! see back of 3

Trans. AMS 293 (1986) 511-531 !

So, we know: $\Lambda = R*G \simeq \mathrm{End}({}_A R)$

$$\Longrightarrow \quad CM(A) \simeq \text{proj-}(R*G)$$

Claim: $\left[\quad P \otimes_{R*G} R \simeq P^G \quad \right]$

$$p \otimes r \xmapsto{\alpha} \cancel{\frac{1}{n} \sum_\sigma p[\sigma]\sigma^{-1}(r)} = \frac{1}{n}\sum_\sigma pr[\sigma]$$

$$p \otimes 1 \xleftarrow{\beta} p$$

we can

$$\alpha\beta(p) = \frac{1}{n}\sum_\sigma p[\sigma] = p \qquad \text{since } p[\sigma] = p$$

$$\beta\alpha(p \otimes r) = \frac{1}{n}\sum_\sigma p[\sigma]\sigma^{-1}(r) \otimes 1 = \frac{1}{n}\sum_\sigma pr[\sigma] \otimes 1 =$$
$$= \frac{1}{n}\sum_\sigma p \otimes r = p \otimes r$$

semisimple

$$R*G / m(R*G) \simeq kG$$

[and all idempotents lifts (complete!)]

$$\Longrightarrow$$

$\text{proj-}(R*G) \Leftrightarrow \text{proj-}kG = \text{mod-}kG$

$P \longmapsto P/mP$

$V \otimes_k R \longleftarrow V$

$\left[\cancel{P \longleftarrow V \quad \text{s.t. } P/mP \simeq V} \right]$

One can set $\quad P = V \otimes_k R$

with the action: $a \in R$ $\quad \left[\begin{array}{l} \sigma(v \otimes r) = \sigma(v) \otimes \sigma(r) \\ a(v \otimes r) = v \otimes ar \end{array} \right.$

(if $r \in k$! $= rv \otimes a$)

$$mP = V \otimes_k m \qquad \underline{\underline{P/mP \simeq V}}$$

$P_0 = R$

So, ind. proj. $= V_i \otimes_k R = P_i$

where $V_0, \ldots, V_s$ $=$ irr. repr. of G

and ind. $CM(A) = (V_i \otimes_k R)^G = M_i$

$$M_0 = A$$

Aus

What is the sense of McKay quiver for A?

Let V be the fund. repr. $\quad \otimes = \otimes_k$

$$\text{I} \quad V\otimes V_i \simeq \sum_j V_j^{m_{ij}} \quad \text{I}$$

$$\boxed{\overset{\bullet}{j} \overset{m_{ij}}{\rightrightarrows} \overset{\bullet}{i}}$$

$\angle 2,5$

Koszul complex for $R[[x,y]]$:

$$0 \to \Lambda^2 V\otimes R \to V\otimes R \to R \to k \to 0$$

$V = \langle x, y\rangle$
$x\otimes y - y\otimes x$

$$(x\wedge y)\otimes r \to \boxed{x\otimes yr - y\otimes xr}$$

$$\boxed{\text{"determinant repr."} \quad \sigma \mapsto \det\sigma}$$

$$\left(\begin{array}{l} x\otimes r_1 + y\otimes r_2 \to xr_1 + yr_2 = 0 \\ \Rightarrow r_1 = ys \quad r_2 = -xs \end{array}\right)$$ не писать

$$V_i \otimes \Lambda^2 V = \tau V_i \quad (\text{irr.})$$

[$V_i\otimes -$ $\quad \tau P_i$

$$0 \to \tau V_i\otimes R \to V_i\otimes V\otimes R \to V_i\otimes R \to V_i \to 0$$

apply G: $\quad 0 \to (\tau P_i)^G \to (\oplus P_j^{m_{ij}})^G \to P_i^G \to V_i^G \to 0$

$\boxed{\begin{array}{c} \tau V_i \\ \tau P_i \\ \tau M_i \end{array}}$

$V_i \neq V_0$, get $V_i^G = 0$

$$0 \to (\tau P_i)^G \to (\oplus P_j^{m_{ij}})^G \to P_i^G \to 0$$

$$0 \to \tau M_i \to \underbrace{\oplus M_j^{m_{ij}}}_{E_i} \to M_i \to 0$$

Claim $\varphi : M \to M_i$ is not a split epi $\Rightarrow$ it factors throu E_i

Proof: $M = P^G$ $\quad \varphi$ comes from $P \xrightarrow{\psi} P_i$ $\quad$ ($R*G$-homom.)

$$\begin{array}{ccccccc} V_i\otimes V\otimes R & \to & P_i & \to & V_i & \to & 0 \\ & & \uparrow \psi & & & & \\ & & P & & & & \end{array}$$

$[V_i = P_i/mP_i]$

$\operatorname{Im}\psi \subseteq mP_i$ $\Rightarrow$ factors thru

$$0 \to \tau A \to (V\otimes R)^G \to A \to k \to 0$$

$V_i = V_0$ $\quad 0 \to \tau P_0 \to V\otimes R \to R \to k \to 0$

$$\cancel{0 \to \tau A \to (V\otimes R)^G} \, ((V\otimes V_0)\otimes R)^G \to A \cancel{\to k \to 0}$$

$\oplus M_i^{m_i}$ $\quad$ same!

$\angle 2{,}5$ Aus

AR quiver : $M_j \overset{m_{ij}}{\Rrightarrow} M_i$

explain!

It describes all maps!

