Invitation to the Arbeitsgemeinschaft at Oberwolfach on Hyperbolic groups.
April 4 – April 10, 1999

A story in 18 parts
compiled by Brian H. Bowditch and Thomas Delzant.

Introduction.

Hyperbolic groups were introduced by Gromov in his seminal article [Gr], published in 1987. In itself, this subject constitutes a significant proportion of the geometric theory of groups. Geometric group theory has grown into a major field over the last two decades or so, though its roots can be traced back to the early years of this century. For example, Dehn made use of hyperbolic geometry to solve the word problem for surface groups. Out of this developed small cancellation theory, which is still important today, and which might be viewed as a kind of precursor to the theory of hyperbolic groups. Since then, hyperbolic geometry has been one of the principal sources of ideas in combinatorial group theory. One should also mention the work of Aleksandrov, Busemann and Toponogov starting in the late 40s, who developed synthetic methods in Riemannian geometry. These techniques have exerted a strong influence on the evolution of the subject.

Another important source of ideas has been low-dimensional topology, in particular 3-manifold theory. These were translated into group theory through the work of Stalling, Dunwoody etc. Since the work of Thurston towards the end of the 70s, 3-dimensional topology and hyperbolic geometry have become intimately linked. This convergence of ideas produced a flurry of activity which has never lost pace. Among the more recent developments to spring from this source has been the theory of group actions on $\mathbf{R}$-trees, introduced by Morgan and Shalen, and developed by Rips, Bestvina, Feighn, Levitt, Paulin and others. It is now one of the most powerful tools in the subject.

It was against this background that Gromov published his article [Gr] on hyperbolic groups. As a typical example of such a group, one might consider the fundamental group of a compact negatively curved manifold. The entire subject is founded on an axiom, commonly termed the "thin triangles" axiom, which is a coarse version of a comparison axiom introduced by Aleksandrov. It is remarkable how such a simple axiom can capture so much of the large scale geometry of negatively curved spaces. The large number of equivalent natural formulations of this notion bear witness to its central importance. Today there is an extensive literature on the subject, both elaborating on ideas already set out in [Gr], and exploring many new directions.

A number of expositions of Gromov's work were published shortly after the original appeared, in particular, [GhH], [CoDP] and the articles of Short et al. and Bowditch in [GhHV]. Inevitably there is appreciable overlap between these accounts, but each offers its own perspective, and each discusses some results not contained in any of the others. Since that time, the subject has grown enormously, and it is still developing very rapidly. We are still waiting for more a up-to-date general exposition of the field to materialise...

We have had to be very selective in the choice of topics. Clearly many important results will not be discussed. We aim to give a brief overview of the current state of play in the first lecture.

Note. Throughout, we shall use (X, ρ) to denote a "geodesic space". That is, we assume that every pair of points, $x, y \in X$, are connected by a "geodesic" — a rectifiable path of length $\rho(x, y)$. In some lectures, we will want to assume also that X is "proper", i.e. every closed bounded subset is compact, though in general this would rule out many interesting and important examples.

Lecture 1 : Introduction.

A survey lecture: what will and what will not be done in the programme:

-Small cancellation theory and hyperbolic geometry: Olshanskii's work on the Burnside problem, generic groups.

-The geodesic flow and its (potential) applications (e.g. bounded cohomology).

-The Novikov conjecture and related problems.

-Symbolic dynamics of the geodesic flow and zeta function for hyperbolic groups.

-Decision problems in group theory (word problem, conjugacy problem and isomorphism problem).

-Automatic groups and other generalisations.

-Combination theorems.

-Relatively hyperbolic groups.

Lecture 2 : Definitions, first examples.

(a) Equivalent definitions of hyperbolicity, in particular, some variant of the "thin triangles" definition (e.g. given every geodesic triangle, there is some point a bounded distance from all three sides), and the "four point" definition (e.g. $(\exists k \geq 0)(\forall x, y, z, w \in X)(\max\{\rho(x, y) + \rho(z, w), \rho(x, z) + \rho(y, w)\} \geq \rho(x, w) + \rho(y, z) - k.)$

(b) Some examples, finite trees, hyperbolic n-space $\mathbf{H}^n$.

