Hyperbolic Balance Laws: interplay between scales and randomness

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Abstract: Hyperbolic balance laws are fundamental in the mathematical modeling of transport-dominated processes in natural, socio-economic and engineering sciences. An iconic example is the *Euler system of gas dynamics*, leveraging the development of powerful numerical methods and recently open fundamental questions on the well-posedness and admissible solution concepts. A prominent theoretical question is related to recent results on ill-posedness of the weak entropy solutions for the multidimensional Euler system (hyperbolic conservation laws in general) and rise to infinitely many weak entropy solutions, the so-called wild solutions. Interestingly, even the presence of regularized (stochastic or dissipative terms) can fail to rule out such wild solutions. The recent discovery of intriguingly wild non-uniqueness cases within the class of weak entropy solutions and non-convergence results of highly-resolving numerical simulations is strongly related to the turbulence effects arising in the high Reynolds number compressible flow. At the same time, new solution concepts, such as statistical or probabilistic (measure-valued) solutions, give a driving force to the development and rigorous convergence/error analysis of novel structure-preserving numerical methods.

Nonlinear transport phenomena leading to a system of hyperbolic balance laws arise not only in fluid dynamics, but also in material science, particle physics, multi-phase flows, hemodynamics, and social sciences. In the latter nonlocal interactions (i.e. interactions, at the microscopic level, between particles/agents located in different places) play an important role and give rise to consider also equations with nonlocal terms, typically described by convolution kernels. Some examples for modern research direction treated by hyperbolic equations are population dynamics and epidemic models, traffic and biological networks, supply chains, crowd dynamics, opinion formation and consensus problems.

The aim of the workshop is to discuss open questions in the area of nonlinear hyperbolic conservation and balance laws. We focus on a delicate interplay between scale hierarchies and random/stochastic effects and discuss them from analytical, numerical and modeling point of view. This leads to questions of admissibility criteria connecting to ill-posedness of weak entropy solutions, hyperbolic problems with non-local terms, mean field theory, multiscale and structure preserving numerical methods, random solutions and uncertainty quantification methods, as well as data-based methods.

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