Available snapshots of modern mathematics from Oberwolfach

1/2014 John E. McCarthy
Dirichlet Series

Mathematicians are very interested in prime numbers. In this snapshot, we will discuss some problems concerning the distribution of primes and introduce some special infinite series in order to study them.

2/2014 Wolfgang Lerche
Matrixfaktorisierungen

Im Folgenden soll ein kurzer Abriss des Themas Matrixfaktorisierungen gegeben werden. Wir werden darlegen, warum dieses recht simple Konzept zu erstaunlich tiefen mathematischen Gedankengängen führt und auch in der modernen theoretischen Physik wichtige Anwendungen hat.

3/2014 Harald Helfgott
The ternary Goldbach problem

Leonhard Euler (1707–1783) – one of the greatest mathematicians of the eighteenth century and of all times – often corresponded with a friend of his, Christian Goldbach (1690–1764), an amateur and polymath who lived and worked in Russia, just like Euler himself. In a letter written in June 1742, Goldbach made a conjecture – that is, an educated guess – on prime numbers:

> *Es scheinet wenigstens, dass eine jede Zahl, die größer ist als* 2, *ein aggregatum trium numerorum primorum sey.*
>
> It seems [...] that every positive integer greater than 2 can be written as the sum of three prime numbers.

In this snapshot, we will describe to what extent the mathematical community has resolved Goldbach's conjecture, with some emphasis on recent progress.

Mathematisches Forschungsinstitut Oberwolfach
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4/2014 Bruce Reznick

What does ">" really mean?

This Snapshot is about the generalization of ">" from ordinary numbers to so-called *fields*. At the end, I will touch on some ideas in recent research.

5/2014 Brian Harbourne and Tomasz Szemberg

Arrangements of lines

We discuss certain open problems in the context of arrangements of lines in the plane.

6/2014 Howell Tong

Statistics and dynamical phenomena

A friend of mine, an expert in statistical genomics, told me the following story: At a dinner party, an attractive lady asked him, "What do you do for a living?" He replied, "I model." As my friend is a handsome man, the lady did not question his statement and continued, "What do you model?" "Genes." She then looked at him up and down and said, "Mh, you must be very much in demand." "Yes, very much so, especially after I helped discover a new culprit gene for a common childhood disease." The lady looked puzzled. In this snapshot, I will give you an insight into Statistics, the field that fascinated my friend (and myself) so much. I will concentrate on phenomena that change over time, in other words, dynamical events.

7/2014 Ragnar-Olaf Buchweitz and Eleonore Faber

Swallowtail on the shore

Platonic solids, Felix Klein, H.S.M. Coxeter and a flap of a swallowtail: The five Platonic solids tetrahedron, cube, octahedron, icosahedron and dodecahedron have always attracted much curiosity from mathematicians, not only for their sheer beauty but also because of their many symmetry properties. In this snapshot we will start from these symmetries, move on to groups, singularities, and finally find the connection between a tetrahedron and a "swallowtail". Our running example is the tetrahedron, but every construction can be carried out with any other of the Platonic solids.

8/2014 Alain Valette

The Kadison-Singer problem

In quantum mechanics, unlike in classical mechanics, one cannot make precise predictions about how a system will behave. Instead, one is concerned with mere probabilities. Consequently, it is a very important task to determine the basic probabilities associated with a given system. In this snapshot we will present a recent uniqueness result concerning these probabilities.

9/2014 Greg Knese

Operator theory and the singular value decomposition

This is a snapshot about operator theory and one of its fundamental tools: the singular value decomposition (SVD). The SVD breaks up linear transformations into simpler mappings, thus unveiling their geometric properties. This tool has become important in many areas of applied mathematics for its ability to organize information. We discuss the SVD in the concrete situation of linear transformations of the plane (such as rotations, reflections, etc.).

10/2014 Peter Benner, Hermann Mena and René Schneider

Drugs, herbicides, and numerical simulation

The Colombian government sprays coca fields with herbicides in an effort to reduce drug production. Spray drifts at the Ecuador-Colombia border became an international issue. We developed a mathematical model for the herbicide aerial spray drift, enabling simulations of the phenomenon.