(c) The lemma of approximation by finite trees: $(\forall n)(\exists k(n))$ such that if $F \subseteq X$ has cardinality n, then we can embed F in a finite tree, (T, d) such that for all $x, y \in F$, $|\rho(x, y) - d(x, y)| \leq k(n)$.

(Remark: the four point definition is equivalent this with $n = 4$.)

(d) As a corollary, if Γ acts properly discontinuously on a hyperbolic space (X, ρ) and $G \leq \Gamma$ is a finite subgroup, then there is some $x \in X$ with $\mathrm{diam}(Gx)$ bounded in terms of the hyperbolicity constant.

(e) Definition of a hyperbolic group as one which acts properly discontinuously cocompactly on a proper hyperbolic space.

Corollary of (d): A hyperbolic groups has finitely many conjugacy classes of finite subgroups.

Refs: [Gr], [GhH], [CoDP].

Lecture 3 : CAT(χ) spaces.

(a) Definition of the CAT(χ) property.

Examples of CAT(0) and CAT(-1) spaces: Cartan-Hadamard manifolds, buildings.

(b) The Cartan-Hadamard Theorem: For $\chi \leq 0$, locally CAT(χ) and simply connected implies (globally) CAT(χ), (which in turn implies contractibility).

(c) Theorem : If Γ acts properly discontinuously cocompactly on a proper CAT(0) space, X, then either Γ is hyperbolic or X contains an embedded euclidean plane.

(d) Some open questions:

In the second case of (c) can one always take the euclidean plane to be periodic (or equivalently, must Γ contain a free abelian subgroup of rank 2)?

Does every hyperbolic group act properly discontinuously (and cocompactly) on a CAT(-1) space?

Refs: [Gr], [GhH], [BrH].

Lecture 4 : Geodesics, quasi-geodesics and quasi-isometries.

Let (X, ρ) and (Y, d) be two metric spaces, and suppose $\lambda \geq 1$, $K \geq 0$. A "(λ, K)-quasi-isometry" between X and Y is a map $f : X \longrightarrow Y$ such that for all $x, x' \in X$

$$\lambda^{-1}\rho(x, x') - K \leq d(f(x), f(x')) \leq \lambda\rho(x, x') + K$$

and

$$(\forall y \in Y)(\exists x \in X)(d(f(x), y) \leq K).$$

The notion of quasiisometry thus gives an equivalence relation between metric spaces.

A fundamental property of hyperbolicity is its quasi-isometry invariance (for geodesic spaces). The most direct way to prove this is to use quasigeodesics.

A "(λ, K)-quasigeodesic" is a map $f : [a, b] \longrightarrow X$ from an interval $[a, b] \subseteq \mathbf{R}$ into X such that for all $x, x' \in X$,

$$\lambda^{-1}|x - x'| - K \leq \rho(f(x), f(x')) \leq \lambda|x - x'| + K.$$

A basic property of a hyperbolic space, X, is that there is a constant, D (depending only on λ, K and the constant of hyperbolicity), such that any (λ, K)-quasigeodesic remains a bounded distance from any geodesic segment connecting $f(a)$ to $f(b)$.

¿From this, it is easily seen (using the thin triangles property) that hypebolicity is a quasi-isometry invariant. In particular, hyperbolicity of a finitely generated group is independent of the generating set.

Refs: 7.2 of [Gr], Chapter 3 of [CoDP], the papers of Short et. al. and of Bowditch in [GhHV].

Lecture 5 : Isoperimetric inequalities and hyperbolicity.

One can characterise hyperbolicity in terms of isoperimetric inequalities. In order to avoid technical definitions, it is convenient to describe this in the context of Riemannian manifolds.

