, Springer Science & Business Media, 2009.

[11] ———, *Well-posed systems-the LTI case and beyond*, Automatica, 50 (2014), pp. 1757–1779.

[12] M. Winkler, *Stabilization in a two-dimensional chemotaxis-Navier–Stokes system*, Archive for Rational Mechanics and Analysis, 211 (2014), pp. 455–487.