Theorem $\boxed{AR(A) \simeq MK(G)}$

$A \leftrightarrow k$

Example $\begin{pmatrix} \varepsilon & 0 \\ 0 & \varepsilon^k \end{pmatrix}$

$\varepsilon = e^{\frac{2\pi i}{(n+1)}}$

$\gcd(k,(n+1)) = 1$

$kG = \langle V_0, V_1, \dots, V_n \rangle$

$g \longmapsto \varepsilon^i \qquad (0 \leq i \leq n)$

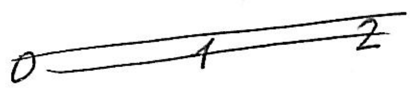
$(i+k \text{ modulo } n+1)$

$V_i \otimes V_0 = V_{i+1} \oplus V_{i+k}$

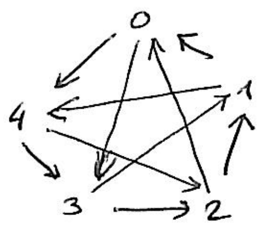
$i \mapsto$

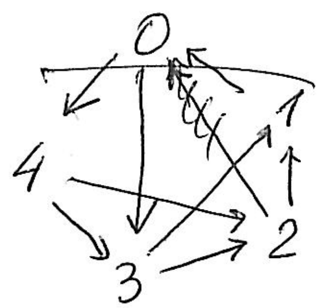
$1, k$

$2, k+1$



$n = 4$ $\qquad k = 2$





$k = -1$

$i \rightleftarrows i+1$

?

$\begin{pmatrix} \varepsilon & 0 \\ 0 & \varepsilon^k \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad \boxed{\varepsilon^{2n} = 1}$

!

M. Auslander

Trans. AMS 293 (1986) 511-531

$$\begin{pmatrix} \varepsilon & 0 \\ 0 & \varepsilon^{k} \end{pmatrix} \qquad \begin{pmatrix} 0 & x \\ y & 0 \end{pmatrix} \qquad xy = -1$$

$$\begin{pmatrix} 0 & \varepsilon x \\ \varepsilon^{k} y & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & \varepsilon^{k+1} x \\ \varepsilon^{k+1} y & 0 \end{pmatrix}$$

$\varepsilon^{k+1} = 1$!

---

(1) $1 \neq \sigma \in G \Rightarrow \exists a \in R \quad \sigma(a) \not\equiv a \mod \mathfrak{p}$

---

(2) $\bar{\mathfrak{p}} = A \cap \mathfrak{p}$ $\quad R_{\mathfrak{p}} / \mathfrak{p} R_{\mathfrak{p}}$ is ~~n~~ r-dim. over $A_{\bar{\mathfrak{p}}} / \bar{\mathfrak{p}} A_{\bar{\mathfrak{p}}}$ $(r \leq n)$

and G acts effectively on $R_{\mathfrak{p}} / \mathfrak{p} R_{\mathfrak{p}}$

$\Longrightarrow r = n$ so $\boxed{\mathfrak{p} R_{\mathfrak{p}} = \bar{\mathfrak{p}} A_{\mathfrak{p}}}$

and $R_{\mathfrak{p}} / \mathfrak{p} R_{\mathfrak{p}}$ is Galois ext. of $A_{\bar{\mathfrak{p}}} / \bar{\mathfrak{p}} A_{\bar{\mathfrak{p}}}$

OK	!

L2,5] $a = \begin{pmatrix} \varepsilon^{1} & 0 \\ 0 & \varepsilon^{-1} \end{pmatrix}$ $b = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ $\boxed{\varepsilon = e^{\frac{\pi k i}{n}}}$ -4-

$\cancel{V_{\pm 1}}$ $V_{\pm i}$ Ans

1-dim: $a \mapsto \pm 1$ $b \mapsto \begin{cases} \pm 1 \\ \pm i \end{cases}$

$V(1,1)$

2-dim

V_k $\begin{cases} a \mapsto \begin{pmatrix} \varepsilon^{k} & 0 \\ 0 & \varepsilon^{-k} \end{pmatrix} & b \mapsto \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad k \text{ odd} < n \\ a \mapsto \begin{pmatrix} \varepsilon^{k} & 0 \\ 0 & \varepsilon^{-k} \end{pmatrix} & b \mapsto \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad k \text{ even} < n \end{cases}$

$$\begin{tikzcd}[row sep=small]
V(1,1) \arrow[dr, dash] & & & & V(-1,i) \\
 & V_1 \arrow[r, dash] & V_2 \arrow[r, dash] & \cdots \arrow[r, dash] & V_{n-1} \arrow[u, dash] \arrow[d, dash] \\
V(1,-1) \arrow[ur, dash] & & & & V(-1,-i)
\end{tikzcd}$$

	$\chi(a)$	$\chi(b)$
$V(1,1)$	1	1
$V(1,-1)$	1	-1
$V(-1,i)$	-1	i
$V(-1,-i)$	-1	$-i$
V_k	$2\cos\frac{\pi k}{n}$	0
$V_k \otimes V_1$	$4\cos\frac{\pi k}{n}\cos\frac{\pi}{n}$	0

$\boxed{2\cos\frac{\pi(k+1)}{n} + 2\cos\frac{\pi(k-1)}{n}}$

$k=1$ $V(1,1) + V(1,-1)$

$k=n-1$

$V(-1,i) + V(-1,-i)$

!

$\cancel{V_k \otimes V_1}$:

Dihedral