Let M be a compact Riemannian manifold. The "isoperimetric function" (or "profile") of M is defined as $A(L) = \max_{L(\partial D)=L} A(D)$, where D is a 2-dimensional disc in M with area $A(D)$, and length of the boundary $L(\partial D) = L$. It is known that if all the sectional curvatures of M are non-positive, $A(L) \leq L^2/4\pi$.

Theorem: The following are equivalent:

i) The universal cover of M (or equivalently $\pi_1(M)$) is hyperbolic.

ii) There is some $K > 0$ such that $A(L) \leq KL$ for all L.

iii) There exist, $A_0 > 0$, $c > 1$ and $a > 2$ with $\frac{2a^{1/c}}{a-2} < 1$ such that given any $L \geq 0$, we have $A(L) \leq \frac{L^2}{16c\pi}$ provided $A_0 \leq A(L) \leq a^2 A_0$.

In particular, we see that there can be no isometric profile between quadratic and linear. The approach of Gromov is differential geometric, and gives the best known constants. Other (combinatorial) approaches have been proposed by several authors (Olshanskii, Short, Papasoglu, Bowditch) either in the context of Cayley graphs of groups, or for general geodesic spaces.

Here, we describe a combinatorial analogue of the isoperimetric profile called the "Dehn function". Let $\Gamma = \langle a_1, ..., a_n; R_1, ...R_k \rangle$ be a finitely presented group. A closed curve in the Cayley graph of Γ is a word, W, in the letters $a_i^{\pm 1}$ which equals the identity in Γ. Thus, in the free group generated by $a_1, \ldots, a_k$, the word W can be written as a product of conjugates of the relators $R_i^{\pm 1}$: $W = \prod_{1 \leq k \leq a(W)} m_k R_{i_k} m_k^{-1}$. The minimal value of $a(W)$ involved in such an expression is interpreted as the combinatorial area, $A(W)$, of W. The Dehn function is defined by $D(n) = \max_{|W|=n} A(W)$ for all natural numbers, n. One can show that the Dehn function of a group, Γ, has the same asymptotic behaviour as the isoperimetric profile of a compact manifold whose fundamental group is Γ. Hence, hence a group is hyperbolic if and only if its Dehn function is linear.

Refs: [Gr], Chapter 6 of [CoDP].

Lecture 6 : The Rips complex.

To any finitely generated group, Γ, together with a finite generating set, and any natural number, d, is associated a (finite-dimensional) simplicial complex, $P_d(\Gamma)$, which admits a discrete cocompact action of Γ. If Γ is hyperbolic, and d is large enough (in relation to the hyperbolicity constant), then $P_d(\Gamma)$ is contractable. Moreover, it satisfies a linear isometric inequality in each dimension.

This construction has many corollaries, for example:

Corollary: If Γ is a torsion-free hyperbolic group, then Γ has a finite $K(\Gamma, 1)$. In particular, Γ has finite cohomological dimension.

Refs: Section 1.7 of [Gr], [GhH], Chapter 5 of [CoDP], Section 4 of the paper of Short et. al. in [GhHV]

Lecture 7 : Boundaries.

(a) Definition of the boundary, ∂X, of a hyperbolic space, X (for example, as parallel classes of (quasi)geodesic rays). The topology of ∂X. (Remark: ∂X is metrisable.)

(b) Examples: convex subsets of (constant curvature) hyperbolic space etc.

(c) Restrict to the case where X is proper.

Then, ∂X is compact. The connected components of ∂X are the topological ends of X.

(d) Invariance of ∂X by quasiisometry.

Definition of the boundary, $\partial\Gamma$, of a hyperbolic group, Γ. If Γ is not virtually cyclic, then $\partial\Gamma$ is perfect (no isolated points).

(e) By Stallings's theorem, $\partial\Gamma$ is connected if and only if Γ does not split over any finite subgroup.

(f) Remark: Kaimanovich has shown that $\partial\Gamma$ is the same as the Poisson boundary.

Refs: [Gr], [CoDP], paper of Short et al in [GhHV]. For (f), see [Kai].

Lecture 8 : Convergence groups.