1/2015 Diana Davis

Billiards and flat surfaces

Billiards, the study of a ball bouncing around on a table, is a rich area of current mathematical research. We discuss questions and results on billiards, and on the related topic of flat surfaces.

2/2015 Christine Breiner

Minimizing energy

What is the most efficient way to fence land when you've only got so many metres of fence? Or, to put it differently, what is the largest area bounded by a simple closed planar curve of fixed length?

We consider the answer to this question and others like it, making note of recent results in the same spirit.

3/2015 George Willis

Zero-dimensional symmetry

This snapshot is about *zero-dimensional symmetry*. Thanks to recent discoveries we now understand such symmetry better than previously imagined possible. While still far from complete, a picture of zero-dimensional symmetry is beginning to emerge.

4/2015 Thorsten Holm

Friezes and tilings

Friezes have occured as architectural ornaments for many centuries. In this snapshot, we consider the mathematical analogue of friezes as introduced in the 1970s by Conway and Coxeter. Recently, infinite versions of such friezes have appeared in current research. We are going to describe them and explain how they can be classified using some nice geometric pictures.

5/2015 Tom Solomon

Chaos and chaotic fluid mixing

Very simple mathematical equations can give rise to surprisingly complicated, *chaotic* dynamics, with behavior that is sensitive to small deviations in the initial conditions. We illustrate this with a single recurrence equation that can be easily simulated, and with mixing in simple fluid flows.

6/2015 Raluca Eftimie

Modeling communication and movement: from cells to animals and humans

Communication forms the basis of biological interactions. While the use of a single communication mechanism (for example visual communication) by a species is quite well understood, in nature the majority of species communicate via multiple mechanisms. Here, I review some mathematical results on the unexpected behaviors that can be observed in biological aggregations where individuals interact with each other via multiple communication mechanisms.

7/2015 Ben Schweizer

Darcy's law and groundwater flow modelling

Formulations of natural phenomena are derived, sometimes, from experimentation and observation. Mathematical methods can be applied to expand on these formulations, and develop them into better models. In the year 1856, the French hydraulic engineer Henry Darcy performed experiments, measuring water flow through a column of sand. He discovered and described a fundamental law: the linear relation between pressure difference and flow rate – known today as Darcy's law. We describe the law and the evolution of its modern formulation. We furthermore sketch some current mathematical research related to Darcy's law.

8/2015 Valentina Kiritchenko, Evgeny Smirnov and Vladlen Timorin
Ideas of Newton-Okounkov bodies

In this snapshot, we will consider the problem of finding the number of solutions to a given system of polynomial equations. This question leads to the theory of Newton polytopes and Newton-Okounkov bodies of which we will give a basic notion.

9/2015 Victoria Powers
How to choose a winner: the mathematics of social choice

Suppose a group of individuals wish to choose among several options, for example electing one of several candidates to a political office or choosing the best contestant in a skating competition. The group might ask: what is the best method for choosing a winner, in the sense that it best reflects the individual preferences of the group members? We will see some examples showing that many voting methods in use around the world can lead to paradoxes and bad outcomes, and we will look at a mathematical model of group decision making. We will discuss *Arrow's impossibility theorem*, which says that if there are more than two choices, there is, in a very precise sense, **no** good method for choosing a winner.

10/2015 Jürg Kramer and Anna-Maria von Pippich
Special values of zeta functions and areas of triangles

In this snapshot we give a glimpse of the interplay of special values of zeta functions and volumes of triangles. Special values of zeta functions and their generalizations arise in the computation of volumes of moduli spaces (for example of Abelian varieties) and their universal spaces. As a first example, we compute the special value of the Riemann zeta function $\zeta(s) = \sum_{n=1}^{\infty} n^{-s}$ at $s = 2$ and give its interpretation as the volume of the moduli space of elliptic curves. As a second example, we calculate a special value of the Mordell–Tornheim zeta function using the Stern–Brocot tree. This example allows a geometric interpretation related to current research.

11/2015 Christopher Sangwin
Curriculum development in university mathematics: where mathematicians and education collide

This snapshot looks at educational aspects of the design of curricula in mathematics. In particular, we examine choices textbook authors have made when introducing the concept of the completness of the real numbers. Can significant choices really be made? Do these choices have an effect on how people learn, and, if so, can we understand what they are?

