MINI WORKSHOP ON "GROWTH AND EXPANSION IN GROUPS" APRIL 7-12, 2024

The study of growth and expansion in groups has uncovered in the last decades many spectacular results, connecting group theory with various other fields of mathematics such as number theory, graph theory, combinatorics, and analysis (to mention a few). In the 15-year span since the proof of the celebrated Babai's conjecture [1, Conj. 1.7] by Helfgott [6] in the case of $\mathrm{SL}_2(\mathbb{F}_p)$, p a prime, using the sum-product phenomenon over finite fields, the nascent ideas developed into a mature theory of growth in finite simple groups culminating with the works of Helfgott-Seress [7], Breuillard-Green-Tao [3], and Pyber-Szabó [13]. Its techniques passed also into the theory of expanders, for instance through the work of Bourgain-Gamburd [2], and provided concrete and sophisticated new tools to attack various problems in complexity theory and computer science. The workshop aims to present the field's state of the art by gathering leading experts and passing the knowledge to the next generation.

Each of the aspects of our general questions shows connections to some different areas. The study of growth in groups of Lie type necessitated results from algebraic geometry. Growth in permutation groups naturally links our problems to combinatorics. Probabilistic questions as in Eberhard-Jezernik [4], as well as questions of growth up to conjugation [11, 10, 5], make use of tools from representation theory and are tied to word problems in groups. The expansion property for families of groups is closely related to Property (T), which is an important concept in the context of topological groups: see the works of Lubotzky [12] and Kassabov [9, 8] both on groups of Lie type and on permutation groups. We intend to organize a series of courses focusing on these lines of research, so as to present both an overview of each of the topics and recent important advances; additional short talks from the other participants will provide further insight into other major problems and results.

We plan to have four short courses on four different themes on the topic by leading experts, as well as a series of invited talks by the participants during the week. We adopt the format of a mini-workshop, so as to allow every participant to be an invited speaker. According to this format, there will be 16 participants including the 2 organizers and 4 short course speakers.

References

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