(a) Let M be a perfect compact metrisable space, and suppose that Γ acts by homeomorphism on M.

Definition: Γ is a (uniform) convergence group if the induced action on the space of distinct triples of M is properly discontinuous (and cocompact).

(b) The definition of convergence group is equivalent to the original definition of Gehring and Martin in terms of convergent subsequences.

(c) Classification of elements of a convergence group (elliptic, parabolic, loxodromic).

(d) If Γ acts properly discontinuously (cocompactly) on a proper hyperbolic space, X, then it acts as a (uniform) convergence group on ∂X.

(e) Thus, a hyperbolic group Γ acts as a uniform convergence group on $\partial\Gamma$.

Remark: There is a converse due to Bowditch.

Refs: [GeM], [Tu2], [Bo3], [Tu3]. For (e), see [Bo5].

Lecture 9 : Classification of isometries and elementary subgroups.

(a) Let X be any hyperbolic geodesic space. Any non-identity isometry of X is one of three types, elliptic, parabolic or loxodromic (or hyperbolic). If X is proper, and Γ acts properly discontinuously on X, then this agrees with the classifaction of elements of Γ as a convergence group acting on $\partial\Gamma$.

(b) If Γ acts cocompactly on X, then Γ cannot contain any parabolic element. Put another way, every infinite order element of a hyperbolic group is loxodromic.

(c) Definition of the translation length of an element of a hyperbolic group, Γ. Given Γ, there is some natural number, L, such that the translation length of each element of Γ lies in $\frac{1}{L}\mathbf{N}$.

(d) A subgroup of Γ is two-ended (i.e. virtually cyclic) if and only if it fixes an (unordered) pair of points in $\partial\Gamma$. Every infinite order element lies in a unique maximal two-ended subgroup.

(e) Every soluble subgroup of Γ is finite or two-ended.

Refs: 8.1, 8.2 of [Gr], Chapters 9,10 of [CoDP].

Lecture 10 : Free and quasiconvex subgroups of hyperbolic groups.

(a) The Tits alternative for a hyperbolic space: Suppose that Γ is neither finite nor two-ended, then it contains a nonabelian free subgroup.

(b) If g is a loxodromic isometry of X, and h doesn't fix either of the fixed points of g, then there is some natural number, n, such that g^n and hg^nh^{-1} generate a free subgroup. If X happens to be Cayley graph of a hyperbolic group, Γ, then n can be chosen to be independent of g.

(c) Definition of a quasiconvex subgroup. Any quasiconvex subgroup of a hyperbolic group is hyperbolic.

(d) Any 2-generator subgroup of Γ is free and quasiconvex unless it contains an element of small translation length.

Refs: 5.3 and 8.2 of [Gr]; Chapter 10, sections 2 and 3 of [CoDP]; Delzant's and Short's papers in [GhHV]; [Kou].

Lecture 11 : The theorem of Bestvina and Mess I.

Let Γ be a hyperbolic group, and let P be a (finite-dimensional) contractable simplicial complex on which Γ acts properly discontinuously and cocompactly (for example, the Rips complex, $P_d(\Gamma)$ for sufficiently large d). Thus, $P \cup \partial\Gamma$ has a natural topology as a compact metrisable space.

The main result of Bestvina and Mess in [BeM] states that this is, in fact, a Z-set compactification (in the sense of shape theory).

In particular, it follows that $P \cup \partial\Gamma$ is contractible.

It has a number of important corollaries. For example, for all $n \geq 1$, we have $H^n(\Gamma; \mathbf{Z}\Gamma) \cong \check{H}^{n-1}(\partial\Gamma)$, where $\check{H}$ denotes the Čech cohomology.

Refs: [BeM], [Be2].

Lecture 12. The theorem of Bestvina and Mess II

(a) Let M be a closed irreducible 3-manifold such that $\Gamma = \pi_1(M)$ is hyperbolic. Then the universal cover $\tilde{M}$ is homeomorphic to $\mathbf{R}^3$, and $\tilde{M} \cup \partial\Gamma$ is a closed 3-ball.

