# Abstract

Oberwolfach Workshop:

## Non-commutative Function Theory and Free Probability

Dates:

**28 April - 3 May 2024**     (Code: 2418)

Organizers:

**Kelly Bickel, Lewisburg**
**Michael Hartz, Saarbrücken**
**John E. McCarthy, Saint Louis**
**Roland Speicher, Saarbrücken**

The goal of this workshop is to bring together researchers in operator theory/complex analysis on the one hand and free probability on the other hand. The two communities have been studying related objects, but from a somewhat different point of view, and concrete interactions have been limited so far. The aim is to understand each others' perspectives, develop a common language and build a basis for future collaboration. We expect this to lead to advances in both areas.

In particular, non-commutative (also known as free) holomorphic functions form a key link between the two areas. Even though non-commutative holomorphic functions have been studied and applied successfully in both subjects, concrete interactions between the two subjects have been somewhat limited. To give a concrete example, realization formulas originate in control theory and automata theory and are used in operator theory to represent bounded holomorphic functions. These were discovered independently in free probability by Haagerup and Thorbjornsen, but the connection was only realized much later.

Loosely speaking, non-commutative holomorphic functions have been studied using complex analysis and non-selfadjoint operator algebra techniques, whereas free probability relies more on self adjoint operator algebra methods. Just as classical analysis has benefited from viewing complex and real analysis as complementary tools that can be used simultaneously, we believe that non-commutative analysis will benefit from a fusion of techniques. We expect that the Oberwolfach workshop will lead to more interactions between the two communities that unearth similar connections.

To achieve our goal of developing a common language, we will have two mini courses, consisting of 3-5 lectures each. The planned topic of the first

mini course is "Free probability for operator theorists" and the planned topic of the second mini course is "Non-commutative function theory for free probabilists."