12/2015 J. S. Marron
Visual Analysis of Spanish Male Mortality

Statistical visualization uses graphical methods to gain insights from data. Here we show how a technique called principal component analysis is used to analyze mortality in Spain over about the last hundred years. This data decomposition both reflects expected historical events and reveals some perhaps less expected trends in mortality over the years.

13/2015 Amanda Swan and Albert Murtha
Modelling the spread of brain tumours

The study of mathematical biology attempts to use mathematical models to draw useful conclusions about biological systems. Here, we consider the modelling of brain tumour spread with the ultimate goal of improving treatment outcomes.

14/2015 Antti Knowles
Quantum diffusion

If you place a drop of ink into a glass of water, the ink will slowly dissipate into the surrounding water until it is perfectly mixed. If you record your experiment with a camera and play the film backwards, you will see something that is never observed in the real world. Such diffusive and irreversible behaviour is ubiquitous in nature. Nevertheless, the fundamental equations that describe the motion of individual particles – Newton's and Schrödinger's equations – are reversible in time: a film depicting the motion of just a few particles looks as realistic when played forwards as when played backwards.

In this snapshot, we discuss how one may try to understand the origin of diffusion starting from the fundamental laws of quantum mechanics.

15/2015 Sebastian Funk
The mystery of sleeping sickness – why does it keep waking up?

Sleeping sickness is a neglected tropical disease that affects rural populations in Africa. Deadly when untreated, it is being targeted for elimination through case finding and treatment. Yet, fundamental questions about its transmission cycle remain unanswered. One of them is whether transmission is limited to humans, or whether other species play a role in maintaining circulation of the disease.

In this snapshot, we introduce a mathematical model for the spread of *Trypanosoma brucei*, the parasite responsible for causing sleeping sickness, and present some results based on data collected in Cameroon. Understanding how important animals are in harbouring Trypanosoma brucei that can infect

humans is important for assessing whether the disease could be reintroduced in human populations even after all infected people have been successfully treated.

16/2015 Éric Fusy and Juanjo Rué
Domino tilings of the Aztec Diamond

Imagine you have a cutout from a piece of squared paper and a pile of dominoes, each of which can cover exactly two squares of the squared paper. How many different ways are there to cover the entire paper cutout with dominoes? One specific paper cutout can be mathematically described as the so-called *Aztec Diamond*, and a way to cover it with dominoes is a *domino tiling*. In this snapshot we revisit some of the seminal combinatorial ideas used to enumerate the number of domino tilings of the Aztec Diamond. The existing connection with the study of the so-called *alternating-sign matrices* is also explored.

17/2015 Ulrich Krähmer
From computer algorithms to quantum field theory: an introduction to operads

An operad is an abstract mathematical tool encoding operations on specific mathematical structures. It finds applications in many areas of mathematics and related fields. This snapshot explains the concept of an operad and of an algebra over an operad, with a view towards a conjecture formulated by the mathematician Pierre Deligne. Deligne's (by now proven) conjecture also gives deep inights into mathematical physics.

1/2016 Magnus Egerstedt
Swarming robots

When lots of robots come together to form shapes, spread in an area, or move in one direction, their motion has to be planned carefully. We discuss how mathematicians devise strategies to help swarms of robots behave like an experienced, coordinated team.

2/2016 Éric Fusy
Random sampling of domino and lozenge tilings

A *grid region* is (roughly speaking) a collection of "elementary cells" (squares, for example, or triangles) in the plane. One can "tile" these grid regions by arranging the cells in pairs. In this snapshot we review different strategies to generate random tilings of large grid regions in the plane. This makes it possible to observe the behaviour of large random tilings, in particular the occurrence of boundary phenomena that have been the subject of intensive recent research.

3/2016 Tomasz Szemberg and Justyna Szpond
On the containment problem

Mathematicians routinely speak two languages: the language of geometry and the language of algebra. When translating between these languages, curves and lines become sets of polynomials called "ideals". Often there are several possible translations. Then the mystery is how these possible translations relate to each other. We present how geometry itself gives insights into this question.