Remark: This potentially represents a major step towards Thurston's hyperbolisation conjecture.

(b) Remark: Another result of [BeM] states that if Γ is a hyperbolic group such that $\partial\Gamma$ is connected and has no (global) cut point, then $\partial\Gamma$ is locally connected.

Remark: It follows from the recent work of Bowditch, Levitt and Swarup that $\partial\Gamma$ can never contain a global cut point (see Lecture 17).

Refs: [BeM], [Bo4].

Lecture 13 : The Besvina-Paulin theorem; first applications.

(a) Equivariant Gromov-Hausdorff convergence for metric spaces.

(b) Theorem: Let Γ be a given finitely generated group with a preferred system of generators $\{g_i \mid 1 \leq i \leq n\}$, and let (X_n, ρ_n) be a sequence of $\delta-$hyperbolic metric spaces each with an action of a Γ. Suppose that $\lambda_n = \min_{x \in X_n} \max_{1 \leq i \leq n} \rho_i(g_i x, x)$ is a not bounded. Then the family of spaces $\frac{1}{\lambda_n} X_n$ converges in the Gromov-Haudorff equivariant topology to an $\mathbf{R}$-tree without any global fixed point.

This fact has many corollaries. For example:

(b) We say that a group is "rigid" if it does not admit a non-trivial action on an $\mathbf{R}$-tree. (An action of an $\mathbf{R}$-tree is, rather confusingly, termed "trivial" if it has a global fixed point.)

Let Γ be a rigid hyperbolic group. Then, the outer automorphism group, $\mathrm{Out}(\Gamma)$ is finite. Morever Γ is Hopfian and co-Hopfian.

(c) Remark: Recently the theory of assymptotic cones, as introduced by Gromov, has recently emerged as a technically more powerful tool than Gromov-Hausdorff convergence for these sorts of issues.

Refs: [Be1], [Be3], [Pa1], [Pa2], [Kap], [RS], [Sel].

Lecture 14. Bass-Serre theory, and the Rips machinery I.

(a) Serre's theorem: The following are equivalent for a group, Γ:

(1) Γ is isomorphic to an amalgam $\Gamma \cong A *_C B$ (respectively an HNN extension $\Gamma \cong A *_C$), and

(2) Γ acts on a simplicial tree, T, with just one edge orbit, and no edge inversions, such that for some edge $e = [a, b]$ we have $\Gamma_a = A$, $\Gamma_b = B$ and $\Gamma_e = C$, and with a and b in different (respectively the same) Γ-orbit. (Here Γ_x denotes the stabiliser of x.)

(b) Bass-Serre theory is a generalisation of Serre's theorem to the case where T/Γ might be any connected graph.

(c) Remark: A group is free if and only if it acts freely on a simplicial tree. Corollary: a subgroup of a free group is free.

(d) The "Rips machinery" can be viewed as a generalisation of Bass-Serre theory to group actions on $\mathbf{R}$-trees (which arise naturally, for example, as in Lecture 13).

This is indeed more general. In particular, surface groups can act freely on $\mathbf{R}$-trees.

Some discussion of actions of surface groups, for example, via singular measured foliations on surfaces.

Refs: [Pa2], [Ser], [Be3].

Lecture 15 : Bass-Serre theory and the Rips machinery II.

(a) Theorem (Rips). Any finitely generated group acing freely isometrically on an $\mathbf{R}$-tree can be expressed as a free product of surface groups and free abelian groups. (Note that any finitely generated free abelian group acts freely on the real line.)

(b) Definition of a stable action of a group on an $\mathbf{R}$-tree. Statement of the theorem of Bestvina and Feighn [BeF] concerning splittings of finitely presented groups acting stably on $\mathbf{R}$-trees.

(c) Foliated 2-complexes. The resolution of a finitely presented group acting on an $\mathbf{R}$-tree. A sketch of the some of the ideas involved in the proof of (a) in the case of finitely presented groups.

Refs: [Be3], [BeF], [KaP], [Pa2].

