

SEMINAR PROGRAM

Title: Moduli Spaces of Canonical Metrics: Metric Riemannian Geometry and Topology

Organizers:

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1. CONTENTS

How to "visualize" the shape of a manifold? More technically, how can one describe the topological and geometric structures of manifolds in a quantitative way? In modern differential geometry, a fundamental strategy is to appropriately "normalize" the shape by canonical metrics — the Riemannian metrics that satisfy certain nonlinear partial differential equations.

The week-long seminar will be devoted to investigating canonical metrics and address several basic questions, such as

- regularity theory: how well do the canonical metrics perform?
- degeneration theory: which kinds of complications may appear?
- moduli space theory: what does the space of all canonical metrics look like?

To deal with these questions, the seminar will focus on degenerations and moduli space of Einstein metrics, global structures and compactifications of moduli space, moduli space and global topological invariants, etc. In particular, the seminar lectures will cover topics such as Ricci and sectional curvature comparison, the Gromov-Hausdorff (GH) distance and GH-convergence for smooth and non-smooth spaces, elliptic regularity theory (Schauder estimates, bootstrapping, etc) and regularity theory for non-collapsing manifolds with Ricci or sectional curvature bounds, basic geometry and characterizations of metric cones, structures of non-collapsing and collapsing Einstein 4-manifolds, bubble trees, index theory, and moduli space theory of special Einstein metrics.

2. SCHEDULE

We plan to give 3 lectures of 60 minutes in every morning with breaks of 10 minutes according to the schedule 9:00-10:00 am, 10:10-11:10 am, 11:20 am -12:20 pm. After lunch there should be a long break until 4 pm to give the participants the possibility to read their notes, to check the references and to get in contact with the other participants in order to discuss difficult points of the subject and to do group work on tutorial problems. In the afternoon we plan to have 4 interactive rounds of 30 minutes of exercises, questions, discussion and presentations, such that the participants can practice the theoretical methods. The schedule is 4-4:30 pm, 4:40-5:10 pm, 5:20-5:50 pm and 6-6:30 pm. Talks of the participants on own results in the subject of the seminar are also possible but should be an exception (e.g., after dinner). As the seminar will start on Monday morning, arrival day at Oberwolfach should be Sunday afternoon or evening. Ideally, a short round (about 30 minutes) of self-introduction

of all participants could be planned for Sunday evening after dinner. According to the tradition in Oberwolfach, there should be an excursion on Wednesday afternoon. The seminar would end on Friday after lunch with departure afternoon or evening.

3. PREREQUISITES

Graduate participants would be expected to be familiar with the following:

- (1) *Riemannian geometry*, including basic comparison geometry of Ricci and sectional curvature;
- (2) *Differential topology*, including basic theory of fiber bundles, cohomology, etc;
- (3) *Basic analysis of differential equations*, including Sobolev spaces, existence and regularity theory of linear elliptic equations.