4/2016 Maria Dostert, Stefan Krupp and Jan Hendrik Rolfes
Das Problem der Kugelpackung

möglichst dicht in eine Kiste packen stapeln? Oder allgemeiner formuliert: Wie dicht lassen sich identische 3-dimensionale Objekte überschneidungsfrei anordnen? Das Problem, welches auch Anwendungen in der digitalen Kommunikation hat, hört sich einfach an, ist jedoch für Kugeln in höheren Dimensionen noch immer ungelöst. Sogar die Berechnung guter Näherungslösungen ist für die meisten Dimensionen schwierig.

5/2016 Eugenio Giannelli and Jay Taylor
Symmetry and characters of finite groups

Over the last two centuries mathematicians have developed an elegant abstract framework to study the natural idea of symmetry. The aim of this snapshot is to gently guide the interested reader through these ideas. In particular, we introduce finite groups and their representations and try to indicate their central role in understanding symmetry.

6/2016 Anthony T. Patera and Karsten Urban
High performance computing on smartphones

Nowadays there is a strong demand to simulate even real-world engineering problems on small computing devices with very limited capacity, such as a smartphone. We explain, using a concrete example, how we can obtain a reduction in complexity – to enable such computations – using mathematical methods.

7/2016 Robert Altmann and Jan Heiland
Wie steuert man einen Kran?

Die Steuerung einer Last an einem Kran ist ein technisch und mathematisch schwieriges Problem, da die Bewegung der Last nur indirekt beeinflusst werden kann. Anhand eines Masse-Feder-Systems illustrieren wir diese Schwierigkeiten und zeigen wie man mit einem zum konventionellen Lösungsweg

alternativen Optimierungsansatz die auftretenden Komplikationen teilweise umgehen kann.

8/2016 Manfred Deistler and Andreas Graef
Fokus-Erkennung bei Epilepsiepatienten mithilfe moderner Verfahren der Zeitreihenanalyse

Viele epileptische Anfälle entstehen in einer begrenzten Region im Gehirn, dem sogenannten Anfallsursprung. Eine chirurgische Entfernung dieser Region kann in vielen Fällen zu Anfallsfreiheit führen. Aus diesem Grund ist die Frage nach der Lokalisation des Anfallsursprungs aus EEG-Aufzeichnungen wichtig. Wir beschreiben hier ein Verfahren zur Lokalisation des Anfallsursprungs mittels Zeitreihenanalyse, das auf der Schätzung von Spektren im EEG beruht.

9/2016 Rafael Guglielmetti and Matthieu Jacquemet
Polyhedra and commensurability

This snapshot introduces the notion of commensurability of polyhedra. At its bottom, this concept can be developed from constructions with paper, scissors, and glue. Starting with an elementary example, we formalize it subsequently. Finally, we discuss intriguing connections with other fields of mathematics.

10/2016 Lior Bary-Soroker
Prime Tuples in Function Fields

How many prime numbers are there? How are they distributed among other numbers? These are questions that have intrigued mathematicians since ancient times. However, many questions in this area have remained unsolved, and seemingly unsolvable in the forseeable future.

In this snapshot, we will discuss one such problem, the Twin Prime Conjecture, and a quantitative version of it known as the Hardy–Littlewood Conjecture. We will also see that these and other questions about prime numbers can be extended to questions about *function fields*, and discuss recent progress which has been made to answer them in this context.

11/2016 Nikolai Nowaczyk
The Willmore Conjecture

The Willmore problem studies which *torus* has the least amount of *bending energy*. We explain how to think of a torus as a donut-shaped surface and how the intuitive notion of bending has been studied by mathematics over time.

12/2016 Steven Klee

Footballs and donuts in four dimensions

In this snapshot, we explore connections between the mathematical areas of counting and geometry by studying objects called simplicial complexes. We begin by exploring many familiar objects in our three dimensional world and then discuss the ways one may generalize these ideas into higher dimensions.

13/2016 Dietmar Gallistl

The adaptive finite element method

Computer simulations of many physical phenomena rely on approximations by models with a finite number of unknowns. The number of these parameters determines the computational effort needed for the simulation. On the other hand, a larger number of unknowns can improve the precision of the simulation. The adaptive finite element method (AFEM) is an algorithm for optimizing the choice of parameters so accurate simulation results can be obtained with as little computational effort as possible.