Lecture 16 : The topological characterisation of fuchsian groups.

(a) The celebrated theorem of Tukia, Gabai and Casson and Jungreis states that if a group, Γ, acts as a convergence group on the circle, then it also admits a properly discontinuous action on the hyperbolic plane (such that the induced action on the circle at infinity is topologically conjugate to the original). The converse is an immediate consequence of Lecture 7.

Remark: The detailed analysis of Tukia left open the case of (putative) virtual semitriangle groups. This case was resolved independently by Gabai and by Casson and Jungreis. A peculiar feature of virtual semitriangle groups is that they do not split over any two-ended subgroup — a fact which also arises in consideration of the JSJ spittings (Lecture 18).

(b) If Γ is a uniform convergence group on the circle then it follows that it acts properly discontinuously cocompactly on the hyperbolic plane, and is thus a virtual surface group.

Remark: In fact, the work of Gabai gives another proof of the Nielsen realisation problem (proved by Kerchoff).

(c) Remark: An important open problem is whether a uniform convergence group acting on the 2-sphere must also act properly discontinuously cocompactly on hyperbolic 3-space (cf. Lecture 12). Some progress in this direction has been made by Cannon and his coworkers.

Refs: [Tu1], [Ga], [CaJ]. (Perhaps [Ga] is the most complete account for a general survey.)

Lecture 17 : Cut points an local connectivity.

The cut point theorem states that if Γ is a one-ended hyperbolic group, then $\partial\Gamma$ has no global cut point.

It follows from Bestvina-Mess (Lecture 12) that $\partial\Gamma$ is locally connected.

The proof is by contradiction. Suppose that $\partial\Gamma$ has a global cut point. The steps are as follows:

(a) Use the separation properties of the set of cut points to construct an $\mathbf{R}$-tree on which Γ acts by homeomorphism.

(b) Use this to construct another $\mathbf{R}$-tree on which Γ acts by isometry.

(c) Use the Rips machinery to derive a contradiction.

Remark: This theorem has a number of consequences, for example, (as observed by Geoghegen) it follows that hyperbolic groups are semistable at infinity (as described by Mihalik). Also, it gives us a topological construction of the JSJ splitting (see Lecture 18).

Refs: (a) [Bo1], [Swe]. (b) [Lev], [BoC] (c) [Swa], [Bo4]

Lecture 18 : The JSJ splitting.

(a) Remark: certain splittings of a hyperbolic group, Γ, can be recognised from the topology of its boundary, $\partial\Gamma$.

As observed in Lecture 7, $\partial\Gamma$ is connected if and only if Γ one-ended, and hence does not split over any finite subgroup.

In this case, if Γ splits over a two-ended subgroup, G, then $\partial\Gamma \setminus \{x, y\}$ is disconnected, where x and y are the two limit points of G. In particular, x and y are both local cut points of $\partial\Gamma$.

(b) The JSJ splitting of a one-ended hypebolic group is a canonical representation of Γ as (the fundamental group of) a finite graph of groups with two-ended edge groups, such that every splitting of Γ over a two-ended subgroup can be read off from this picture.

Remark: It was first described by Sela (for splitting over cyclic groups) who made use of the Rips machinery. It was inspired by the analogous characteristic submanifold construction for irreducible 3-manifolds due to Waldhausen, Johannson, Jaco and Shalen.

Remark: It gives rise to a fairly complete description of the outer automorphism group of a hyperbolic group.

A topological construction of the JSJ splitting using $\partial\Gamma$ has been given by Bowditch.

(c) The JSJ splitting has been generalised to finitely presented groups over two-ended subgroups by Rips and Sela, and to finitely presented groups over slender subgroups by Dunwoody and Sageev and by Fujiwara and Papasoglou.

Refs: We suggest focusing on one of the accounts [Bo2], [DuS] or [FuP] — the methods of all three are quite different.

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I would like to take part in the Arbeitsgemeinschaft on Hyperbolic Groups and give a lecture, preferably No. ...... or No. ......