14/2016 Laurent Bartholdi

Profinite groups

Profinite objects are mathematical constructions used to collect, in a uniform manner, facts about infinitely many finite objects. We shall review recent progress in the theory of profinite groups, due to Nikolov and Segal, and its implications for finite groups.

15/2016 Jacob Bedrossian

Towards a Mathematical Theory of Turbulence in Fluids

Fluid mechanics is the theory of how liquids and gases move around. For the most part, the basic physics are well understood and the mathematical models look relatively simple. Despite this, fluids display a dazzling mystery to their motion. The random-looking, chaotic behavior of fluids is known as *turbulence*, and it lies far beyond our mathematical understanding, despite a century of intense research.

1/2017 Felix Günther

Winkeltreue zahlt sich aus

Nicht nur Seefahrerinnen, auch Computergrafikerinnen und Physikerinnen wissen Winkeltreue zu schätzen. Doch beschränkte Rechenkapazitäten und Vereinfachungen in theoretischen Modellen erfordern es, winkeltreue Abbildungen nur mit einer überschaubaren Datenmenge zu beschreiben. Entsprechende Theorien werden in der diskreten Mathematik untersucht. Im Fol-

genden lade ich Sie auf eine Reise in die faszinierende Welt der winkeltreuen Abbildungen ein.

2/2017 Lukas Pottmeyer
News on quadratic polynomials

Many problems in mathematics have remained unsolved because of missing links between mathematical disciplines, such as algebra, geometry, analysis, or number theory. Here we introduce a recently discovered result concerning quadratic polynomials, which uses a bridge between algebra and analysis. We study the iterations of quadratic polynomials, obtained by computing the value of a polynomial for a given number and feeding the outcome into the exact same polynomial again. These iterations of polynomials have interesting applications, such as in fractal theory.

3/2017 Michael Baake, David Damanik and Uwe Grimm
Aperiodic Order and Spectral Properties

Periodic structures like a typical tiled kitchen floor or the arrangement of carbon atoms in a diamond crystal certainly possess a high degree of order. But what is order *without* periodicity? In this snapshot, we are going to explore highly ordered structures that are substantially nonperiodic, or *aperiodic*. As we construct such structures, we will discover surprising connections to various branches of mathematics, materials science, and physics. Let us catch a glimpse into the inherent beauty of aperiodic order!

4/2017 Christian Engwer und Markus Knappitsch
Mathematische Modellierung von Krebswachstum

Krebs ist eine der größten Herausforderungen der modernen Medizin. Der WHO zufolge starben 2012 weltweit 8,2 Millionen Menschen an Krebs. Bis heute sind dessen molekulare Mechanismen nur in Teilen verstanden, was eine erfolgreiche Behandlung erschwert. Mathematische Modellierung und Computersimulationen können helfen, die Mechanismen des Tumorwachstums besser zu verstehen. Sie eröffnen somit neue Chancen für zukünftige Behandlungsmethoden. In diesem Schnappschuss steht die mathematische Modellierung von Glioblastomen im Fokus, einer Klasse sehr aggressiver Tumore im menschlichen Gehirn.

5/2017 Marco Radeschi
Closed geodesics on surfaces and Riemannian manifolds

Geodesics are special paths in surfaces and so-called *Riemannian manifolds* which connect close points in the shortest way. *Closed geodesics* are geodesics which go back to where they started. In this snapshot we talk about these special paths, and the efforts to find closed geodesics.

6/2017 George A. Hagedorn and Caroline Lasser
Molecular Quantum Dynamics

We provide a brief introduction to some basic ideas of Molecular Quantum Dynamics. We discuss the scope, strengths and main applications of this field of science. Finally, we also mention open problems of current interest in this exciting subject.

7/2017 Justyna Szpond
Unexpected and imposed

The topic of this snapshot is *interpolation*. In the ordinary sense, interpolation means to insert something of a different nature into something else. In mathematics, interpolation means constructing new data points from given data points. The new points usually lie in between the already-known points. The purpose of this snapshot is to introduce a particular type of interpolation, namely, polynomial interpolation. This will be explained starting from basic ideas that go back to the ancient Babylonians and Greeks, and will arrive at subjects of current research activity.

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